Partial Fractions HW

#17

$$\frac{4}{x(x-1)} = \frac{A}{x} + \frac{B}{(x-1)}$$

$$\Rightarrow \frac{x(x-1)}{1}\left[\frac{4}{x(x-1)} = \frac{A}{x} + \frac{B}{(x-1)}\right]$$

$$\Rightarrow 4 = A(x-1) + B(x)$$

Let $x = 0$

$\Rightarrow 4 = A(0-1) + 0$

$\Rightarrow 4 = A(-1)$

$\Rightarrow -4 = A$

Let $x = 1$

$\Rightarrow 4 = 0 + B(1)$

$\Rightarrow 4 = B$

so we have:

$$\frac{-4}{x} + \frac{4}{(x-1)}$$

#18

$$\frac{3x}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

$$\Rightarrow \frac{(x+2)(x-1)}{1}\left[\frac{3x}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}\right]$$

$$\Rightarrow 3x = A(x-1) + B(x+2)$$

Let $x = 1$

$\Rightarrow 3(1) = 0 + B((1)+2)$

$\Rightarrow 3 = B(3)$

$\Rightarrow B = 1$

Let $x = -2$

$3(-2) = A((-2)-1) + 0$

$\Rightarrow -6 = A(-3)$

$\Rightarrow 2 = A$

so we have:

$$\frac{2}{(x+2)} + \frac{1}{(x-1)}$$

#19 category III

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{(Bx+C)}{(x^2+1)}$$

$$\Rightarrow \frac{x(x^2+1)}{1}\left[\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{(Bx+C)}{(x^2+1)}\right]$$

$$\Rightarrow 1 = A(x^2+1) + (Bx+C)x$$

$$\Rightarrow 1 = Ax^2 + A + Bx^2 + Cx$$

$$= (A+B)x^2 + Cx + A$$

EQUATE Coefficients

$\Rightarrow$

$A + B = 0$

$C = 0$

$A = 1$

$A + B = 0$ $\quad C = 0$

$A = 1 \Rightarrow A = 1$

$1 + B = 0 \Rightarrow B = -1$

so we have:

$$\frac{1}{x} + \frac{(-1)x + 0}{(x^2+1)}$$

$$= \frac{1}{x} + \frac{-x}{x^2+1}$$

(#20) CATEGORY III

$$\frac{1}{(x+1)(x^2+4)} = \frac{A}{(x+1)} + \frac{(Bx+C)}{(x^2+4)}$$

$$\Rightarrow \frac{(x+1)(x^2+4)}{1}\left[\frac{1}{(x+1)(x^2+4)} = \frac{A}{(x+1)} + \frac{(Bx+C)}{(x^2+4)}\right]$$

$$\Rightarrow 1 = A(x^2+4) + (Bx+C)(x+1)$$

$$\Rightarrow 1 = Ax^2 + 4A + Bx^2 + Bx + Cx + C$$

$$\Rightarrow 1 = (A+B)x^2 + (B+C)x + (4A+C)$$

EQUATE COEFFICIENTS:

$$\Rightarrow \begin{cases} A+B = 0 \\ B+C = 0 \\ 4A+C = 1 \end{cases} \Rightarrow \begin{cases} A-C=0 \\ 4A+C=1 \end{cases} \Rightarrow 5A = 1 \Rightarrow A = \frac{1}{5}$$

So $\frac{1}{5} + B = 0 \Rightarrow B = -\frac{1}{5}$

So $-\frac{1}{5} + C = 0 \Rightarrow C = \frac{1}{5}$

$\Rightarrow$ so we have:

$$\boxed{\frac{1/5}{(x+1)} + \frac{\left(\left(-\frac{1}{5}\right)x + \frac{1}{5}\right)}{(x^2+4)}}$$

(#21) Category I

$$\frac{x}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$$

$$\Rightarrow \frac{(x-1)(x-2)}{1}\left[\frac{x}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}\right]$$

$$\Rightarrow x = A(x-2) + B(x-1)$$

Let $x=2$

$\Rightarrow 2 = 0 + B(2-1)$

$\Rightarrow 2 = B(1)$

$\Rightarrow 2 = B$

Let $x=1$

$\Rightarrow 1 = A(1-2) + 0$

$\Rightarrow 1 = A(-1)$

$\Rightarrow -1 = A$

so we have:

$$\boxed{\frac{-1}{(x-1)} + \frac{2}{(x-2)}}$$

(22.) CATEGORY I

$$\frac{3x}{(x+2)(x-4)} = \frac{A}{(x+2)} + \frac{B}{(x-4)}$$

$$\Rightarrow \frac{(x+2)(x-4)}{1}\left[\frac{3x}{(x+2)(x-4)} = \frac{A}{(x+2)} + \frac{B}{(x-4)}\right]$$

$$\Rightarrow \quad 3x = A(x-4) + B(x+2)$$

so we have

$$\boxed{\frac{1}{(x+2)} + \frac{2}{(x-4)}}$$

Let $x = 4$

$\Rightarrow 3(4) = 0 + B(4+2)$

$\Rightarrow 12 = B(6)$

$\Rightarrow 2 = B$

Let $x = -2$

$\Rightarrow 3(-2) = A(-2-4) + B(0)$

$\Rightarrow -6 = A(-6)$

$\Rightarrow 1 = A$

(23.) CATEGORY II

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

$$\Rightarrow \frac{(x-1)^2(x+1)}{1}\left[\frac{x^2}{(x-1)^2(x+1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}\right]$$

$$\Rightarrow \quad x^2 = A(x-1)^2 + B(x-1)(x+1) + C(x+1)$$

Let $x = 1$

$\Rightarrow (1)^2 = 0 + 0 + C((1)+1)$

$\Rightarrow 1 = C(2)$

$\Rightarrow \frac{1}{2} = C$

Let $x = -1$

$\Rightarrow (-1)^2 = A(-1-1)^2 + 0 + 0$

$\Rightarrow 1 = A(-2)^2$

$\Rightarrow 1 = A(4)$

$\Rightarrow \frac{1}{4} = A$

Let $x = 0$

$\Rightarrow (0)^2 = \left(\frac{1}{4}\right)(0-1)^2 + B(-1)(1) + \left(\frac{1}{2}\right)(0+1)$

$\Rightarrow 0 = \frac{1}{4}(1) + B(-1) + \frac{1}{2}$

$\Rightarrow 0 = \frac{3}{4} + B(-1) \Rightarrow -\frac{3}{4} = B(-1)$

$\Rightarrow 3/4 = B$

so we have

$$\boxed{\frac{1/4}{(x+1)} + \frac{3/4}{(x-1)} + \frac{1/2}{(x-1)^2}}$$

(24) CATEGORY II

$$\frac{(x+1)}{x^2(x-2)} = \frac{A}{(x-2)} + \frac{B}{x} + \frac{C}{x^2}$$

$$\Rightarrow \frac{x^2(x-2)}{1}\left[\frac{(x+1)}{x^2(x-2)} = \frac{A}{(x-2)} + \frac{B}{x} + \frac{C}{x^2}\right]$$

$$\Rightarrow (x+1) = A(x^2) + B(x)(x-2) + C(x-2)$$

Let $x=0$

$\Rightarrow (0)+1 = 0+0+C(0-2)$

$\Rightarrow 1 = C(-2)$

$\Rightarrow -\frac{1}{2} = C$

Let $x=2$

$(2)+1 = A(2)^2 + 0 + 0$

$3 = A(4)$

$\frac{3}{4} = A$

Let $x=1$

$\rightarrow (1)+1 = (\frac{3}{4})(1)^2 + B(1)(1-2) + (-\frac{1}{2})(1-2)$

$\Rightarrow 2 = \frac{3}{4} + B(-1) + (-\frac{1}{2})(-1)$

$\Rightarrow 2 = \frac{3}{4} + B(-1) + \frac{1}{2}$

$\Rightarrow 2 = \frac{5}{4} + B(-1)$

$\Rightarrow \frac{3}{4} = B(-1) \Rightarrow B = -\frac{3}{4}$

so we have

$$\frac{\frac{3}{4}}{(x-2)} + \frac{(-\frac{3}{4})}{x} + \frac{(-\frac{1}{2})}{x^2}$$

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(29) CATEGORY II

$$\frac{(x+2)(x+1)^2}{1}\left[\frac{(x-3)}{(x+2)(x+1)^2} = \frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}\right]$$

$$\Rightarrow x-3 = A(x+1)^2 + B(x+2)(x+1) + C(x+2)$$

Let $x=-1$

$\Rightarrow (-1)-3 = 0+0+C(-1+2)$

$\Rightarrow -4 = C(1)$

$\Rightarrow -4 = C$

Let $x=-2$

$\Rightarrow (-2)-3 = A(-2+1)^2 + 0 + 0$

$\Rightarrow -5 = A(1)$

$\Rightarrow -5 = A$

Let $x=0$

$\Rightarrow (0)-3 = (-5)(0+1)^2 + B(0+2)(0+1) + (-4)(0+2)$

$\Rightarrow -3 = -5 + B(2)(1) + (-8) \quad \Rightarrow -3 = -13 + B(2)$

$\Rightarrow 10 = B(2) \Rightarrow B = 5$

so we have

$$\frac{(-5)}{(x+2)} + \frac{5}{(x+1)} + \frac{(-4)}{(x+1)^2}$$

#30 ✓

$$\frac{(x+1)(x-1)^2}{1}\left[\frac{x^2+x}{(x+2)(x-1)^2}=\frac{A}{(x+2)}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}\right]$$

$$\Rightarrow \quad x^2+x = A(x-1)^2+B(x+2)(x-1)+C(x+2)$$

Let $x=1$

$\Rightarrow (1)^2+1 = 0+0+C(1+2)$

$\Rightarrow 2 = C(3)$

$\Rightarrow 2/3 = C$

Let $x=-2$

$\Rightarrow (-2)^2+(-2) = A((-2)-1)^2+0+0$

$\Rightarrow 2 = A(9)$

$\Rightarrow 2/9 = A$

Let $x=0$

$\Rightarrow (0)^2+0 = 2/9(0-1)^2+B(0+2)(0-1)+(2/3)((0)+2)$

$\Rightarrow 0 = 2/9+B(-2)+4/3$

$\Rightarrow 0 = 14/9+B(-2)$

$\Rightarrow -14/9 = B(-2)$

$\Rightarrow 7/9 = B$

so we have

$$\boxed{\frac{(2/9)}{(x+2)}+\frac{(7/9)}{(x-1)}+\frac{(2/3)}{(x-1)^2}}$$

(31) CATEGORY III

$$\frac{x^2(x^2+4)}{1}\left[\frac{(x+4)}{x^2(x^2+4)}=\frac{A}{x}+\frac{B}{x^2}+\frac{(Cx+D)}{(x^2+4)}\right]$$

$$\Rightarrow \quad (x+4) = A(x)(x^2+4)+B(x^2+4)+(Cx+D)(x^2)$$

$\Rightarrow x+4 = A(x^3+4x)+B(x^2+4)+Cx^3+Dx^2$

$\Rightarrow x+4 = Ax^3+4Ax+Bx^2+4B+Cx^3+Dx^2$

$\Rightarrow x+4 = (A+C)x^3+(B+D)x^2+(4A)x+4B$

EQUATE COEFFICIENTS:

$A + C = 0 \Rightarrow C = -1/4$

$B + D = 0 \Rightarrow D = -1$

$4A = 1 \Rightarrow A = 1/4$

$4B = 4 \Rightarrow B = 1$

so we have

$$\boxed{\frac{(1/4)}{x}+\frac{1}{x^2}+\frac{(-1/4)x+(-1)}{(x^2+4)}}$$

(32) Category III

$$\frac{(x-1)^2(x^2+2)}{1}\left[\frac{10x^2+2x}{(x-1)^2(x^2+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{(Cx+D)}{(x^2+2)}\right]$$

$$\Rightarrow 10x^2+2x = A(x-1)(x^2+2) + B(x^2+2) + (Cx+D)(x-1)^2$$

(FOIL; Foil; Ask about this on Monday)

$$\Rightarrow 10x^2+2x = A(x^3+2x-x^2-2) + Bx^2+2B + (Cx+D)(x^2-2x+1)$$

("Double Distribute")

$$\Rightarrow 10x^2+2x = Ax^3+2Ax-Ax^2-2A+Bx^2+2B+Cx^3-2Cx^2+Cx+Dx^2-2Dx+D$$

$$\Rightarrow 10x^2+2x = (A+C)x^3+(-A+B-2C+D)x^2+(2A+C-2D)x+(-2A+2B+D)$$

NOTE: SAME AS $0x^3+10x^2+2x+0$

$$\Rightarrow \begin{cases} A+C=0 \\ -A+B-2C+D=10 \\ 2A+C-2D=2 \\ -2A+2B+D=0 \end{cases}$$

ADD EQ III & EQ IIII

$\Rightarrow 2B+C-D=2$

ADD EQ I & EQ II

$B-C+D=10$

MULT EQ II BY 2

$-2A+2B-4C+2D=20$

ADD THIS TO EQ III

$2B-3C=22$

NOW WE HAVE:

$$\begin{cases} 2B+C-D=2 \\ B-C+D=10 \\ 2B-3C=22 \end{cases} \Rightarrow \text{MULTIPLY EQ II BY } -2 \Rightarrow \begin{cases} 2B+C-D=2 \\ -2B+2C-2D=-20 \\ 2B-3C=22 \end{cases} \begin{array}{l} \rightarrow \text{ADD} \rightarrow 3C-3D=-18 \\ \rightarrow \text{ADD} \rightarrow -C-2D=2 \end{array}$$

AT LAST!

NOW WE HAVE

$3C-3D=-18$

$-C-2D=2 \rightarrow$ MULTIPLY BY 3 $\Rightarrow$

$3C-3D=-18$

$-3C-6D=6$ $\rightarrow$ ADD $\rightarrow -9D=-12 \Rightarrow D=\frac{12}{9}$

$D=\frac{4}{3}$

So we have: $3C-3\left(\frac{4}{3}\right)=-18$

$\Rightarrow 3C-4=-18$

$\Rightarrow C=\frac{-14}{3}$

THUS:

$2B-3\left(\frac{-14}{3}\right)=22$

$\Rightarrow 2B+14=22$

$\Rightarrow B=4$

$0=A+\frac{-14}{3}=0$

$\Rightarrow A=14/3$

AND

$\frac{14}{3}-2D=2$

$\Rightarrow -2D=-8/3$

$\Rightarrow D=4/3$

So we have:

$$\boxed{\frac{14/3}{(x-1)} + \frac{4}{(x-1)^2} + \frac{(-14/3x+4/3)}{(x^2+2)}}$$

(33) ✓ Category III since x^2+2x+4 does not factor.

$$\frac{(x+1)(x^2+2x+4)}{1}\left[\frac{x^2+2x+3}{(x+1)(x^2+2x+4)} = \frac{A}{(x+1)} + \frac{(Bx+C)}{(x^2+2x+4)}\right]$$

$$\Rightarrow x^2+2x+3 = A(x^2+2x+4) + (Bx+C)(x+1)$$

$$\Rightarrow x^2+2x+3 = Ax^2+2Ax+4A+Bx^2+Cx+Bx+C$$

$$\Rightarrow x^2+2x+3 = (A+B)x^2+(2A+B+C)x+(4A+C)$$

EQUATE COEFFICIENTS

$$\begin{aligned} A+B &= 1 \\ 2A+B+C &= 2 \\ 4A+C &= 3 \end{aligned}$$

$A+B=1 \Rightarrow A+B=1$

$2A-B=1$ (EQ 3 minus EQ 2) $\Rightarrow 3A=2 \Rightarrow A=2/3$ ←

So $2/3+B=1 \Rightarrow B=1/3$ ←

Now $4(2/3)+C=3 \Rightarrow 8/3+C=9/3 \Rightarrow C=1/3$ ←

So we have: $$\left[\frac{(2/3)}{(x+1)} + \frac{(1/3)x+(1/3)}{(x^2+2x+4)}\right]$$

(35) ✓ CATEGORY I

$$\frac{(3x-2)(2x+1)}{1}\left[\frac{x}{(3x-2)(2x+1)} = \frac{A}{(3x-2)} + \frac{B}{(2x+1)}\right]$$

$$\Rightarrow x = A(2x+1)+B(3x-2)$$

Let $x=2/3$

$\Rightarrow 2/3 = A(2(2/3)+1)+0$

$\Rightarrow 2/3 = A(7/3)$

$\Rightarrow 2/7 = A$

Let $x=-1/2$

$\Rightarrow -1/2 = 0+B(3(-1/2)-2)$

$\Rightarrow -1/2 = B(-7/2)$

$\Rightarrow 1/7 = B$

So we have

$$\left[\frac{(2/7)}{(3x-2)} + \frac{(1/7)}{(2x+1)}\right]$$

pg 8 right

(37) ✓ $\frac{x}{x^2+2x-3} = \frac{x}{(x-3)(x+1)}$ CATEGORY I

$\frac{(x-3)(x+1)}{1}\left[\frac{x}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}\right]$

$\Rightarrow \quad x = A(x+1) + B(x-3)$

Let $x=-1$

$\Rightarrow -1 = 0 + B((-1)-3)$

$\Rightarrow -1 = B(-4)$

$\Rightarrow \frac{1}{4} = B$

Let $x=3$

$\Rightarrow 3 = A(3+1) + 0$

$\Rightarrow 3 = A(4)$

$\Rightarrow \frac{3}{4} = A$

so we have

$$\frac{(3/4)}{(x-3)} + \frac{(1/4)}{(x+1)}$$

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(39) ✓ CATEGORY IIII

$\frac{(x^2+4)^2}{1}\left[\frac{x^2+2x+3}{(x^2+4)^2} = \frac{(Ax+B)}{(x^2+4)} + \frac{(Cx+D)}{(x^2+4)^2}\right]$

$\Rightarrow x^2+2x+3 = (Ax+B)(x^2+4) + (Cx+D)$

$\Rightarrow x^2+2x+3 = Ax^3 + 4Ax + Bx^2 + 4B + Cx + D$

$= Ax^3 + Bx^2 + (4A+C)x + (4B+D)$

EQUATE COEFFICIENTS:

$A = 0 \Rightarrow A = 0$

$B = 1 \Rightarrow B = 1$

$4A + C = 2 \Rightarrow C = 2$

$4B + D = 3 \Rightarrow 4 + D = 3 \Rightarrow D = -1$

so we have:

$$\frac{1}{(x^2+4)} + \frac{(2x-1)}{(x^2+4)^2}$$