Solutions Exam #2wo

I

1. D $f(x) = \sec(5x) \rightarrow f'(x) = \sec(5x)\tan(5x)(5)$

2. B $f(x) = -3(x+2)^2 + 20x \Rightarrow f'(x) = -6(x+2)^1(1) + 20$
$f'(2) = -6(4) + 20 = -4$

3. A $2x^4 + 2x^2y = 4y^2 \Rightarrow 8x^3 + 2x^2\frac{dy}{dx} + y(4x) = 4y\frac{dy}{dx}$ $x=1$ $y=0$
$\Rightarrow 8(1) + 2(1)\frac{dy}{dx} + 0 = 0 \Rightarrow \frac{dy}{dx} = \frac{8}{2} = -4$

4. E $f(x) = \cos(x) - \tan(x) \Rightarrow f'(x) = -\sin(x) - \sec^2(x)$

5. C $f(x) = 2x^3 - 4x + 2 \Rightarrow f'(x) = 6x^2 - 4 \Rightarrow f'(1) = 6(1) - 4 = 2$
$y - 0 = 2(x-1) \Rightarrow y = 2x - 2$

6. B $f(x) = \frac{(2x-1)}{(x^2+x)} \Rightarrow f'(x) = \frac{(x^2+x)(2) - (2x-1)(2x+1)}{(x^2+x)^2}$

7. A

8. D

9. E

II 10. $\frac{d}{dx}[uv] = \boxed{uv' + vu'}$ 11. $\frac{d}{dx}[\sec(u)] = \boxed{\sec(u)\tan(u)u'}$

12. $\frac{d}{dx}[\tan(u)] = \boxed{\sec^2(u)u'}$ 13. $\frac{d}{dx}\left[\frac{u}{v}\right] = \boxed{\frac{vu' - uv'}{v^2}}$

3 pts ea

14. $\frac{d}{dx}[k] = \boxed{0}$ 15. $\frac{d}{dx}[\cos(u)] = \boxed{-\sin(u)u'}$

III

3 pts ea

16. $f(x) = x^4 \cdot \sec(x) \Rightarrow \boxed{f'(x) = (x^4)(\sec(x)\tan(x)) + (\sec(x))(4x^3)}$

17. $f(x) = (-x^{-5} - x^{-2})^{-3} \Rightarrow \boxed{f'(x) = -3(-x^{-5} - x^{-2})^{-4}(5x^{-6} + 2x^{-3})}$

18. $f(x) = \frac{4x}{x^3 - \tan(x)} \Rightarrow \boxed{f'(x) = \frac{(x^3 - \tan(x))(4) - (4x)(3x^2 - \sec^2(x))}{(x^3 - \tan(x))^2}}$

(19). $f(x) = \sin^2(2x)$

Rewrite $f(x) = (\sin(2x))^2 \Rightarrow \boxed{f'(x) = 2(\sin(2x))^1(\cos(2x))(2)}$

(20). $f(x) = x^6(4x^3+7x)^3 \Rightarrow \boxed{f'(x) = (x^6)(3)(4x^3+7x)^2(12x^2+7) + (4x^3+7x)^3(6x^5)}$

(21). $f(x) = \frac{x^2+5}{x^5+2} \Rightarrow \boxed{f'(x) = \frac{(x^5+2)(2x) - (x^2+5)(5x^4)}{(x^5+2)^2}}$

(22). $f(x) = 2x^4 + 3x^{-2} - 12x \Rightarrow \boxed{f'(x) = 8x^3 - 6x^{-3} - 12}$

(23). Rewrite $f(x) = -3(9x^2+1)^{1/4} \Rightarrow f'(x) = \boxed{(-3)(\frac{1}{4})(9x^2+1)^{-3/4}(18x)}$

(24). $f(x) = 9 \Rightarrow f'(x) = \boxed{0}$

(25). $5x^2y^2 + x^3 = 3y \Rightarrow (5x^2)(2y\frac{dy}{dx}) + (y^2)(10x) + 3x^2 = 3\frac{dy}{dx}$

$\Rightarrow 10x^2y\frac{dy}{dx} + 10xy^2 + 3x^2 = 3\frac{dy}{dx}$

$\Rightarrow 10x^2y\frac{dy}{dx} - 3\frac{dy}{dx} = -10xy^2 - 3x^2$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-10xy^2 - 3x^2}{10x^2y - 3}}$

(26). $-\cos(y) = 2x^4 - y^2$

$\xrightarrow{\frac{d}{dx}} -(-\sin(y)\frac{dy}{dx}) = 8x^3 - 2y\frac{dy}{dx}$

$\Rightarrow \sin(y)\frac{dy}{dx} + 2y\frac{dy}{dx} = 8x^3$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{8x^3}{\sin(y) + 2y}}$

III Find: $\frac{dr}{dt} =$ ____ cm/sec

(27) given: $\frac{dV}{dt} = -12$ $\quad V = \pi r^2 h$ ← diff count

$h = 3r$

$r = 4$

$\frac{dV}{dt}$ ← given

$\frac{dr}{dt}$ ← desired

$\frac{dh}{dt}$ NO ⇒ must eliminate h using this

$\Rightarrow V = \pi r^2 \cdot 3r$

$\Rightarrow V = 3\pi r^3$

$\Rightarrow \frac{dV}{dt} = 9\pi r^2 \frac{dr}{dt}$

$\Rightarrow -12 = 9\pi (4)^2 \frac{dr}{dt}$

$\Rightarrow -12 = 144\pi \frac{dr}{dt}$

$\Rightarrow \frac{-12}{144\pi} = \frac{dr}{dt}$

so

$\frac{dr}{dt} = \frac{-12}{144\pi}$ cm/sec

$= \boxed{\frac{-1}{12\pi}}$

(28) Find $\frac{dy}{dt} =$ ____ m/s

given:

$\frac{dx}{dt} = 2$

$\frac{dz}{dt} = -1$

$z = 5$

$y = 4$

$x^2 + y^2 = z^2$ ← differential count

$\frac{dx}{dt}$ ← given

$\frac{dy}{dt}$ ← desired

$\frac{dz}{dt}$ ← given

$\Rightarrow x^2 + y^2 = z^2$

$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$

$\Rightarrow (3)(2) + (4)\frac{dy}{dt} = (5)(-1)$

$\Rightarrow 6 + 4\frac{dy}{dt} = -5$

$\Rightarrow 4\frac{dy}{dt} = -11 \Rightarrow \boxed{\frac{dy}{dt} = \frac{-11}{4} \text{ m/s}}$

NEED VALUES FOR x:

$x^2 + (4)^2 = (5)^2$

$x^2 = 9$

$x = 3$ ←

BONUS Find $\frac{dr}{dt}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 72\pi$

$V = 36\pi$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

need value for r

$36\pi = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{36\pi}{1} \cdot \frac{3}{4\pi} = r^3$

$\Rightarrow 27 = r^3$

$\Rightarrow 3 = r$

so $72\pi = 4\pi (3)^2 \frac{dr}{dt}$

$\Rightarrow 72\pi = 36\pi \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{72\pi}{36\pi} = 2$

now $A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi (3)(2)$

$= \boxed{48\pi}$

$\approx 150.796 \text{ cm}^2/\text{sec}$

(#29) $f(x) = x^2 + 6x - 3$

$$\Rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 6(x+\Delta x) - 3 - (x^2 + 6x - 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 + 6x + 6\Delta x - 3 - x^2 - 6x + 3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2 + 6\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x + 6)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x + 6)$$

$$= 2x + (0) + 6$$

$$= \boxed{2x + 6}$$