Solutions

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1) $f(x) = \tan(h(x))$ chain rule

$\Rightarrow f'(x) = \boxed{\sec^2(h(x))\left(\frac{1}{x}\right)}$

2. $f(x) = x^4 e^{1/\sqrt{x}} = x^4 e^{(x^{-1/2})}$ product & chain rules

$\Rightarrow f'(x) = \boxed{(x^4)\left(e^{(x^{-1/2})}\left(-\tfrac{1}{2}x^{-3/2}\right)\right) + \left(e^{(x^{-1/2})}\right)(4x^3)}$

3. $f(x) = \dfrac{x + h(x)}{e^x - 1}$ quotient rule

$\Rightarrow f'(x) = \boxed{\dfrac{(e^x - 1)\left(1 + \frac{1}{x}\right) - (x + h(x))(e^x)}{(e^x - 1)^2}}$

4. $f(x) = e^{\sin(x)}$

$\Rightarrow f'(x) = \boxed{e^{\sin(x)}(\cos(x))}$

5. $f(x) = h\left(\sqrt[4]{x^3 + 1}\right) = h\left((x^3+1)^{1/4}\right) = \frac{1}{4} h(x^3 + 1)$

$\Rightarrow f'(x) = \boxed{\frac{1}{4}\left(\frac{1}{x^3+1}(3x^2)\right)}$ OR $f'(x) = \boxed{\frac{1}{(x^3+1)^{1/4}}\left(\frac{1}{4}(x^3+1)^{-3/4}(3x^2)\right)}$

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(6). $f(x) = x^3 \ln(x^2+2)$

$\Rightarrow f'(x) = \boxed{(x^3)\left(\left(\frac{1}{x^2+2}\right)(2x)\right) + \ln(x^2+2)(3x^2)}$

(7). $f(x) = \ln(\ln(x^2)) = \ln(2\ln(x))$

$\Rightarrow f'(x) = \boxed{\frac{1}{\ln(x^2)}\left(\frac{1}{x^2}\right)(2x)} \overset{\text{or}}{\Longleftrightarrow} f'(x) = \boxed{\frac{1}{2\ln(x)}\left(2\left(\frac{1}{x}\right)\right)}$

(8). $f(x) = \ln(x^3)e^{\sqrt{1-3x}} = 3\ln(x)\cdot e^{(1-3x)^{1/2}}$

$\Rightarrow f'(x) = \boxed{3\left[\ln(x)\left(e^{(1-3x)^{1/2}}\right)\left(\frac{1}{2}(1-3x)^{-1/2}(-3)\right) + \left(e^{(1-3x)^{1/2}}\right)\left(\frac{1}{x}\right)\right]}$

(9). $f(x) = \dfrac{e^{(4x)} + x^2}{\sec(x) + \ln(x)}$

$\Rightarrow f'(x) = \boxed{\dfrac{[\sec(x)+\ln(x)]\left(e^{(4x)}(4) + 2x\right) - \left(e^{(4x)} + x^2\right)\left(\sec(x)\tan(x) + \frac{1}{x}\right)}{(\sec(x)+\ln(x))^2}}$

(10) $f(x) = \ln(\sec(x))$

$\Rightarrow f'(x) = \boxed{\frac{1}{\sec(x)}(\sec(x)\tan(x))}$ By the way $= \boxed{\tan(x)}$

(11). $f(x) = \dfrac{x^2(x-3)^{1/4}}{(x^2+3)}$ $\Rightarrow$ use logarithmic differentiation.

$$\Rightarrow y = \frac{x^2(x-3)^{1/4}}{(x^2+3)} \Rightarrow \ln(y) = \ln\left(\frac{x^2(x-3)^{1/4}}{(x^2+3)}\right)$$

$$= \ln(x^2) + \ln((x-3)^{1/4}) - \ln(x^2+3)$$

$$= 2\ln(x) + \tfrac{1}{4}\ln(x-3) - \ln(x^2+3)$$

so $\dfrac{1}{y}\cdot y' = \dfrac{2}{x} + \dfrac{1}{4(x-3)} - \dfrac{2x}{(x^2+3)}$

$$\Rightarrow \boxed{y' = \left[\frac{2}{x} + \frac{1}{4(x-3)} - \frac{2x}{(x^2+3)}\right]\left[\frac{x^2(x-3)^{1/4}}{(x^2+3)}\right]}$$

(12). $f(x) = (\tan(x))^{2x}$ NOTE: THERE SHOULD BE PARENTHESES HERE

use logarithmic differentiation.

$$\Rightarrow y = (\tan(x))^{2x} \Rightarrow \ln(y) = \ln(\tan(x))^{2x}$$

$$= (2x)(\ln(\tan(x)))$$

$$\Rightarrow \frac{1}{y}\cdot y' = (2x)\left(\frac{1}{\tan(x)}(\sec^2(x))\right) + (\ln(\tan(x)))(2)$$

$$\Rightarrow \boxed{y' = \left[\frac{2x\sec^2(x)}{\tan(x)} + 2\ln(\tan(x))\right]\left[(\tan(x))^{2x}\right]}$$

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(13) $\int \frac{1}{x+5}\,dx = \int \frac{1}{u}\,du$

Let $u = x+5$
$\Rightarrow du = dx$

$= \ln|u| + C$

$= \boxed{\ln|x+5| + C}$

(14). $\int \frac{\ln(x)}{x}\,dx = \int \ln(x)\cdot\frac{1}{x}\,dx$

Let $u = \ln(x)$
$\Rightarrow du = \frac{1}{x}\,dx$

$= \int u\,du$

$= \frac{u^2}{2} + C = \boxed{\frac{1}{2}(\ln(x))^2 + C}$

(15). $\int e^x\cos(e^x)\,dx = \int \cos(u)\,du$

Let $u = e^x$
$\Rightarrow du = e^x dx$

$= \sin(u) + C = \boxed{\sin(e^x) + C}$

(16). $\int x^2 e^{(x^3-4)}\,dx = \frac{1}{3}\int e^u\,du$

Let $u = x^3 - 4$
$\Rightarrow du = 3x^2\,dx$
$\Rightarrow \frac{1}{3}du = x^2 dx$

$= \frac{1}{3}e^u + C$

$= \boxed{\frac{1}{3}e^{(x^3-4)} + C}$

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(17) $\int \frac{x^2-2}{x^3-6x+1}\,dx = \frac{1}{3}\int \frac{1}{u}\,du$

Let $u = x^3-6x+1$

$\Rightarrow du = (3x^2-6)\,dx$

$\Rightarrow du = 3(x^2-2)\,dx$

$\Rightarrow \frac{1}{3}du = (x^2-2)\,dx$

$= \frac{1}{3}\ln|u|+C$

$= \boxed{\frac{1}{3}\ln|x^3-6x+1|+C}$

(18) $\int \frac{\sec(x)\tan(x)}{\sqrt{1+3\sec(x)}}\,dx = \int(\sec(x)\tan(x))(1+3\sec(x))^{-1/2}\,dx$

Let $u = 1+3\sec(x)$

$\Rightarrow du = 3\sec(x)\tan(x)\,dx$

$\Rightarrow \frac{1}{3}du = \sec(x)\tan(x)\,dx$

$= \frac{1}{3}\int u^{-1/2}\,du$

$= \frac{1(2)}{3}u^{1/2}+C = \boxed{\frac{2}{3}(1+3\sec(x))^{1/2}+C}$

(19) $\int \frac{8e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int 8e^{(x^{1/2})}x^{-1/2}\,dx = \int 8x^{-1/2}e^{(x^{1/2})}\,dx$

Let $u = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$\Rightarrow 16\,du = 8x^{-1/2}\,dx$

$= 16\int e^u\,du$

$= 16e^u + C = \boxed{16e^{(x^{1/2})}+C}$

(20) $\int \frac{e^{(5x)}}{5+5e^{(5x)}}\,dx \longrightarrow = \frac{1}{25}\int \frac{1}{u}\,du$

Let $u = 5+5e^{(5x)}$

$\Rightarrow du = 5e^{(5x)}(5)\,dx$

$\Rightarrow du = 25e^{(5x)}\,dx$

$\Rightarrow \frac{1}{25}\,du = e^{(5x)}\,dx$

$= \frac{1}{25}\ln|u| + c$

$= \boxed{\frac{1}{25}\ln|5+5e^{(5x)}| + C}$

(III)

(21) $\int_1^e \frac{1}{x(1+\ln(x))}\,dx = \int_1^2 \frac{1}{u}\,du$

Let $u = 1+\ln(x)$

$\Rightarrow du = \frac{1}{x}\,dx$

Limits

upper $u = 1+\ln(e) = 2$

lower $u = 1+\ln(1) = 1$

$= \left[\ln|u|\right]_1^2$

$= [\ln(2)] - [\ln(1)]$

$= \ln(2) - 0 = \boxed{\ln(2)}$

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(22) $\int_0^{\pi/2} 2\sin(x)e^{(\cos(x))}\,dx = -2\int_1^0 e^u\,du$

Let $u = \cos(x)$

$\Rightarrow du = -\sin(x)\,dx$

$\Rightarrow -du = \sin(x)\,dx$

$\Rightarrow -2du = 2\sin(x)\,dx$

Limits

upper $u = \cos(\pi/2) = 0$

lower $u = \cos(0) = 1$

$= -2\left[e^u\right]_1^0$

$= -2\left[[e^{(0)}] - [e^{(1)}]\right]$

$= -2[1-e]$

$= \boxed{2e-2}$