#13a. Hw Solutions pg 157 pg 1ne

Find: $\boxed{\frac{dV}{dt} = \quad in^3/min}$

Given: $\frac{dr}{dt} = 3$

Part 1 $r=9$

part 2 $r=36$

START:

$V = \frac{4}{3}\pi r^3$

* Differential count

$\frac{dV}{dt}$ desired ✓

$\frac{dr}{dt}$ given ✓

*Differentiate w.r.t. time

$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right)$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

PART 1ne

$\frac{dV}{dt} = 4\pi(9)^2(3)$

$= \boxed{972\pi \; in^3/min}$

Part 2wo

$\frac{dV}{dt} = 4\pi(36)^2(3)$

$= \boxed{15552\pi \; in^3/min}$

#14a Find: $\boxed{\frac{dr}{dt} = \quad cm/min}$

Given:

$\frac{dV}{dt} = 800$

Part 1: $r=30$

Part 2: $r=85$

START:

$V = \frac{4}{3}\pi r^3$

* Differential count:

$\frac{dV}{dt}$ given ✓

$\frac{dr}{dt}$ given ✓

* Differentiate

$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right)$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

Part 1ne

$800 = 4\pi(30)^2\frac{dr}{dt}$

$\Rightarrow 800 = 3600\pi\frac{dr}{dt}$

$\Rightarrow \boxed{\frac{800}{3600\pi} = \frac{dr}{dt}}$

OR $\boxed{\frac{dr}{dt} = \frac{2}{9\pi} \; cm/min}$

Part 2wo

$800 = 4\pi(85)^2\frac{dr}{dt}$

$800 = 28900\pi\frac{dr}{dt}$

$\Rightarrow \boxed{\frac{800}{28900\pi} = \frac{dr}{dt}}$

OR $\boxed{\frac{dr}{dt} = \frac{8}{289\pi} \; cm/min}$

#17

Find: $\boxed{\frac{dh}{dt} = \quad ft/min}$

pg 2w

Given: $\frac{dV}{dt} = 10$

$2r = 3h$

$h = 15$

START: $V = \frac{1}{3}\pi r^2 h$

⊛ Differential count:

$\frac{dV}{dt}$ given ✓

$\frac{dr}{dt} \rightarrow$ NO $\Rightarrow$ MUST ELIMINATE "r"

$\frac{dh}{dt}$ desired ✓

$2r = 3h$

$\Rightarrow r = \frac{3}{2}h$

so $V = \frac{4}{3}\pi\left(\frac{3}{2}h\right)^2 h$

$\Rightarrow V = \frac{36}{12}\pi h^3$

$\Rightarrow V = 3\pi h^3$

⊛ Differential count ✓

$\frac{dV}{dt} = 3\pi(3h^2)\frac{dh}{dt}$

$\Rightarrow \frac{dV}{dt} = 27\pi h^2 \frac{dh}{dt}$

$\Rightarrow 10 = 27\pi(15)^2 \frac{dh}{dt}$

$\Rightarrow 10 = 6075\pi \frac{dh}{dt}$

$\Rightarrow \boxed{\frac{dh}{dt} = \frac{10}{6075\pi} \; ft/min}$

(18)

Find: $\boxed{\frac{dh}{dt} = \quad in/sec}$

Given $\frac{dV}{dt} = 150$

$h = 10r$

$h = 35$

START:

$V = \pi r^2 h$

⊛ Differential count.

$\frac{dV}{dt}$ given ✓ *

$\frac{dr}{dt}$ NO $\Rightarrow$ MUST ELIMINATE "r"

$\frac{dh}{dt}$ desired ✓

$h = 10r \Rightarrow \frac{h}{10} = r$

$\Rightarrow V = \pi\left(\frac{h}{10}\right)^2 h$

$\Rightarrow V = \frac{\pi}{100}h^3$

⊛ Differential count ✓

⊛ Differentiate w.r.t. time

$\frac{dV}{dt} = \frac{\pi}{100}(3h^2)\frac{dh}{dt}$

*SOLVE

$150 = \frac{\pi}{100}\left(3(35)^2\right)\frac{dh}{dt}$

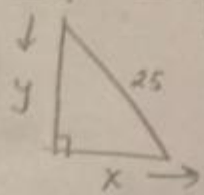
$\Rightarrow 150 = \frac{3675\pi}{100}\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{(150)(100)}{3675\pi}$

$\boxed{= \frac{15000}{3675\pi} \; in/sec}$

#21 ⓐ Find: $\boxed{\frac{dy}{dt} = \quad ft/sec}$

Given $\frac{dx}{dt} = 2$



Note: $\frac{dy}{dt}$ will be a NEGATIVE value

Part 1ne $x = 7$

Part 2wo $x = 15$

Part 3hree $x = 24$

START:

$x^2 + y^2 = 25^2$

ⓑ Differential Const:

$\frac{dx}{dt}$ given – $\frac{dy}{dt}$ desired –

ⓒ Differentiate

$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

ⓓ multiply thru by $\frac{1}{2}$

$\Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

Part 1ne

$(7)(2) + (\,?\,)\frac{dy}{dt} = 0$

need value for y

GO BACK BEFORE THE DERIVATIVE:

$x^2 + y^2 = 25^2$

$\Rightarrow (7)^2 + y^2 = (25)^2$

$\Rightarrow y^2 = 625 - 49 = 576$

$\Rightarrow y = 24$ (part 2: $y = 20$)
(part 3: $y = 7$)

$\Rightarrow (7)(2) + (24)\frac{dy}{dt} = 0$

$\Rightarrow \frac{dy}{dt} = -\frac{14}{24} = \boxed{-\frac{7}{12}\ ft/sec}$

Part 2wo

$(7)(2) + (20)\frac{dy}{dt} = 0$

$\Rightarrow 14 + 20\frac{dy}{dt} = 0$

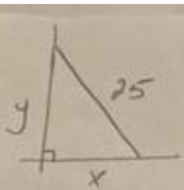
$\Rightarrow \frac{dy}{dt} = -\frac{14}{20} = \boxed{-\frac{7}{10}\ ft/sec}$

Part 3hree

$\Rightarrow (7)(2) + (7)\frac{dy}{dt} = 0$

$\Rightarrow \frac{dy}{dt} = \frac{-14}{7} = \boxed{-2\ ft/sec}$

(21)(b)



Find $\boxed{\frac{dA}{dt} = \quad ft^2/sec}$

pg 4 ans:

Given: $\frac{dx}{dt} = 2$ ⊛ Note that these are the conditions of part 1 for part a

$x = 7$

so we also have

$\frac{dy}{dt} = -\frac{7}{12}$ and $y = 24$

⊛

⊛ START

$A = \frac{1}{2}xy$

⊛ differential count

$\frac{dA}{dt}$ desired ✓

$\frac{dx}{dt}$ given ✓

$\frac{dy}{dt}$ found in part a ✓

so $\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + y \cdot \frac{1}{2} \cdot \frac{dx}{dt}$ ← product rule

$= \frac{1}{2}(7)\left(-\frac{7}{12}\right) + (24)\left(\frac{1}{2}\right)(2)$

$= \frac{-49}{24} + 24$

$= \frac{-49}{24} + \frac{576}{24} = \boxed{\frac{527}{24}\ ft^2/sec}$

OR

If you did not notice that the given conditions for part b were the same as part a, part 1:

=> START

$A = \frac{1}{2}xy$

⊛ Differential Count:

$\frac{dA}{dt}$ → desired ✓

$\frac{dx}{dt}$ → given ✓

$\frac{dy}{dt}$ No — MUST ELIMINATE y

=> $y = \sqrt{625 - x^2}$

From the triangle and pythagoras

$A = \frac{1}{2}x(625 - x^2)^{1/2}$

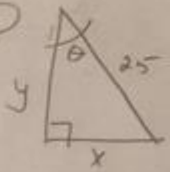
$\Rightarrow \frac{dA}{dt} = \left(\frac{1}{2}x\right)\left(\frac{1}{2}(625 - x^2)^{-1/2}\right)\left(-2x\frac{dx}{dt}\right) + (625 - x^2)^{1/2}\left(\frac{1}{2}\frac{dx}{dt}\right)$

$= \left[\left(\frac{1}{2}(7)\right)\left(\frac{1}{2}(625 - (7)^2)^{-1/2}(-2(7)(2)\right)\right] + \left[(625 - (7)^2)^{1/2}\left(\frac{1}{2}\right)(2)\right]$

$= \frac{-49}{24} + \frac{24}{1}$

$= \frac{-49}{24} + \frac{576}{24} = \boxed{\frac{527}{24}\ ft^2/sec}$

(#21) (c)



Find $\boxed{\frac{d\theta}{dt} = \quad \text{rad}/\text{sec}}$

Given: $\frac{dx}{dt} = 2$, $x = 7$ → AGAIN, we have $y = 24$, $\frac{dy}{dt} = -\frac{7}{12}$

⇒ START: WE MAY choose any function: (I'll do two)

$\Rightarrow \sin(\theta) = \frac{x}{25}$

$\Rightarrow 25\sin(\theta) = x$

⇒ Differential Count

$\frac{d\theta}{dt}$ desired ✓

$\frac{dx}{dt}$ given ✓

so $25\sin(\theta) = x$

$\Rightarrow 25\cos(\theta)\frac{d\theta}{dt} = \frac{dx}{dt}$

$\Rightarrow 25\left(\frac{24}{25}\right)\frac{d\theta}{dt} = 2$

$\Rightarrow 24\frac{d\theta}{dt} = 2 \Rightarrow \frac{d\theta}{dt} = \frac{2}{24}$

$= \boxed{\frac{1}{12}\ \text{rad}/\text{sec}}$

OR $\tan\theta = \frac{x}{y}$

Differential Count

$\frac{d\theta}{dt}$ desired ✓

$\frac{dx}{dt}$ given ✓

$\frac{dy}{dt}$ found in part a

$\Rightarrow \tan(\theta) = \frac{x}{y}$

$\Rightarrow y\tan(\theta) = x$

$\Rightarrow y\sec^2(\theta)\frac{d\theta}{dt} + \tan(\theta)\frac{dy}{dt} = \frac{dx}{dt}$

$\Rightarrow (24)\left(\frac{25}{24}\right)^2\frac{d\theta}{dt} + \left(\frac{7}{24}\right)\left(-\frac{7}{12}\right) = 2$

$\Rightarrow \frac{625}{24}\frac{d\theta}{dt} + \frac{-49}{288} = 2$

$\Rightarrow \frac{625}{24}\frac{d\theta}{dt} = 2 + \frac{49}{288}$

$\Rightarrow \frac{625}{24}\frac{d\theta}{dt} = \frac{625}{288}$

$\Rightarrow \frac{d\theta}{dt} = \frac{625}{288}\cdot\frac{24}{625} = \boxed{\frac{1}{12}\ \frac{\text{rad}}{\text{sec}}}$

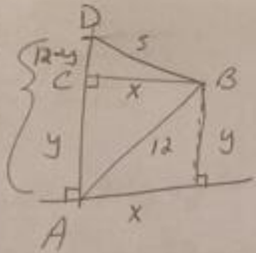
(*) Note: IF it is desired in degrees per sec:

$\Rightarrow \frac{\pi}{1/12} = \frac{180}{x}$

$\Rightarrow x\cdot\pi = (180)\left(\frac{1}{12}\right)$

$\Rightarrow x = \frac{(180)(1/12)}{\pi} \approx \boxed{4.77\ \text{deg}/\text{sec}}$

#23



Find Part 1ne: $\boxed{\frac{dy}{dt} = \quad m/s}$

Part 2wo: $\boxed{\frac{dx}{dt} = \quad m/s}$

given: $\frac{ds}{dt} = -.2 \quad y = 6$

Part 1ne:

START:

in $\triangle BCD$

$x^2 + (12-y)^2 = s^2$

⊛ Differential Count

$\frac{dx}{dt}$ No MUST ELIMINATE "x"

$\frac{dy}{dt}$ desired ✓

$\frac{ds}{dt}$ given ✓

use $\triangle ABC$

$x^2 + y^2 = 144$

$\Rightarrow x^2 = 144 - y^2$

$\Rightarrow$ so:

$144 - y^2 + (12-y)^2 = s^2$

$\Rightarrow 144 - y^2 + 144 - 24y + y^2 = s^2$

$\Rightarrow 288 - 24y = s^2$

$\Rightarrow 0 - 24\frac{dy}{dt} = 2s\frac{ds}{dt}$

$\Rightarrow -24\frac{dy}{dt} = 2(\quad)\frac{ds}{dt}$

Need value for s:

$\Rightarrow x^2 + (12-y)^2 = s^2$

$\Rightarrow$ NEED VALUE FOR x^2!

$\Rightarrow$ Again in $\triangle ABC$

$x^2 = 144 - y^2$

$= 144 - (6)^2$

$= 108$

so $(108) + (12 - (6))^2 = s^2$

$\Rightarrow 108 + 36 = s^2$

$\Rightarrow s = 12$

so finally:

$-24\frac{dy}{dt} = 2(12)(-.2)$

$\Rightarrow -24\frac{dy}{dt} = -4.8$

$\Rightarrow \frac{dy}{dt} = \frac{-4.8}{-24}$

$= \boxed{.2 \ m/s}$

Part 2wo: Note for part 2 we may use the info found in part 1

start:

$x^2 + (12-y)^2 = s^2$

⊛ Differential Count

ok

$x^2 + 144 - 24y + y^2 = s^2$

CONT

#23 part two

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CONT:

$$x^2+144-24y+y^2=s^2$$

$$\Rightarrow 2x\frac{dx}{dt}+0-24\frac{dy}{dt}+2y\frac{dy}{dt}=2s\frac{ds}{dt}$$

$$\Rightarrow 2(\sqrt{108})\frac{dx}{dt}-24(.2)+2(6)(.2)=2(12)(-.2)$$

$$\Rightarrow 2\sqrt{108}\,\frac{dx}{dt}-4.8+2.4=-4.8$$

$$\Rightarrow 2\sqrt{108}\,\frac{dx}{dt}=-2.4$$

$$\Rightarrow \frac{dx}{dt}=\frac{-2.4}{2\sqrt{108}}\ m/s$$

$$\approx \boxed{-.115\ m/s}$$