Solutions CALC I
EXAM I FALL 2023

I

① C

② B

③ A → $\lim_{x\to 7^-} f(x) \overset{\text{TOP}}{=} \lim_{x\to 7^-}(x^2-6) = (7)^2-6 = 43$

$\lim_{x\to 7^+} f(x) \overset{\text{BOT}}{=} \lim_{x\to 7^+}(3x+4) = 3(7)+4 = 25$

$\Rightarrow \lim_{x\to 7} f(x)$ D.N.E.

④ A

⑤ E

⑥ C → $x^2-7x+12=0 \Rightarrow (x-3)(x-4)=0 \Rightarrow x=3 \quad x=4$

$x=5$ IS NOT a discontinuity

⑦ C

⑧ B → $f(x)=\sec(x)$ has vertical asymptotes at odd multiples of $\pi/2$ $\Rightarrow \frac{(2k+1)\pi}{2}$

⑨ D → $\lim_{x\to 7} \frac{x^2-49}{x-7} = \lim_{x\to 7} \frac{(x+7)(x-7)}{(x-7)} = \lim_{x\to 7}(x+7) = (7)+7 = 14$

⑩ C

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⑪ $\lim_{x\to a}[f(x)] \pm \lim_{x\to a}[g(x)]$

⑫ $k\left[\lim_{x\to a} f(x)\right]$

⑬ $\left[\lim_{x\to a} f(x)\right] \cdot \left[\lim_{x\to a} g(x)\right]$

⑭ $\boxed{k}$

⑮ $\dfrac{\lim_{x\to a}[f(x)]}{\lim_{x\to a}[g(x)]}$

⑯ $\boxed{1}$

II

(17) 2 / 2 / 2 / NO

(18) -5 / 5 / DNE / NO

(19) DNE / DNE / DNE / NO

III

(20) (i) E, Z, K

(ii) E, H, K

(iii) M,

III

(21) $\lim_{x \to (-1)} (2x^3 - 4x + 5) = 2(-1)^3 - 4(-1) + 5$

$= -2 + 4 + 5$

$= \boxed{7}$

(22) $\lim_{x \to 3} \frac{x^2 - 10x + 21}{x^2 - 9} = \lim_{x \to 3} \frac{(x-3)(x-7)}{(x+3)(x-3)}$

$= \lim_{x \to 3} \frac{(x-7)}{(3+3)}$

$= \frac{3-7}{3+3}$

$= \boxed{\frac{-4}{6}} = \boxed{\frac{-2}{3}}$

(23) $\lim_{x\to 4^-}\left(\cos\left(\frac{\pi}{x}\right)\right) = \cos\left(\frac{\pi}{4}\right)$

$= \boxed{\frac{\sqrt{2}}{2}} \approx .7071$

(24) $\lim_{x\to 0}\left(\frac{\sin(x)}{4x}\right) = \frac{1}{4}\lim_{x\to 0}\left(\frac{\sin(x)}{x}\right)$

$= \frac{1}{4}(1)$

$= \boxed{\frac{1}{4}}$

(25) $\lim_{x\to 5}\frac{(\sqrt{x+4}-3)}{(x-5)} = \lim_{x\to 5}\frac{(\sqrt{x+4}-3)(\sqrt{x+4}+3)}{(x-5)(\sqrt{x+4}+3)}$

$= \lim_{x\to 5}\frac{x+4-9}{(x-5)(\sqrt{x+4}+3)}$

$= \lim_{x\to 5}\frac{(x-5)}{(x-5)(\sqrt{x+4}+3)}$

$= \lim_{x\to 5}\frac{1}{\sqrt{x+4}+3}$

$= \frac{1}{\sqrt{(5)+4}+3} = \boxed{\frac{1}{6}}$

(26) $\lim_{x\to(-6)^+} f(x) \overset{\text{Bot}}{=} \lim_{x\to(-6)^+}(5x+4)$

$= 5(-6)+4$

$= -30+4 = \boxed{-26}$

(27) $x^2-16=0 \Rightarrow x=4; x=-4$

$\boxed{f(x) \text{ is Discontinuous at } x=4;\ x=-4}$

(28) f is a polynomial so it has NO DISCONTINUITIES

$\boxed{\text{NONE}}$

(29). We Require

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$

$$\Leftrightarrow \lim_{x\to 1^-} (3x^2) = \lim_{x\to 1^-} (kx - 7)$$

$$\Leftrightarrow 3(1)^2 = k(1) - 7)$$

$$\Leftrightarrow 3 = k - 7 \Longleftrightarrow \boxed{k = 10.}$$

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(30). Prove $\lim_{x\to 3} (5x+2) = 17$

(I) Let $\epsilon > 0$

(II) Search for δ

we require

$|f(x) - L| < \epsilon \Rightarrow$

$\to |(5x+2) - 17| < \epsilon$

$\Rightarrow |5x - 15| < \epsilon$

$\Rightarrow |5(x-3)| < \epsilon$

$\Rightarrow |5||x-3| < \epsilon$

$\to |x-3| < \frac{\epsilon}{|5|}$

Choose

$\boxed{\delta = \frac{\epsilon}{|5|}}$

(III) Verify δ:

$0 < |x-a| < \delta \Rightarrow |x-3| < \frac{\epsilon}{|5|}$

$\Rightarrow |5||x-3| < \epsilon$

$\to |5(x-3)| < \epsilon$

$\Rightarrow |5x-15| < \epsilon$

$\to |5x+2-2-15| < \epsilon$

$\to |(5x+2)-17| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$ ✓