**Solution** The denominator contains the repeated irreducible quadratic factor $(x^2 + 4)^2$, so by Case 4,

$$\frac{x^3 + x^2}{(x^2 + 4)^2} = \frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2} \tag{11}$$

Clear fractions to obtain

$$x^3 + x^2 = (Ax + B)(x^2 + 4) + Cx + D$$

Collecting like terms yields the identity

$$x^3 + x^2 = Ax^3 + Bx^2 + (4A + C)x + 4B + D$$

Equating coefficients results in the system

$$\begin{cases} A = 1 \\ B = 1 \\ 4A + C = 0 \\ 4B + D = 0 \end{cases}$$

The solution is $A = 1, B = 1, C = -4, D = -4$. From equation (11),

$$\frac{x^3 + x^2}{(x^2 + 4)^2} = \frac{x + 1}{x^2 + 4} + \frac{-4x - 4}{(x^2 + 4)^2}$$

**Now Work** PROBLEM 39

## 11.5 Assess Your Understanding

**'Are You Prepared?'** *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. ***True or False*** The equation $(x - 1)^2 - 1 = x(x - 2)$ is an example of an identity. (p. A44) **True**
2. ***True or False*** The rational expression $\frac{5x^2 - 1}{x^3 + 1}$ is proper. (p. 203) **True**
3. Reduce to lowest terms: $\frac{3x - 12}{x^2 - 16}$ (pp. A35–A36) $\frac{3}{x + 4}$
4. ***True or False*** Every polynomial with real numbers as coefficients can be factored into a product of linear and/or irreducible quadratic factors. (p. 249) **True**

### Skill Building

*In Problems 5–16, determine whether the given rational expression is proper or improper. If the expression is improper, rewrite it as the sum of a polynomial and a proper rational expression.*

5. $\frac{x}{x^2 - 1}$ **Proper**
6. $\frac{5x + 2}{x^3 - 1}$ **Proper**
*7. $\frac{x^2 + 5}{x^2 - 4}$
*8. $\frac{3x^2 - 2}{x^2 - 1}$
*9. $\frac{x^3 + x^2 - 12x + 9}{x^2 + 2x - 15}$
*10. $\frac{6x^3 - 5x^2 - 7x - 3}{2x - 5}$
11. $\frac{5x^2 - 7x - 6}{x + x^3}$ **Proper**
12. $\frac{x^3 + 12x^2 - 9x}{9x^2 - x^4}$ **Proper**
*13. $\frac{5x^3 + 2x - 1}{x^2 - 4}$
*14. $\frac{3x^4 + x^2 - 2}{x^3 + 8}$
*15. $\frac{x(x - 1)}{(x + 4)(x - 3)}$
*16. $\frac{2x(x^2 + 4)}{x^2 + 1}$

*In Problems 17–50, find the partial fraction decomposition of each rational expression.*

17. $\frac{4}{x(x - 1)}$ $\frac{-4}{x} + \frac{4}{x - 1}$
*18. $\frac{3x}{(x + 2)(x - 1)}$
19. $\frac{1}{x(x^2 + 1)}$ $\frac{1}{x} + \frac{-x}{x^2 + 1}$
*20. $\frac{1}{(x + 1)(x^2 + 4)}$
*21. $\frac{x}{(x - 1)(x - 2)}$
*22. $\frac{3x}{(x + 2)(x - 4)}$
*23. $\frac{x^2}{(x - 1)^2(x + 1)}$
*24. $\frac{x + 1}{x^2(x - 2)}$
*25. $\frac{1}{x^3 - 8}$
*26. $\frac{2x + 4}{x^3 - 1}$
*27. $\frac{x^2}{(x - 1)^2(x + 1)^2}$
*28. $\frac{x + 1}{x^2(x - 2)^2}$
*29. $\frac{x - 3}{(x + 2)(x + 1)^2}$
*30. $\frac{x^2 + x}{(x + 2)(x - 1)^2}$
*31. $\frac{x + 4}{x^2(x^2 + 4)}$
*32. $\frac{10x^2 + 2x}{(x - 1)^2(x^2 + 2)}$

*Due to space restrictions, answers to these exercises may be found in the Answers in the back of the text.

*33. $\frac{x^2 + 2x + 3}{(x + 1)(x^2 + 2x + 4)}$

*34. $\frac{x^2 - 11x - 18}{x(x^2 + 3x + 3)}$

*35. $\frac{x}{(3x - 2)(2x + 1)}$

*36. $\frac{1}{(2x + 3)(4x - 1)}$

*37. $\frac{x}{x^2 + 2x - 3}$

*38. $\frac{x^2 - x - 8}{(x + 1)(x^2 + 5x + 6)}$

*39. $\frac{x^2 + 2x + 3}{(x^2 + 4)^2}$

*40. $\frac{x^3 + 1}{(x^2 + 16)^2}$

*41. $\frac{7x + 3}{x^3 - 2x^2 - 3x}$

*42. $\frac{x^3 + 1}{x^5 - x^4}$

*43. $\frac{x^2}{x^3 - 4x^2 + 5x - 2}$

*44. $\frac{x^2 + 1}{x^3 + x^2 - 5x + 3}$

*45. $\frac{x^3}{(x^2 + 16)^3}$

*46. $\frac{x^2}{(x^2 + 4)^3}$

*47. $\frac{4}{2x^2 - 5x - 3}$

*48. $\frac{4x}{2x^2 + 3x - 2}$

49. $\frac{2x + 3}{x^4 - 9x^2}$ $\frac{-\frac{2}{9}}{x} + \frac{-\frac{1}{3}}{x^2} + \frac{\frac{1}{6}}{x - 3} + \frac{\frac{1}{18}}{x + 3}$

50. $\frac{x^2 + 9}{x^4 - 2x^2 - 8}$ $\frac{\frac{13}{24}}{x - 2} + \frac{\frac{-13}{24}}{x + 2} + \frac{-\frac{7}{6}}{x^2 + 2}$

***Mixed Practice*** *In Problems 51–58, use the division algorithm to rewrite each improper rational expression as the sum of a polynomial and a proper rational expression. Finally, express the improper rational expression as the sum of a polynomial and a partial fraction decomposition.**

10. $\frac{x^3}{x^2 + 1}$

54. $\frac{x^3 + x}{x^2 + 4}$

58. $\frac{x^5 - x^3 + x^2 + 1}{x^4 + 6x^2 + 9}$

a substitution and partial fraction

s $\frac{2}{x - \sqrt[3]{x}}$ in terms of $\sqrt[3]{x}$.

ms are given at the end of the text.

Pg 783 & 784

17, 18, 19, 21

17–24

29–33, 35, 37

(2 points each)

*s to keep the material fresh in your*

$^{-3} - 4$ is one-to-one. Find $f^{-1}$.

e hyperbola with vertices $(0, -5)$ t $(0, 13)$.

$\left\{x \middle| x \geq \frac{6}{5}\right\}$ or $\left[\frac{6}{5}, \infty\right)$

$- 4y + 2yD = D$

line that is perpendicular to the nt of tangency. If $y = -2x + 2$ is

find its rectangular coordinates.

the tangent line to $f(x) = \frac{1}{3}x^2 - 4x + 5$, find an equation of the normal line to $f$ at the point of tangency.

65. Determine whether $f(x) = -\frac{3x}{x^2 - 10}$ is even, odd, or neither. **Odd**

## 'Are You Prepared?' Answers

1. True 2. True 3. $\frac{3}{x + 4}$ 4. True