

Solutions:

① B compare degrees $\left. \begin{array}{l} \text{deg num} \rightarrow 1 \\ \text{deg denom} \rightarrow 1 \end{array} \right\} \Rightarrow \text{case 2} \Rightarrow y = \frac{4}{2}$

② E set denom = 0 $\Rightarrow x+5=0 \Rightarrow \boxed{x=-5} \leftarrow \Rightarrow \boxed{y=2}$

③ A set num = 0 $\Rightarrow x-5=0 \Rightarrow \boxed{x=5} \leftarrow$

④ A set $f=0 \Rightarrow x^3-4x^2-4x+16=0$ Factor by grouping
 $\Rightarrow x^3-4x^2-4x+16=0$
 $\Rightarrow x^2(x-4)-4(x-4)=0$
 $\Rightarrow (x-4)(x^2-4)=0$
 $\Rightarrow (x-4)(x+2)(x-2)=0$
 $\Rightarrow \boxed{x=4} \quad \boxed{x=-2} \quad \boxed{x=2}$

⑤ C $(x+3)^3=0 \rightarrow \text{mult} = \underline{3}$

⑥ A $x+7=0 \rightarrow \text{mult} = \underline{1}$

⑦ B compare degrees $\left. \begin{array}{l} \text{deg num} \rightarrow 2 \\ \text{deg denom} \rightarrow 2 \end{array} \right\} \rightarrow \text{fraction of coefficients} \Rightarrow y = \frac{-10}{1} = \boxed{y=-10}$

⑧ C set denom = 0 $\Rightarrow x^2-6x+5=0$ Factor (FOIL)
 $\Rightarrow (x-5)(x-1)=0$
 $\Rightarrow x-5=0 \quad x-1=0$
 $\Rightarrow \boxed{x=5} \quad \boxed{x=1}$

⑨ A set numerator = 0 $\Rightarrow 5x+10=0$
 $\Rightarrow 5x=-10$
 $\Rightarrow x=-2 \Rightarrow (-2, 0)$

⑩ C compare degrees $\left. \begin{array}{l} \text{deg num} \rightarrow 0 \\ \text{deg denom} \rightarrow 2 \end{array} \right\} \Rightarrow \text{case 1} \Rightarrow \boxed{x\text{-axis is asymptote}}$

① D. we must have a function with degree of numerator less than degree of denominator pg 240
 $A \rightarrow \frac{2}{2}$ NO $B \rightarrow \frac{2}{2}$ NO $C \rightarrow \frac{2}{2}$ NO $D \rightarrow \frac{1}{2}$ yes

② ① $f(x) = x^3 + x^2 - 25x - 25$ HIGHER DEGREE POLYNOMIAL

* Degree $\Rightarrow 3$

* Lead Coef $\Rightarrow$ pos

* Intercepts

$\rightarrow$ y-int: set $x=0$

$$\Rightarrow f(0) = (0)^3 + (0)^2 - 25(0) - 25$$

$$= -25 \rightarrow (0, -25)$$

$\rightarrow$ x-int: set $f=0$

$$\Rightarrow x^3 + x^2 - 25x - 25 = 0 \quad \text{Factor by grouping}$$

$$\Rightarrow x^2(x+1) - 25(x+1) = 0$$

$$\Rightarrow (x+1)(x^2 - 25) = 0$$

$$\Rightarrow (x+1)(x+5)(x-5) = 0$$

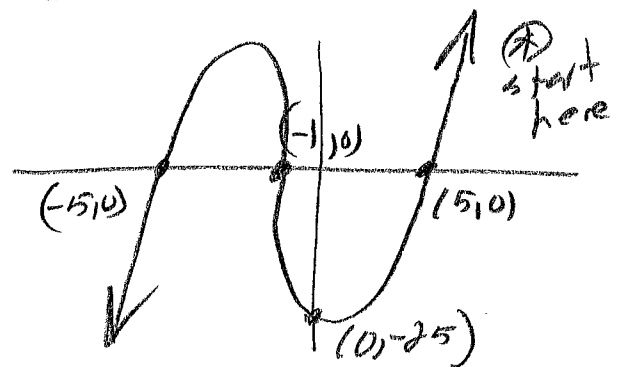
$$\Rightarrow x+1=0 \quad x+5=0 \quad x-5=0$$

$$\Rightarrow x=-1 \quad x=-5 \quad x=5$$

$$\Rightarrow (-1, 0) \quad (-5, 0) \quad (5, 0)$$

* Multiplicity

1 1 1



② $f(x) = \frac{2x-6}{x+3}$

* Vert. asymp. set denom = 0
 $\Rightarrow x+3=0$
 $\Rightarrow x=-3$

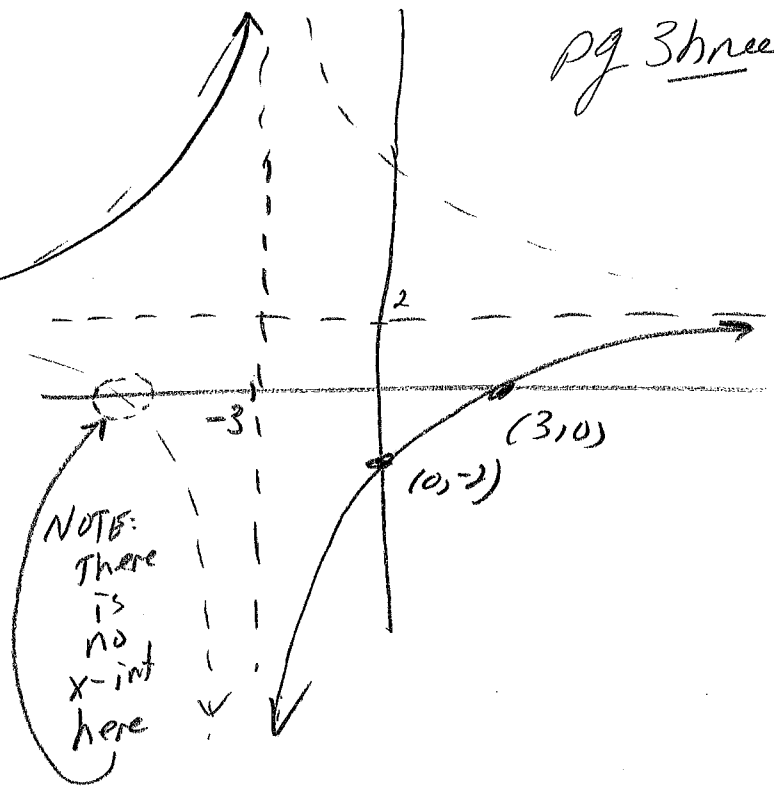
* horiz. asymp.: compare degrees
 deg num $\rightarrow 1$ } $\rightarrow$ case 2
 deg denom $\rightarrow 1$ }
 fraction of coefficients $\rightarrow y = \frac{2}{1}$
 $\Rightarrow y=2$

* y-int: set $x=0$

$f(0) = \frac{2(0)-6}{(0)+3} = \frac{-6}{3} = -2$
 $\rightarrow (0, -2)$

* x-int: set num = 0

$\Rightarrow 2x-6=0 \Rightarrow 2x=6 \Rightarrow x=3$
 $\Rightarrow (3, 0)$



③ $f(x) = (-x+2)(x+1)^2$

* degree: $1+2=3$

* lead coef.

$(-)(+) \Rightarrow \text{NEB}$

* y-int: set $x=0$

$f(0) = (- (0) + 2)(0+1)^2$
 $= (2)(1)^2$
 $= 2 \Rightarrow (0, 2)$

* x-int: set $f=0$

$\Rightarrow (-x+2)(x+1)^2=0$

$\Rightarrow \textcircled{1} -x+2=0$ OR $\textcircled{2} (x+1)^2=0$

$\Rightarrow -x = -2$

$\Rightarrow x=2$

$\Rightarrow (2, 0)$

$\Rightarrow x+1=0$

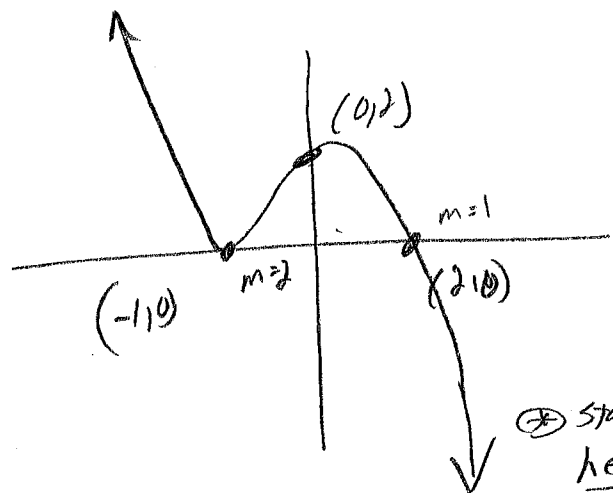
$\Rightarrow x=-1$

$\Rightarrow (-1, 0)$

* Multiplicity

1

2



* start here

④ $f(x) = \frac{x+1}{x^2-16}$

ⓧⓧ Vert. asymptotes set denom = 0

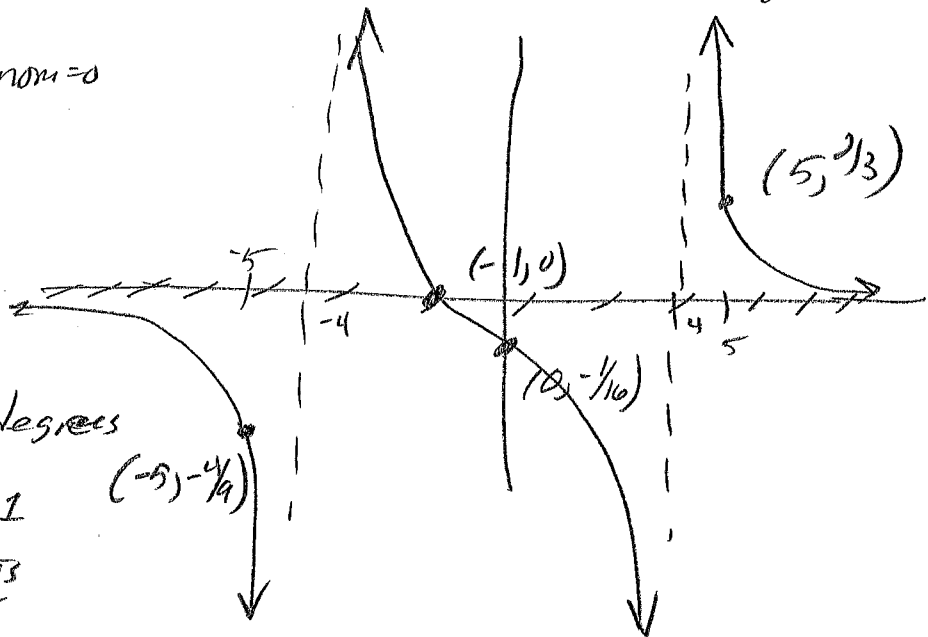
$$\Rightarrow x^2 - 16 = 0$$

$$\Rightarrow x^2 = 16$$

$$\Rightarrow x = \pm \sqrt{16} \Rightarrow x = 4 \text{ or } x = -4$$

ⓧⓧ Horiz. asympt. compare degrees

$$\begin{matrix} \text{deg num} \rightarrow 1 \\ \text{deg denom} \rightarrow 2 \end{matrix} \Rightarrow \begin{matrix} \text{Case 1} \\ \text{x-axis} \end{matrix}$$



ⓧⓧ y-int: set $x=0$

$$f(0) = \frac{(0)+1}{(0)^2-16} = -\frac{1}{16}$$

$$\Rightarrow (0, -\frac{1}{16})$$

ⓧⓧ x-int: set num = 0

$$\Rightarrow x+1=0$$

$$\Rightarrow \underline{x = -1} \Rightarrow (-1, 0)$$

Plot pts Let $x = -5$

$$\begin{aligned} f(-5) &= \frac{(-5)+1}{(-5)^2-16} \\ &= \frac{-4}{9} \Rightarrow (-5, -\frac{4}{9}) \end{aligned}$$

Let $x = 5$

$$\begin{aligned} f(5) &= \frac{(5)+1}{(5)^2-16} = \frac{6}{9} = \frac{2}{3} \\ &\Rightarrow (5, \frac{2}{3}) \end{aligned}$$



⑤ $f(x) = -x^4 - 6x^3 - 5x^2$

pg 5, ve

- ⊗ degree = 4
- ⊗ L.C. → Neg

⊗ Intercepts:

→ y-int: set $x=0$

$$f(0) = (0) - (0) - (0)$$

$$= 0 \Rightarrow (0,0)$$

→ x-int: set $f=0$

$$-x^4 - 6x^3 - 5x^2 = 0$$

Factor out $-x^2$

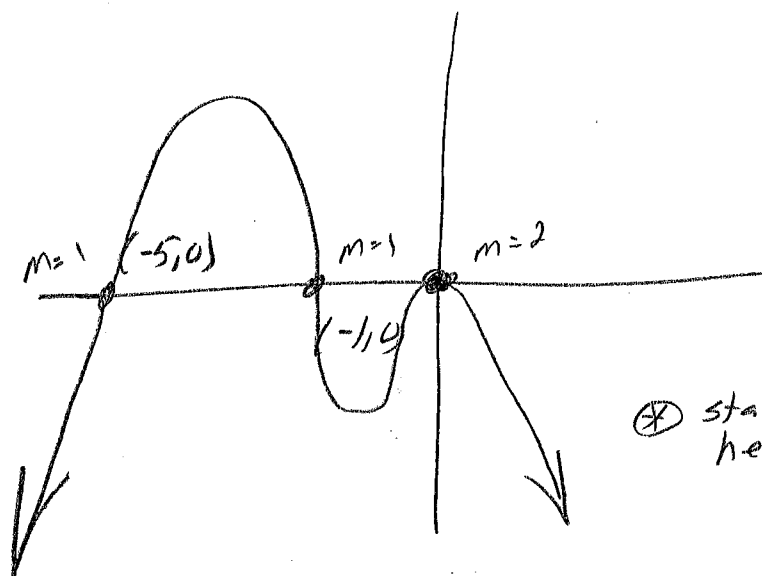
$$\Rightarrow -x^2(x^2 + 6x + 5) = 0$$

Factor more $\Rightarrow (x^2)(x+5)(x+1) = 0$

$$\Rightarrow -x^2 = 0 \quad x+5 = 0 \quad x+1 = 0$$

$$\Rightarrow x = 0 \quad x = -5 \quad x = -1$$

$$\Rightarrow (0,0) \quad (-5,0) \quad (-1,0)$$



⊗ start here

⑥ $f(x) = \frac{x-2}{x^2-9}$

⊗ Vert. asympt = set denom = 0

$$\Rightarrow x^2 - 9 = 0$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \pm\sqrt{9} = \pm 3$$

⊗ Horiz. asympt.

Compare degrees

deg num $\rightarrow 1$ case 1

deg denom $\rightarrow 2$ $\rightarrow$ x-axis

⊗ y-int: set $x=0$

$$f(0) = \frac{(0)-2}{(0)^2-9} = \frac{-2}{-9} = \frac{2}{9} \Rightarrow (0, \frac{2}{9})$$

⊗ x-int: set num = 0

$$\Rightarrow x-2=0 \Rightarrow x=2 \Rightarrow (2,0)$$

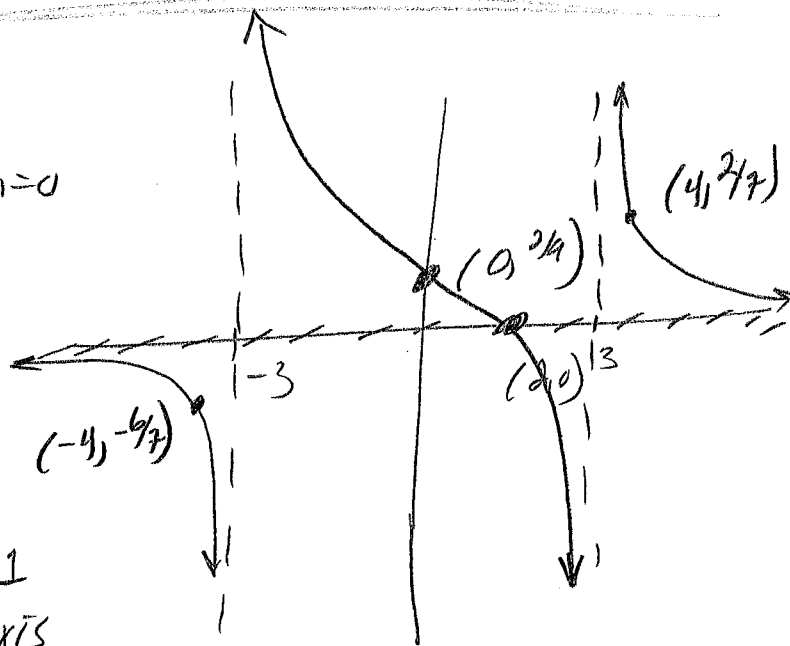
Plot points

⊗ Far left

$$f(-4) = \frac{(-4)-2}{(-4)^2-9} = \frac{-6}{7} \Rightarrow (-4, -\frac{6}{7})$$

⊗ Far Right

$$f(4) = \frac{4-2}{(4)^2-9} = \frac{2}{7} \Rightarrow (4, \frac{2}{7})$$



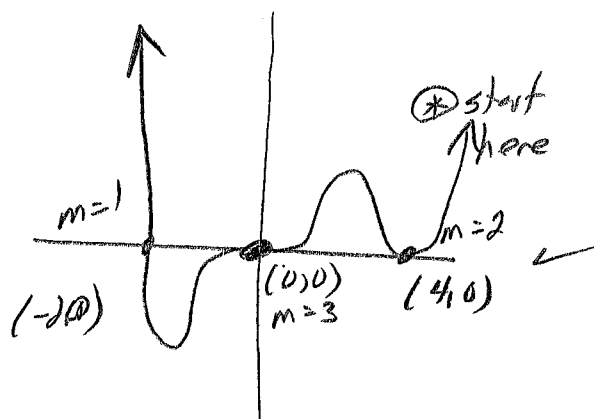
#7, $f(x) = x^3(x-4)^2(x+2)^1$

④ Degree $3+2+1 = 6$

④ Lead Coef (POS)(POS)(POS) $\rightarrow$ POS

④ y-int: set $x=0$

$\rightarrow f(0) = (0)^3(0-4)^2(0+2)^1$
 $= (0)(16)(2)$
 $= 0 \rightarrow (0,0)$



④ x-int: set $f=0$

$x^3=0$ or $(x-4)^2=0$ or $(x+2)^1=0$

$\Rightarrow x=0$

$x=4$

$x=-2$

④ $(0,0)$

$(4,0)$

$(-2,0)$

Multiply

3

2

1

Note
 this was just
 for practice

ONE EXTRA EXAMPLES:

graph $f(x) = x^2 - 8x + 15$

④ QUADRATIC

④ Lead Coef $\Rightarrow$ POS $\rightarrow$ $\uparrow \uparrow$

④ Vertex $x = -\frac{b}{2a}$
 $= -\frac{(-8)}{2(1)}$
 $= 4$

$y = f(4)$

$= (4)^2 - 8(4) + 15$

$= 16 - 32 + 15$

$= -1 \Rightarrow (4, -1)$

④ y-int set $x=0$

$f(0) = (0)^2 - 8(0) + 15$
 $= 15$

$\rightarrow (0, 15)$

④ x-int set $f=0$

$x^2 - 8x + 15 = 0$

Quadratic Formula

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$

$= \frac{8 \pm \sqrt{64 - 60}}{2}$

$= \frac{8 \pm \sqrt{4}}{2}$

$= \frac{8 \pm 2}{2}$

$\frac{8+2}{2}$ or $\frac{8-2}{2}$

$\frac{10}{2}$ or $\frac{6}{2}$

$x=5$ or $x=3$

$(5,0)$ or $(3,0)$

