COLL ALG EXAM II SOLUTIONS.

I. ① C y-int ⇒ set $x=0$
$\Rightarrow f(0) = ((0)-2)^2((0)+1)$
$= (4)(1)$
$= 4 \Rightarrow (0,4)$

② E x-int set $f=0$
$\Rightarrow (x-2)^2(x+1) = 0$
$\Rightarrow x=2$ or $x=-1$
$\Rightarrow (2,0)$ or $(-1,0)$

③ D Largest exponent = 5

④ C shape is similar to $f(x) = x^3$
$\Rightarrow$ multiplicity = 3

⑤ B $(-1,0)$ has multiplicity = 1
⑥ C $(0,0)$ " " = 1
$(2,0)$ " " = 2
$\Big\} (x+1)^1(x)^1(x-2)^2$

Leading coefficient is positive → so NOT A

⑦ B y-int: set $x=0 \Rightarrow f(0) = 0+0-0+5 = 5 \Rightarrow (0,5)$

⑧ B set denominator $=0 \Rightarrow x^2-4=0 \Rightarrow x = \pm 2$

⑨ C Compare degrees: num → 1, denom → 1 ⇒ case II fraction of leading coefficients $= \frac{1}{1} =$
$\Rightarrow y =$

⑩ B Vertical asymptotes: $x = \pm 2 \Rightarrow$ denom $= x^2 - 4$
Horizontal asymptote is not x-axis so degrees are equal.

(1) $f(x) = x^2 - 6x + 5$

- Degree = 2 → ↗↗
- Lead. coef. pos.
- Vertex: $x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$

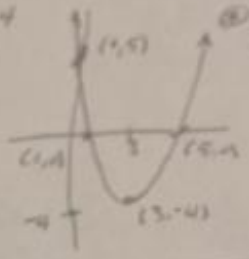
$y = f(3) = (3)^2 - 6(3) + 5 = 9 - 18 + 5 = -4 \Rightarrow (3, -4)$

y-int: set $x = 0$
$\Rightarrow f(x) = (0)^2 - 6(0) + 5 = 5$
$\Rightarrow (0, 5)$

x-int: set $f = 0$
$\Rightarrow x^2 - 6x + 5 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)} = \frac{6 \pm \sqrt{36 - 20}}{2} = \frac{6 \pm 4}{2}$

$\frac{6+4}{2} \nearrow 5 \Rightarrow (5, 0)$
$\frac{6-4}{2} \searrow 1 \Rightarrow (1, 0)$



So we have:

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(2) $f(x) = x^2(x-4)^3$ — Cubic shape

- Degree = 2+3 = 5
- Lead. coef. positive
- y-int: set $x = 0$

$f(0) = (0)^2(0-4)^3 = 0(-64) = 0 \Rightarrow (0, 0)$

x-int: set $f = 0$

$x^2(x-4)^3 = 0$

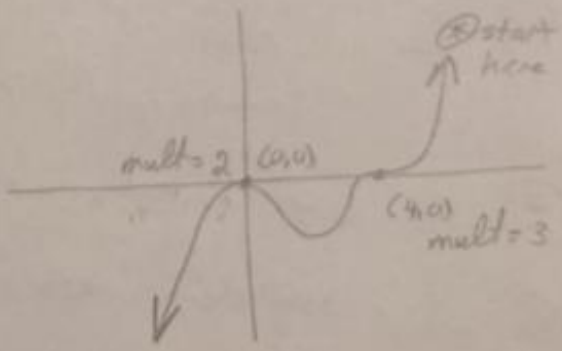
$\Rightarrow x^2 = 0$ or $(x-4)^3 = 0$

$\Rightarrow x = 0$ or $x = 4$

$(0, 0)$ $(4, 0)$

- Multiplicity: 2, 3

So we have:



③ $f(x) = x^3 - x^2 - 9x + 9$

- Degree = 3 ⇒ ↗↘
- Lead coef: positive
- y-int: set $x = 0$

$f(0) = (0)^3 - (0)^2 - 9(0) + 9$

$= 9 \Rightarrow (0, 9)$

x-int: set $f = 0$ & factor by grouping

$\Rightarrow x^3 - x^2 - 9x + 9 = 0$

$\Rightarrow x^2(x-1) - 9(x-1) = 0$

$\Rightarrow (x-1)(x^2 - 9) = 0$

$\Rightarrow (x-1)(x-3)(x+3) = 0$

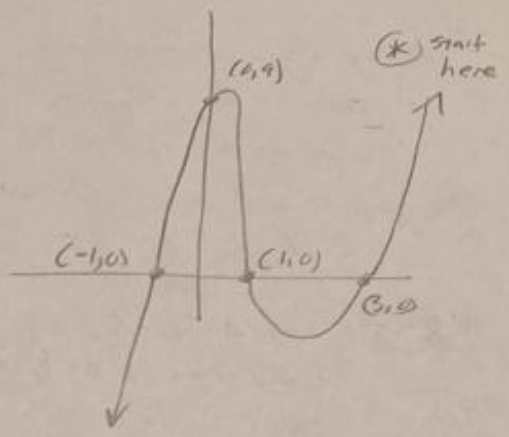
$\Rightarrow x = 1 \quad x = 3 \quad x = -3$

$\Rightarrow (1, 0) \quad (3, 0) \quad (-3, 0)$

- Multiplicity:

1 1 1

so we have



④ $f(x) = \dfrac{x-1}{x^2-16}$

- Asymptotes

Vert: set denom = 0

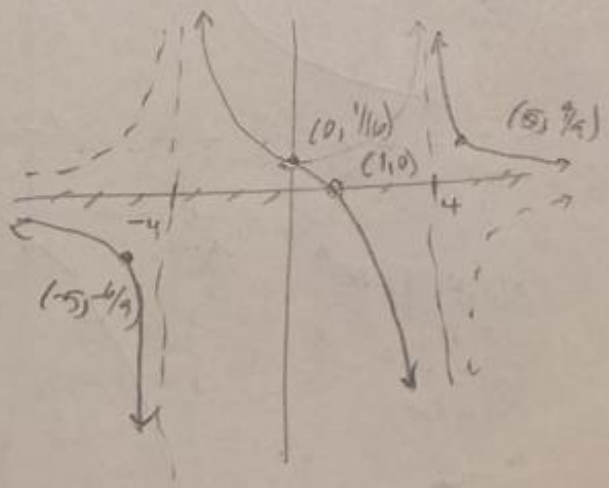
$x^2 - 16 = 0$

$\Rightarrow x = \pm 4$

Horizontal: compare degrees

num = 0 → Case I

denom = 1 → x-axis is the asymp.



- y-int: set $x = 0$

$f(0) = \dfrac{0-1}{(0)^2-16} = \dfrac{1}{16} \Rightarrow (0, 1/16)$

- x-int: Set numerator = 0

$\Rightarrow x - 1 = 0 \Rightarrow x = 1 \quad (1, 0)$

Plot $f(5) = \dfrac{5-1}{(5)^2-16} = \dfrac{4}{9} \Rightarrow (5, 4/9)$ $f(-5) = \dfrac{(-5)-1}{(-5)^2-16} = \dfrac{-6}{9} \Rightarrow (-5, -6/9)$

⑤ $f(x) = \dfrac{2x-1}{x+3}$

* Vert asymp: set denom = 0
  $\Rightarrow x+3=0 \rightarrow x=-3 \Rightarrow (-3, 0)$

* horiz. asymp: compare degrees
  num $\rightarrow$ 1, denom $\rightarrow$ 1 $\Rightarrow$ fraction of coeffs $\rightarrow \frac{2}{1} \Rightarrow y=2$

* y-int: set $x=0$
  $\Rightarrow f(0) = \dfrac{2(0)-1}{0+3} = -\frac{1}{3} \Rightarrow (0, -\frac{1}{3})$

* x-int: set numerator = 0
  $\Rightarrow 2x-1=0 \Rightarrow x=\frac{1}{2} \Rightarrow (\frac{1}{2}, 0)$

So we have