

SOLUTIONS

pg 1ne

①. $2 \cos(x) + 8 = 7$ in $\underline{\underline{[0, 2\pi]}}$

* Isolate

$$\Rightarrow 2 \cos(x) = -1$$

$$\Rightarrow \cos(x) = -\frac{1}{2}$$

* Quad check:

Q II Q III

* Ref $\angle$:

$$x_R = \pi/3$$

* solve

$$x = 2\pi/3 \quad x = 4\pi/3$$

②. $\tan(x) + 1 = 0$ in $\underline{\underline{(-\infty, \infty)}}$

* Isolate:

$$\Rightarrow \tan(x) = -1$$

* Quad check:

Q II Q III

* Ref $\angle$: Note: we want

$$\tan(x) = 1 \Rightarrow \text{that is} \Rightarrow \frac{\sin(x)}{\cos(x)} = 1$$

* The only way a fraction can equal zero is if numerator and denominator are equal:

$\Rightarrow$ the Ref $\angle$ with $\sin(x) = \cos(x)$,

that is, y-coord = x-coord is

$$x_R = \pi/4$$

* solve

$$x = 3\pi/4 \quad x = 7\pi/4$$

#3. $4 \sin^2(x) + 2 = 5$ in $\underline{\underline{[0, 2\pi]}}$

* Isolate

$$\Rightarrow 4 \sin^2(x) = 3$$

$$\Rightarrow \sin^2(x) = \frac{3}{4}$$

$$\Rightarrow \sin(x) = \pm \sqrt{\frac{3}{4}}$$

$$\Rightarrow \sin(x) = \pm \frac{\sqrt{3}}{2}$$

* Quad. check

all 4

Quads.

* Ref. $\angle$

$$x_R = \pi/3$$

* solve

$$x = \pi/3$$

$$x = 2\pi/3$$

$$x = 4\pi/3$$

$$x = 5\pi/3$$

$$x = \frac{3\pi}{4} \pm 2\pi k$$

$$x = \frac{7\pi}{4} \pm 2\pi k$$

#4 $(\cos(x)+1)(\tan(x)-1)=0$ in $[0, 2\pi)$ of two

⇒ NOTE: set each factor

equal to zero:

⇒ $\cos(x)+1=0$ or $\tan(x)-1=0$

⊗ solve each individually

⇒ ⊗ $\cos(x)+1=0$

⊗ ISOLATE:

$\cos(x)=-1$

Quadrantal:

⇒ $\boxed{x=\pi}$

⊗ $\tan(x)-1=0$

⊗ Isolate

$\tan(x)=1$

⊗ Quadr. Check:

QI QIII

⊗ Ref. x:

$x_r = \pi/4$

⊗ solve

$\boxed{\begin{matrix} x = \pi/4 \\ x = 5\pi/4 \end{matrix}}$

so $\boxed{\begin{matrix} x = \pi \\ x = \pi/4 \\ x = 5\pi/4 \end{matrix}}$

NOTE: to solve

$\tan(x)=1$

we want

$\frac{\sin(x)}{\cos(x)}=1$

that is we want

$\sin(x) = \cos(x)$

that is

y-coord = x-coord

⇒ $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

thus $x_r = \pi/4$
etc

#5 $5 \tan(x) + 1.6 = -11.4$ in $[0, 2\pi)$

pg 3 hwee

* Isolate:

$$\Rightarrow 5 \tan(x) = -13$$

$$\Rightarrow \tan(x) = -\frac{13}{5}$$

* Quadrant check

QII QIII

* Ref 4 (calculator!)

$$x_R = \tan^{-1}\left(\frac{13}{5}\right)$$

$$\approx 1.20$$

* Solve

$$QII: x = \pi - 1.20 = 1.94$$

$$QIII: x = 2\pi - 1.20 = 5.08$$

#6 $3 \sec(x) + 10 = 4$

* Isolate:

in $[0, 2\pi)$

$$\Rightarrow 3 \sec(x) = -6$$

$$\Rightarrow \sec(x) = -2$$

RECIPROCAL!

$$\Rightarrow \cos(x) = -\frac{1}{2}$$

* Quadrant check:

QII QIII

* Ref 4:

$$x_R = \pi/3$$

* Solve:

$$x = 2\pi/3$$

$$x = 4\pi/3$$

#7 $\sin(x) = 0$ in $(-\infty, \infty)$

* Quadrantal

$$\Rightarrow x = 0$$

$$x = \pi$$

$\Rightarrow$ so

$$x = 0 \pm 2\pi k$$

$$x = \pi \pm 2\pi k$$

#8 $2 \sin(x) \tan(x) + \tan(x) = 0$ in $[0, 2\pi)$

pg 40 nr

First FACTOR OUT $\tan(x)$

$\Rightarrow \tan(x) (2 \sin(x) + 1) = 0$

* set each equal to zero
and solve separately

$\tan(x) = 0$

Quadrantal

$x = 0$

$x = \pi$

$2 \sin(x) + 1 = 0$

* Isolate

$\Rightarrow \sin(x) = -\frac{1}{2}$

* Quad. check

Q III Q IV

* Ref. $\angle$:

$x_R = \pi/6$

* Solve

$x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6}$

SO we have

$x = 0 \pm 2\pi k$

$x = \pi \pm 2\pi k$

$x = \frac{7\pi}{6} \pm 2\pi k$

$x = \frac{11\pi}{6} \pm 2\pi k$

oops!
Didn't need this!

the solutions ARE

$x = 0, x = \pi, x = \frac{7\pi}{6}, x = \frac{11\pi}{6}$

#9 $4.3 \sin(x) + 2.1 = 3.9$

in $(-\infty, \infty)$

* Isolate

$\Rightarrow 4.3 \sin(x) = 1.8$

$\Rightarrow \sin(x) = \frac{1.8}{4.3}$

* Quad. check

Q I Q II

* Ref. $\angle$:

$x_R = \sin^{-1}\left(\frac{1.8}{4.3}\right)$

$= .43$

* Solve:

Q I $x = .43$

Q II $x = \pi - .43$
 $= 2.71$

SO

$x = .43 \pm 2\pi k$

$x = 2.71 \pm 2\pi k$

~~#10~~ #1 sorry for the numbering issues
 these came from two different sources. pg 51ve

$2\cos(x) + 8 = 7$ in $(-\infty, \infty)$

* Isolate
 $\cos(x) = \frac{-1}{2}$

* Quad check
 QII QIII

* Ref. $\angle$:
 $x_R = \pi/3$

* Solve:
 QII $x = \frac{2\pi}{3}$
 QIII $x = \frac{4\pi}{3}$

so we have
 $x = \frac{2\pi}{3} \pm 2\pi k$
 $x = \frac{4\pi}{3} \pm 2\pi k$

#2 $\tan(x) + 1 = 0$ in $[0, 2\pi)$

* Isolate =
 $\Rightarrow \tan(x) = -1$

* Quad check
 QII QIII

* Ref. $\angle$: Again, this requires $\sin(x) = \cos(x)$
 so $x_R = \pi/4$
 $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

* Solve
 $x = \frac{3\pi}{4}$ $x = \frac{7\pi}{4}$

#3 $3\sec(x) + 10 = 4$ in $[0, 2\pi)$

NOTE: This is the same as #6
 so let's change the problem to:

$3\sec(x) + 11 = 4$

* Isolate:
 $\sec(x) = \frac{-7}{3}$

* Reciprocal
 $\cos(x) = -\frac{3}{7}$

* Quad. check
 QII QIII

* Ref. $\angle$: use calculator!

$x_R = \cos^{-1}(\frac{3}{7})$
 $x_R = 1.13$

* Solve:

QII: $x = \pi - 1.13 \Rightarrow x = 2.01$
 QIII: $x = \pi + 1.13 \Rightarrow x = 4.27$

#4 $\sin(x) = 0$ in $(-\infty, \infty)$

$x = 0$
 $x = \pi$

so $x = 0 \pm 2\pi k$
 $x = \pi \pm 2\pi k$

fix

#5 $2\sin(x)\cos(x) + \cos(x) = 0$ in $[2\pi]$

⊗ FACTOR OUT $\cos(x)$

$\Rightarrow \cos(x)(2\sin(x) + 1) = 0$

$\cos(x) = 0$
Quadrants

$x = \frac{\pi}{2}$
 $x = \frac{3\pi}{2}$

OR $2\sin(x) + 1 = 0$

⊗ ISOLATE =

$\sin(x) = -\frac{1}{2}$

OR ⊗ Quad. check

QIII QIV

⊗ Ref $\angle$:

$x_R = \frac{\pi}{6}$

$x = \frac{7\pi}{6}$
 $x = \frac{11\pi}{6}$

$\Rightarrow$ so we have

$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$

$x = \frac{7\pi}{6}$

$x = \frac{11\pi}{6}$

#6. sorry. Another repeated item. in $(-\infty, \infty)$
solve this instead: $2.7\csc(x) - 5.4 = 3.6$

⊗ $2.7\csc(x) - 5.4 = 3.6$

⊗ Isolate

$\Rightarrow 2.7\csc(x) = 9$

$\Rightarrow \csc(x) = \frac{9}{2.7}$

⊗ Reciprocal

$\sin(x) = \frac{2.7}{9}$

⊗ Quad. check

QI QII

⊗ Ref $\angle$: $x_R = \sin^{-1}\left(\frac{2.7}{9}\right)$
 $= .30$

⊗ solve

QI $x = .30$

QII $x = \pi - .30 = 2.84$

so $x = .30 \pm 2\pi k$
 $x = 2.84 \pm 2\pi k$