SOLUTIONS

Practice Exam #2

(I) $f(x)=3x^2-4x-7$

$$\Rightarrow f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{3(x+\Delta x)^2-4(x+\Delta x)-7-(3x^2-4x-7)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{3(x^2+2x\Delta x+\Delta x^2)-4(x+\Delta x)-7-3x^2+4x+7}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\cancel{3x^2}+6x\Delta x+3\Delta x^2-\cancel{4x}-4\Delta x-\cancel{7}-\cancel{3x^2}+\cancel{4x}+\cancel{7}}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{6x\Delta x+3\Delta x^2-4\Delta x}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\cancel{\Delta x}(6x+3\Delta x-4)}{\cancel{\Delta x}}$$

$$=\lim_{\Delta x\to 0}(6x+3\Delta x-4)$$

$$=6x+3(0)-4$$

$$=\boxed{6x-4}$$

III (#3) $f(x)=5x^5+3x^{-3}-12x^{1/2}$

$$\Rightarrow \boxed{f'(x)=25x^4-9x^{-4}-6x^{-1/2}}$$

(#4) $f(x)=\tan(3x)-\csc(-x)$

$$f'(x)=\boxed{\sec^2(3x)(3)-(-\csc(-x)\cot(-x))(-1)}$$

(#5) $f(x)=5x^{1/3}\cdot\csc(x)\Rightarrow \boxed{f'(x)=(5x^{1/3})(-\csc(x)\cot(x))+(\csc(x))\left(\frac{5}{3}x^{-2/3}\right)}$

product Rule

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#6. $f(x) = -(x^{-4} - x^3)^{-5}$

$\Rightarrow f'(x) = \boxed{-(-5)(x^{-4} - x^3)^{-6}(-4x^{-5} - 3x^2)}$

#7. $f(x) = \dfrac{x + \cot(x)}{4x}$

$\Rightarrow f'(x) = \boxed{\dfrac{(4x)(1 - \csc^2(x)) - (x + \cot(x))(4)}{(4x)^2}}$

#8. $f(x) = \cos^2(x^2)$

$= (\cos(x^2))^2$ ← chain rule twice ← work "outside-in"

$\Rightarrow f'(x) = \boxed{2(\cos(x^2))^1(-\sin(x^2)(2x))}$

#9. $f(x) = \sqrt[4]{2 - x^3}$

$\Rightarrow f(x) = (2 - x^3)^{1/4} \Rightarrow \boxed{f'(x) = \frac{1}{4}(2 - x^3)^{-3/4}(-3x^2)}$

#10. $f(x) = \dfrac{4x^3 - 6x}{3x^4 + 6}$ Quotient Rule

$\Rightarrow f'(x) = \boxed{\dfrac{(3x^4+6)(12x^2 - 6) - (4x^3 - 6x)(12x)}{(3x^4+6)^2}}$

#13. V

This requires PRODUCT RULE

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$$x^3 - 4xy^3 + 3y^4 = 5y$$

$$\Rightarrow 3x^2 + (-4x)\left(3y^2\frac{dy}{dx}\right) + (y^3)(-4) + 12y^3\frac{dy}{dx} = 5\frac{dy}{dx}$$

move terms: (I'm going to simplify as I re-arrange terms).

$$\Rightarrow -12xy^2\frac{dy}{dx} - 12y^3\frac{dy}{dx} - 5\frac{dy}{dx} = -3x^2 + 4y^3$$

$$\Rightarrow \frac{dy}{dx}\left(-12xy^2 - 12y^3 - 5\right) = -3x^2 + 4y^3$$

NOTE: you may omit this step

$$\Rightarrow \frac{dy}{dx} = \frac{-3x^2 + 4y^3}{-12xy^2 - 12y^3 - 5}$$

#14. $3y + \cos(y) = 2x$

$$\Rightarrow 3\frac{dy}{dx} - \sin(y)\frac{dy}{dx} = 2$$

$$\Rightarrow \frac{dy}{dx}\left(3 - \sin(y)\right) = 2$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2}{3 - \sin(y)}}$$

VI #14. Find: $\boxed{\frac{dr}{dt} = \quad ft/min}$

START:

$V = \frac{1}{3}\pi r^2 h$

⊛ differential count

$\frac{dV}{dt}$? ✓ given $\quad \frac{dr}{dt}$? ✓ desired $\quad \frac{dh}{dt}$? ✓ given

so Differential count is good.

Given: $\frac{dV}{dt} = 52\pi$

$\frac{dh}{dt} = 3$

$V = 12\pi$

$r = 2$

* Differentiate:

$$\frac{dV}{dt} = \frac{1}{3}\pi\left[(r^2)\left(\frac{dh}{dt}\right) + (h)\left(2r\frac{dr}{dt}\right)\right]$$

⊛ continued

(*) Continued ⇓

$$\frac{dV}{dt} = \frac{1}{3}\pi\left[(r^2)\left(\frac{dh}{dt}\right) + (h)\left(2r\frac{dr}{dt}\right)\right]$$

(*) Substitute values:

$$\Rightarrow 52\pi = \frac{1}{3}\pi\left[(2)^2(3) + (\quad)\left(2(2)\frac{dr}{dt}\right)\right]$$

(*) Need a value for a variable, so go back BEFORE THE DERIVATIVE:

$$V = \frac{1}{3}\pi r^2 h \Rightarrow \text{with } V = 12\pi \text{ \& } r = 2$$

$$\Rightarrow 12\pi = \frac{1}{3}\pi(2)^2 h$$

$$\Rightarrow 12\pi = \frac{4}{3}\pi h$$

$$\Rightarrow \frac{12\cancel{\pi}}{4/3\cancel{\pi}} = h$$

$$\Rightarrow 9 = h$$

SO NOW:

$$52\pi = 1/3\,\pi\left[(2)^2(3) + (9)\left(2(2)\frac{dr}{dt}\right)\right]$$

$$\Rightarrow 52\pi = 1/3\,\pi\left[12 + 36\frac{dr}{dt}\right]$$

$$\Rightarrow \frac{52\pi}{1/3\,\pi} = 12 + 36\frac{dr}{dt}$$

$$\Rightarrow 156 = 12 + 36\frac{dr}{dt}$$

$$\ {-12} \qquad {-12}$$

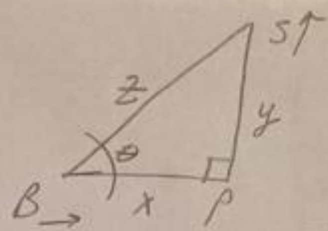
$$\Rightarrow 144 = 36\frac{dr}{dt}$$

$$\Rightarrow \frac{144}{36} = \frac{dr}{dt}$$

$$\Rightarrow \boxed{4 = \frac{dr}{dt}} \Rightarrow \boxed{\frac{dr}{dt} = 4 \text{ ft/min}}$$

#15

Find $\boxed{\frac{d\theta}{dt} = \quad \text{rad/s}}$ $\boxed{= \quad \text{deg/sec}}$



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Given:

$\frac{dy}{dt} = 12$

$\frac{dx}{dt} = -8$ Note: Negative

$y = 120$

$x = 30$

start:

$\tan(\theta) = \frac{y}{x}$

$\Rightarrow x\tan(\theta) = y$

⊛ differential Count:

$\frac{dx}{dt}$?✓ $\frac{d\theta}{dt}$?✓ $\frac{dy}{dt}$?✓

given desired given

so diff. count is ok.

⊛ Differentiate: (product rule)

$\Rightarrow x\sec^2(\theta)\frac{d\theta}{dt} + \tan(\theta)\frac{dx}{dt} = \frac{dy}{dt}$

⊛ Substitute:

$(30)(\quad)\frac{d\theta}{dt} + \left(\frac{120}{30}\right)(-8) = (12)$

must find hypotenuse to get $\sec^2(\theta)$

$z^2 = x^2 + y^2$

$= (30)^2 + (120)^2$

$= 900 + 14400$

$= 15300$

$z = \sqrt{15300}$

$= 123.69$

$\sec^2(\theta) = \left(\frac{hyp}{adj}\right)^2 = \left(\frac{123.69}{30}\right)^2$

so we have:

$(30)\left(\frac{123.69}{30}\right)^2\frac{d\theta}{dt} + \left(\frac{120}{30}\right)(-8) = 12$

$\Rightarrow (510)\frac{d\theta}{dt} - 32 = 12$

$\Rightarrow (510)\frac{d\theta}{dt} = 44$

$\Rightarrow \frac{d\theta}{dt} = .08627$ Rad/sec

convert to degrees

$\pi = 180$

$\Rightarrow \frac{\pi}{.08627} = \frac{180}{x}$

$\Rightarrow x\pi = (.08627)(180)$

$= 4.9^\circ$

so $\boxed{\frac{d\theta}{dt} = 4.9 \text{ deg/sec}}$