SOLUTIONS

① $f(x)=3x^2-4x-7$

$$\Rightarrow f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{3(x+\Delta x)^2-4(x+\Delta x)-7-(3x^2-4x-7)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{3(x^2+2x\Delta x-\Delta x^2)-4x-4\Delta x-7-3x^2+4x+7}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\cancel{3x^2}+6x\Delta x-3\Delta x^2-\cancel{4x}-4\Delta x\cancel{-7}\cancel{-3x^2}\cancel{+4x}\cancel{+7}}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{6x\Delta x-3\Delta x^2-4\Delta x}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\cancel{\Delta x}(6x-3\Delta x-4)}{\cancel{\Delta x}}$$

$$=\lim_{\Delta x\to 0}(6x-3\Delta x-4)$$

$$=6x-3(0)-4$$

$$=\boxed{6x-4}$$

Rewrite

③ $f(x)=5x^5+3x^{-3}-12x^{1/2}$

$$\Rightarrow\boxed{f'(x)=25x^4-9x^{-4}-6x^{-1/2}}$$

④ $f(x)=\tan(3x)-\csc(-x)\Rightarrow\boxed{f'(x)=\sec^2(3x)(3)-(-\csc(-x)\cot(-x))(-1)}$

⑤ $f(x)=(5x^{1/3})(\csc(x))\Rightarrow\boxed{f'(x)=(5x^{1/3})(-\csc(x)\cot(x))+(\csc(x))(\tfrac{5}{3}x^{-2/3})}$

⑥ $f(x)=-(x^{-4}-x^3)^{-5}\Rightarrow\boxed{f'(x)=-(-5)(x^{-4}-x^3)^{-6}(-4x^{-5}-3x^2)}$

⑦ $f(x)=\dfrac{(x+\cot(x))}{(4x)}\Rightarrow\boxed{f'(x)=\dfrac{(4x)(1-\csc^2(x))-(x+\cot(x))(4)}{(4x)^2}}$

Rewrite

⑧ $f(x)=(\cos(x^3))^2\Rightarrow\boxed{f'(x)=2(\cos(x^3))^1(-\sin(x^3))(3x^2)}$

#9. Rewrite $f(x) = (2-x^3)^{1/4} \Rightarrow \boxed{f'(x) = \frac{1}{4}(2-x^3)^{-3/4}(-3x^2)}$

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(10). $f(x) = \frac{(4x^3-6x)}{(3x^4+6)} \Rightarrow \boxed{f'(x) = \frac{(3x^4+6)(12x^2-6)-(4x^3-6x)(3x^4+6)}{(3x^4+6)^2}}$

V

(13). $x^3 - \underbrace{4x\,y^3}_{\text{Product rule}} + 3y^4 = 5y$

$\Rightarrow 3x^2 - 4x\left(3y^2\frac{dy}{dx}\right) + y^3\cdot(-4) + 12y^3\frac{dy}{dx} = 5\frac{dy}{dx}$

$\Rightarrow 3x^2 - 12xy^2\frac{dy}{dx} - 4y^3 + 12y^3\frac{dy}{dx} = 5\frac{dy}{dx}$

$\Rightarrow -12xy^2\frac{dy}{dx} + 12y^3\frac{dy}{dx} - 5\frac{dy}{dx} = -3x^2 + 4y^3$

$\Rightarrow \frac{dy}{dx}\left(-12xy^2 + 12y^3 - 5\right) = -3x^2 + 4y^3$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{(-3x^2+4y^3)}{(-12xy^2+12y^3-5)}}$

(14). $3y + \cos(y) = 2x$

$\Rightarrow 3\frac{dy}{dx} - \sin(y)\frac{dy}{dx} = 2$

$\Rightarrow \frac{dy}{dx}(3 - \sin(y)) = 2$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2}{(3-\sin(y))}}$

V (14). Find $\boxed{\frac{dr}{dt} = \quad ft/min}$

given: $\frac{dV}{dt} = 52\pi$

$\frac{dh}{dt} = 3$

$V = 12\pi$

$r = 2$

$V = \frac{1}{3}\pi r^2 h$

differential Count:

$\frac{dV}{dt}$ ← given

$\frac{dr}{dt}$ ← desired

$\frac{dh}{dt}$ ← given

⇒ so keep all Variables

so $V = \frac{\pi}{3} \underbrace{r^2 h}_{\text{product rule}}$

$\Rightarrow \frac{dV}{dt} = \left(\frac{\pi}{3}r^2\right)\left(\frac{dh}{dt}\right) + (h)\left(\frac{2\pi}{3} r \frac{dr}{dt}\right)$

$\Rightarrow 52\pi = \left(\frac{\pi}{3}(2)^2\right)(3) + (\quad)\left(\frac{2\pi}{3}(2)\frac{dr}{dt}\right)$

Need a value for h so GO BACK BEFORE The DERIVATIVE.

$V = \frac{1}{3}\pi r^2 h \Rightarrow 12\pi = \frac{\pi}{3}(2)^2 h$

$\Rightarrow 12\pi = \frac{4\pi}{3} \cdot h$

$\Rightarrow \frac{3}{4\pi} \cdot \frac{12\pi}{1} = h$

$\Rightarrow 9 = h$

so we have

$52\pi = \left(\frac{\pi}{3}(2)^2\right)(3) + (9)\left(\frac{2\pi}{3}(2)\right)\frac{dr}{dt}$

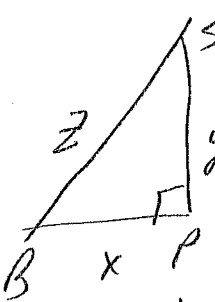
$\Rightarrow 52\pi = 4\pi + 12\pi \frac{dr}{dt}$

$\Rightarrow 48\pi = 12\pi \frac{dr}{dt}$

$\Rightarrow 4 = \frac{dr}{dt} \Longrightarrow \boxed{\frac{dr}{dt} = 4\ ft/min}$

(15). NOTE THIS ITEM IS DIFFERENT FROM THE SHEET HANDED OUT IN CLASS

Find $\boxed{\frac{dz}{dt} = \quad m/s}$



Given:

$\frac{dy}{dt} = 12$

$\frac{dx}{dt} = -8$

NOTE!

$y = 120$

$x = 30$

$\therefore x^2 + y^2 = z^2$

differential Count

$\frac{dx}{dt}$ ← given

$\frac{dy}{dt}$ ← given

$\frac{dz}{dt}$ ← desired

⇒ so Keep all Variables

$\therefore x^2 + y^2 = z^2$

$\Rightarrow \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$

cancel the 2s

⇒ need a value for z

$\Rightarrow x^2 + y^2 = z^2$

$\Rightarrow (30)^2 + (120)^2 = z^2$

$\Rightarrow z \approx 123.69$

$\therefore (30)(-8) + (120)(12) = (123.69)\frac{dz}{dt}$

$\Rightarrow \frac{1200}{123.69} = \frac{dz}{dt} \Rightarrow \boxed{\frac{dz}{dt} \approx 9.70\ m/s}$