Solutions

I

① $\ln(x+2) - \ln(x-4) = \boxed{\ln\left(\frac{x+2}{x-4}\right)}$

② $\ln(3x) + 4\ln(x+3) - \ln(x-1) =$

$= \ln(3x) + \ln(x+3)^4 - \ln(x-1)$

$= \boxed{\ln\left(\frac{(3x)(x+3)^4}{(x-1)}\right)}$

③ $5\ln(x+5) + \frac{1}{5}\ln(x-5) =$

$= \ln(x+5)^5 + \ln(x-5)^{1/5}$

$= \boxed{\ln\left((x+5)^5 (x-5)^{1/5}\right)}$

④ $3\ln(x) - \frac{1}{3}\ln(x-4) - \frac{1}{2}\ln(2x) =$

$= \ln(x^3) - \ln(x-4)^{1/3} - \ln(2x)^{1/2}$

$= \boxed{\ln\left(\frac{x^3}{(x-4)^{1/3}(2x)^{1/2}}\right)}$

II.

(5) $5^{(x-4)} + 17 = 128$

(*) Isolate

$\Rightarrow 5^{(x-4)} = 111$

$\Rightarrow \ln(5^{(x-4)}) = \ln(111)$

$\Rightarrow (x-4)\ln(5) = \ln(111)$

$\Rightarrow x\ln(5) - 4\ln(5) = \ln(111)$

$\Rightarrow x\ln(5) = \ln(111) + 4\ln(5)$

$\Rightarrow x = \boxed{\dfrac{(\ln(111) + 4\ln(5))}{(\ln(5))}}$

$\approx \boxed{6.93}$

(6) $10e^{(3x-8)} = 600$

$\Rightarrow e^{(3x-8)} = 60$

$\Rightarrow \ln(e^{(3x-8)}) = \ln(60)$

$\Rightarrow 3x - 8 = \ln(60)$

$\Rightarrow 3x = \ln(60) + 8$

$\Rightarrow x = \boxed{\dfrac{(\ln(60) + 8)}{3}}$

$\approx \boxed{4.03}$

(7) $5^{(-7x)} = 99$

$\Rightarrow \ln(5^{(-7x)}) = \ln(99)$

$\Rightarrow -7x \cdot \ln(5) = \ln(99)$

$\Rightarrow x = \boxed{\frac{(\ln(99))}{(-7\ln(5))}}$

$\approx \boxed{-.41}$

(8) $e^{(x+17)} - 12 = 488$

$\Rightarrow e^{(x+17)} = 500$

$\Rightarrow \ln(e^{(x+17)}) = \ln(500)$

$\Rightarrow x + 17 = \ln(500)$

$\Rightarrow x = \boxed{\ln(500) - 17}$

$\approx \boxed{-10.79}$

(9) $43^{(x-3)} = 82^{(2x+5)}$

$\Rightarrow \ln(43^{(x-3)}) = \ln(82^{(2x+5)})$

$\Rightarrow (x-3)\cdot\ln(43) = (2x+5)\ln(82)$

$\Rightarrow x\ln(43) - 3\ln(43) = 2x\ln(82) + 5\ln(82)$

$\Rightarrow x\ln(43) - 2x\ln(82) = 5\ln(82) + 3\ln(43)$

$\Rightarrow x = \boxed{\dfrac{(5\ln(82) + 3\ln(43))}{(\ln(43) - 2\ln(82))}}$

$\approx \boxed{-6.59}$

(10) $e^x \cdot e^{(x-8)} + 5 = 6$

$\Rightarrow e^{(x+x-8)} + 5 = 6$

$\Rightarrow e^{(2x-8)} + 5 = 6$

$\Rightarrow e^{(2x-8)} = 1$

$\Rightarrow \ln(e^{(2x-8)}) = \ln(1)$

$\Rightarrow 2x - 8 = \ln(1)$

$\Rightarrow 2x = \ln(1) + 8$

$\Rightarrow x = \boxed{\dfrac{\ln(1) + 8}{2}}$ NOTE: $\ln(1) = 0$

$= \dfrac{8}{2}$

$= \boxed{4}$

(11). $e^{2x} - e^x - 12 = 0$

$\Rightarrow (e^x)^2 - (e^x) - 12 = 0$

Let $u = e^x$

$\Rightarrow u^2 - u - 12 = 0$

$(u-4)(u+3) = 0$ or $u = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-12)}}{2(1)}$

$u = 4$ or $u = -3$ $= \frac{1 \pm \sqrt{49}}{2}$

$= \frac{1 \pm 7}{2}$

$= \frac{8}{2}$ or $\frac{-6}{2}$

$= 4$ or -3

so from the "Let" statement:

$\Rightarrow 4 = e^x$ or $-3 = e^x$

$\Rightarrow \ln(4) = \ln(e^x)$ | $\rightarrow \ln(-3) = \ln(e^x)$

$\Rightarrow \ln(4) = x$ | $\rightarrow$ ~~$\ln(-3) = x$~~

$\boxed{1.39 \approx x}$ | NO VALUE

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(12). $e^{2x} - e^{(x+2)} - 1.2 = 0$

$\Rightarrow (e^x)^2 - e^x \cdot e^2 - 1.2 = 0$

$\Rightarrow (e^x)^2 - e^2(e^x) - 1.2 = 0$

Let $u = e^x$

$\Rightarrow u^2 - e^2 u - 1.2 = 0$ note!

$\Rightarrow u = \frac{-(-e^2) \pm \sqrt{(-e^2)^2 - 4(1)(-1.2)}}{2(1)}$

$\approx \frac{e^2 \pm \sqrt{59.40}}{2}$

$\approx \frac{e^2 \pm 7.71}{2}$

$= \frac{e^2 + 7.71}{2}$ or $\frac{e^2 - 7.71}{2}$

≈ 7.55 or $-.16$

from the Let statement

$\Rightarrow 7.55 = e^x$ or $-.16 = e^x$

$\Rightarrow \ln(7.55) = \ln(e^x)$ | $\Rightarrow \ln(-.16) = \ln(e^x)$

$\ln(7.55) = x$ | $\Rightarrow$ ~~$\ln(-.16) = x$~~

$\boxed{2.02} \approx x$ | NO VALUE

(13) $\ln(x) + \ln(x-7) = 3$

$\Rightarrow \ln[(x)(x-7)] = 3$

$\Rightarrow \ln(x^2 - 7x) = 3$

$\Rightarrow e^{\ln(x^2-7x)} = e^3$

$\Rightarrow x^2 - 7x = e^3$

$\Rightarrow x^2 - 7x - e^3 = 0$

$\Rightarrow x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-e^3)}}{2(1)}$

$= \dfrac{7 \pm \sqrt{49 + 4e^3}}{2}$

$\approx \dfrac{7 \pm 11.37}{2}$

$= \dfrac{7+11.37}{2}$ or $\dfrac{7-11.37}{2}$

≈ 9.19 $\quad -2.19$

Must check!

If $x = 9.19$

$x = 9.19 > 0$ ✓

$x - 7 = 9.19 - 7 = 2.19 > 0$ ✓

so $\boxed{x = 9.19}$

If $x = -2.19$

$x = -2.19$ ~~> 0~~ no

so ~~$x = -2.19$~~

only solution is

$\boxed{x = 9.19}$

(14) $\ln(x-4) - \ln(x-2) = 3$

$\Rightarrow \ln\left(\dfrac{x-4}{x-2}\right) = 3$

$\Rightarrow e^{\ln\left(\frac{x-4}{x-2}\right)} = e^3$

$\Rightarrow \dfrac{x-4}{x-2} = e^3$

cross-multiply

$e^3(x-4) = 1(x-2)$

$e^3 x - e^3 4 = x - 2$

$e^3 \approx 20.09$ so we have

$\Rightarrow 20.09x - (20.09)(4) = x - 2$

$\Rightarrow 20.09x - 80.36 = x - 2$ (subtract x from both sides)

$\Rightarrow 19.09x - 80.36 = -2$ (add 80.36 to both sides)

$\Rightarrow 19.09x = 78.36$

$\Rightarrow x = \dfrac{78.36}{19.09}$

≈ 4.10

must check

If $x = 4.10$

$\Rightarrow x - 4 = 4.10 - 4 = .10 > 0$ ✓

$\Rightarrow x - 2 = 4.10 - 2 = 2.10 > 0$ ✓

so the solution is $x \approx 4.10$

(15) $\ln(x) + \ln(x+2) = 3$

$\Rightarrow \ln\big((x)(x+2)\big) = 3$

$\Rightarrow e^{\ln(x^2+2x)} = e^3$

$\Rightarrow x^2+2x = e^3$

$\Rightarrow x^2+2x-e^3 = 0$

$\Rightarrow x = \dfrac{-(2) \pm \sqrt{(2)^2 - 4(1)(-e^3)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{4+4e^3}}{2}$

$\approx \dfrac{-2 \pm \sqrt{84.36}}{2}$

$\approx \dfrac{-2 \pm 9.18}{2}$

$= \dfrac{7.18}{2}$ or $\dfrac{-11.18}{2}$

$= 3.59$ or -5.59

MUST CHECK!

If $x = 7.18$

$x = 7.18 > 0$ ←

$x+2 = 7.18+2 = 9.18 > 0$ ←

so $\boxed{x = 7.18}$

If $x = -5.59$

$x = -5.59$ ~~so~~ NO

so ~~$x = -5.59$~~

The only solution is $x = 7.18$

(16) $\ln(x+5) + \ln(x) = \ln(x+12)$

$\Rightarrow \ln\big((x+5)(x)\big) = \ln(x+12)$

$\Rightarrow \ln(x^2+5x) = \ln(x+12)$

$\Rightarrow e^{\ln(x^2+5x)} = e^{\ln(x+12)}$

$\Rightarrow x^2+5x = x+12$

$\Rightarrow x^2+4x-12 = 0$

$\Rightarrow (x+6)(x-2) = 0$

$\Rightarrow x = -6$ or $x = 2$

OR

$x = \dfrac{-(4) \pm \sqrt{(4)^2 - 4(1)(-12)}}{2(1)}$

$= \dfrac{-4 \pm \sqrt{64}}{2}$

$= \dfrac{-4 \pm 8}{2}$

$= \dfrac{-4-8}{2}$ or $\dfrac{-4+4}{2} \Rightarrow x = -6$ or $x = 2$

MUST CHECK

If $x = -6$

$x+5 = -6+5 = -1 \not> 0$ NO

so ~~$x = -6$~~

If $x = 2$

$x+5 = 2+5 = 7 > 0$ ←

$x = 2 > 0$ ←

$x+12 = 2+12 = 14 > 0$ ←

so $\boxed{x = 2}$

The only solution is $x = 2$