(I) (1) $\int (4\sqrt{x} - 6x^2 + 7)\,dx = \int (4x^{1/2} - 6x^2 + 7)\,dx$

$= \dfrac{4x^{3/2}}{3/2} - \dfrac{6x^3}{3} + 7x + C$

$= \boxed{\frac{8}{3}x^{3/2} - 2x^3 + 7x + C}$

(2) $\int \left(\dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt[3]{x}}\right)dx = \int (x^{-1/2} - x^{-1/3})\,dx$

$= \dfrac{x^{1/2}}{1/2} - \dfrac{x^{2/3}}{2/3} + C$

$= \boxed{2x^{1/2} - \frac{3}{2}x^{2/3} + C}$

(3) $\int 8x\cos(2x^2)\,dx = 2\int \cos(u)\,du$

Let $u = 2x^2$

$\Rightarrow du = 4x\,dx$

$\Rightarrow 2du = \underline{8x\,dx}$

$= 2\sin(u) + C$

$= \boxed{2\sin(2x^2) + C}$

(4) $\int \left(\dfrac{x-1}{\sqrt[3]{x+5}}\right)dx = \int (x-1)(x+5)^{-1/3}\,dx$

NOTE: Both factors are linear.

Let $u = x+5 \Rightarrow u-5 = x$

$\Rightarrow du = dx$

$= \int (u-5-1)(u)^{-1/3}\,du$

$= \int (u-6)(u)^{-1/3}\,du$

continued

④ CONT.

$$= \int (u-6)(u)^{-1/3}\,du$$

$$= \int \left(u^{2/3} - 6u^{-1/3}\right) du$$

$$= \frac{u^{5/3}}{5/3} - \frac{6u^{2/3}}{2/3} + C$$

$$= \boxed{\frac{3}{5}(x+5)^{5/3} - 9(x+5)^{2/3} + C}$$

⑤ $\int \sec(x)\left(\cos^2(x) + \sec(x)\right) dx =$ ⇐ distribute:

$$= \int \left[\sec(x)\cdot\cos^2(x) + \sec^2(x)\right] dx$$

$$= \int \left[\cos(x) + \sec^2(x)\right] dx$$

$$= \boxed{\sin(x) + \tan(x) + C}$$

⑥ $\int x e^{(2x^2+1)}\,dx = \frac{1}{4}\int e^u\,du$

Let $u = 2x^2+1$

$\Rightarrow du = 4x\,dx$

$\Rightarrow \frac{1}{4}du = x\,dx$

$$= \frac{1}{4}e^u + C$$

$$= \boxed{\frac{1}{4}e^{(2x^2+1)} + C}$$

(7) $\displaystyle\int \frac{x+2}{3x^2+12x+17}\,dx = \frac{1}{6}\int \frac{1}{u}\,du$

Let $u = 3x^2+12x+17$

$\Rightarrow du = (6x+12)\,dx$

$\Rightarrow du = 6(x+2)\,dx$

$\Rightarrow \frac{1}{6}\,du = (x+2)\,dx$

$= \frac{1}{6}\ln|u| + C$

$= \boxed{\frac{1}{6}\ln|3x^2+12x+17| + C}$

(8) $\displaystyle\int \frac{1}{e^{\sqrt{x}}\sqrt{x}}\,dx = \int e^{-x^{1/2}} \cdot x^{-1/2}\,dx = -2\int e^u\,du$

Let $u = -x^{1/2}$

$du = -\frac{1}{2}x^{-1/2}\,dx$

$\Rightarrow -2\,du = x^{-1/2}\,dx$

$= -2e^u + C$

$= \boxed{-2e^{(-x^{1/2})} + C}$

OR

$= \boxed{\frac{-2}{e^{\sqrt{x}}} + C}$

(II)

(1) $\displaystyle\int_{-2}^{1}(8x^3-6x)\,dx = \left[\frac{8x^4}{4} - \frac{6x^2}{2}\right]_{-2}^{1}$

$= \left[2x^4 - 3x^2\right]_{-2}^{1}$

$= (2(1)^4 - 3(1)^2) - (3(-2)^4 - 3(-2)^2)$

$= (2-3) - (3(16) - 3(4))$

$= (-1) - (48-12)$

$= -1 - 36 = \boxed{-37}$

② $\int_1^3 8x\left(\sqrt[3]{x^2-1}\right)dx = \int_1^3 8x(x^2-1)^{1/3}\,dx$

Let $u = x^2-1$

$\Rightarrow du = 2x\,dx$

$\Rightarrow 4du = 8x\,dx$

~~Limits~~ Limits

upper: $u = (3)^2-1 = 8$

Lower: $u = (1)^2-1 = 0$

$= 4\int_0^8 u^{1/3}\,du$

$= \left[4\frac{3}{2}u^{2/3}\right]_0^8$

$= \left[6u^{2/3}\right]_0^8$

$= \left(6(8)^{2/3}\right) - \left(6(0)^{2/3}\right)$

$= (6(4)) - 0$

$= \boxed{24}$

③ $\int_{-1}^{0} (x+2)(3-x)^4\,dx$

NOTE: Both Factors Are Linear

Let $u = 3-x \Rightarrow u-3 = -x$

$\Rightarrow du = -dx \quad \Rightarrow -u+3 = x$

$\Rightarrow -du = dx$

Limits:

upper: $u = 3-(0) = 3$

Lower $u = 3-(-1) = 4$

SO: $= -\int_4^3 (-u+3+2)(u)^4\,du$

continued

③ CONT. $= -\int_4^3 (-u+3+2)\,u^4\,du$

$$= -\int_4^3 (-u^5 + 5u^4)\,du$$

$$= -\left[\frac{-u^6}{6} + \frac{5u^5}{5}\right]_4^3$$

$$= -\left[\left(\frac{-(3)^6}{6} + (3)^5\right) - \left(\frac{-(4)^6}{6} + (4)^5\right)\right]$$

$$= -\left[\left(\frac{-729}{6} + 243\right) - \left(\frac{-4096}{6} + 1024\right)\right]$$

$$= -\left[\frac{3367}{6} - 781\right] \approx -\left[-219.83\right]$$

$$= \boxed{219.83}$$

---

④ $\displaystyle\int_{-\pi/4}^{\pi/4} 4\cos(2x)\,dx$

Let $u = 2x$
$du = 2dx$
$2du = 4dx$

Limits:
upper $u = 2(\pi/4) = \pi/2$
lower $u = 2(-\pi/4) = -\pi/2$

$$= 2\int_{-\pi/2}^{\pi/2} \cos(u)\,du$$

$$= 2\left[\sin(u)\right]_{-\pi/2}^{\pi/2}$$

Continued

(4) cont.

$$= 2\left[\left(\sin(\pi/2)\right) - \left(\sin(-\pi/2)\right)\right]$$

$$= 2\left[(1) - (-1)\right]$$

$$= 2(2)$$

$$= \boxed{4}$$

III (1) $f(x) = \ln(\cos(x))$

$$\Rightarrow f'(x) = \boxed{\frac{1}{\cos(x)} \cdot (-\sin(x))} = \boxed{-\frac{\sin(x)}{\cos(x)}} = \boxed{-\tan(x)}$$

(2) $f(x) = x^{(x^2-3)} e^{2x} \Rightarrow y = x^{(x^2-3)} \cdot e^{2x}$

Logarithmic Diff.

$$\Rightarrow \ln(y) = \ln\left[x^{(x^2-3)} \cdot e^{2x}\right]$$

$$= \ln\left(x^{(x^2-3)}\right) + \ln\left(e^{2x}\right)$$

$$= (x^2-3) \cdot \ln(x) + 2x$$

$$\Rightarrow \frac{1}{y} \cdot y' = \underbrace{(x^2-3)\left(\frac{1}{x}\right) + \ln(x)(2x)}_{\text{product rule}} + 2$$

$$\Rightarrow y' = \left(\frac{x^2-3}{x} + 2x\ln(x) + 2\right)(y)$$

$$= \boxed{\left(\frac{x^2-3}{x} + 2x\ln(x) + 2\right)\left(x^{(x^2-3)} \cdot e^{2x}\right)}$$

pg 7 even

(3) $f(x) = \dfrac{e^x+1}{\ln(x)} \Rightarrow f'(x) = \boxed{\dfrac{\ln(x)(e^x) - (e^x+1)(\frac{1}{x})}{(\ln(x))^2}}$

quotient rule

(4) $f(x) = e^{\sqrt{1+x}}$

chain rule $= e^{(1+x)^{1/2}}$

chain

$\Rightarrow f'(x) = \boxed{e^{(1+x)^{1/2}} \cdot \frac{1}{2}(1+x)^{-1/2}(1)}$

chain

(5) $f(x) = \ln(\sqrt{x^3-1})$

chain rule $= \ln(x^3-1)^{1/2}$

$= \frac{1}{2}\ln(x^3-1)$

so $f'(x) = \boxed{\frac{1}{2} \cdot \frac{1}{x^3-1}(3x^2)}$

or

$= \boxed{\dfrac{3x^2}{2(x^3-1)}}$

(6) $f(x) = x^2 \cdot \ln(x^3)$

$= x^2 \cdot 3\ln(x)$ ← product rule

so $f'(x) = \boxed{x^2 \cdot 3 \cdot \frac{1}{x} + 3\ln(x)(2x)}$

or

$= \boxed{3x + 6x\ln(x)}$