Trigonometry Chapter 1

Section 4

I. Vocabulary

Complete each statement:

1. In order to distinguish between the homonyms “sine” and “sign” we introduce the term ____________________ to be used in lieu of “sign” to avoid confusion.
2. An ____________________ is an equation which is true for all values of the variables for which the equation is defined.
3. The RECIPROCAL IDENTITIES:

$$\sin(\theta) = \frac{1}{\_\_\_\_\_\_} \qquad \csc(\theta) = \frac{1}{\_\_\_\_\_\_}$$

$$\cos(\theta) = \frac{1}{\_\_\_\_\_\_} \qquad \sec(\theta) = \frac{1}{\_\_\_\_\_\_}$$

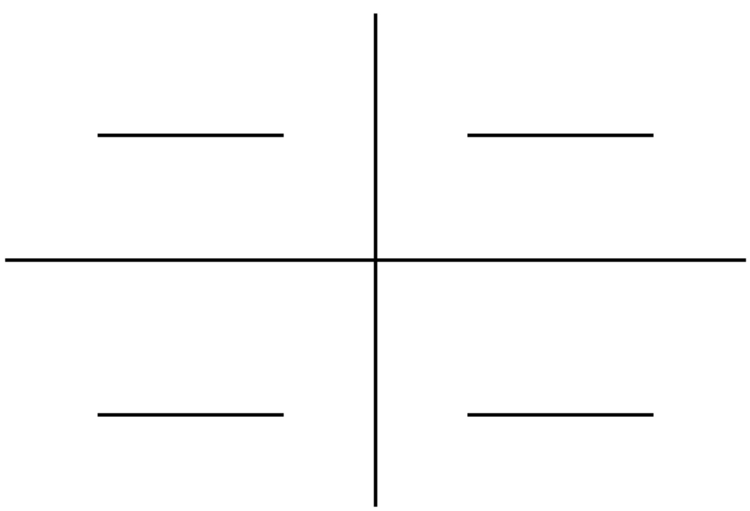
$$\tan(\theta) = \frac{1}{\_\_\_\_\_\_} \qquad \cot(\theta) = \frac{1}{\_\_\_\_\_\_}$$

4. The QUOTIENT IDENTITIES:

$$\frac{\sin(\theta)}{\cos(\theta)} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \qquad \frac{\cos(\theta)}{\sin(\theta)} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

EXTREMELY IMPORTANT NOTE!: The RECIPROCAL and QUOTIENT identities ABSOLUTELY MUST be memorized. Your future success in the course hinges on this!

4. The QUADRANTS of the coordinate plane are:



Note: For clairity reasons which will be discussed in class, I will use QIIII (rather than QIV) to indicate QUADRANT FOUR. You may use either, as long as your writing is unambiguous.

5. An angle in standard position is a QUADRANTAL ANGLE if its terminal side lies on ____________________.
6. An angle in standard position is a NON – QUADRANTAL ANGLE if its terminal side lies in the ____________________ of one of the quadrants.
7. The FUNDAMENTAL PYTHAGOREAN IDENTITY:

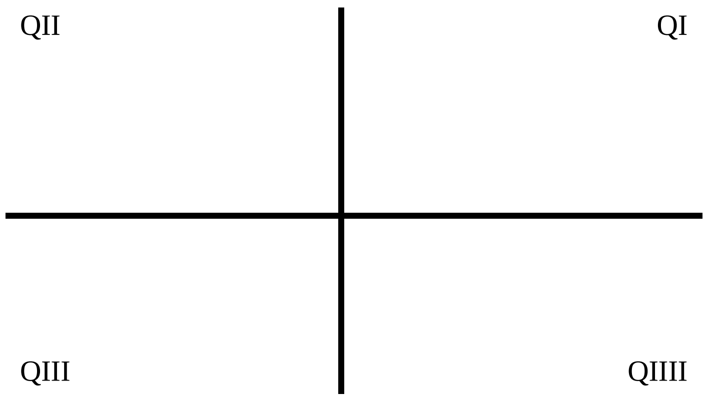
$$\sin^2(\theta)+\cos^2(\theta)=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

8. The PYTHAGOREAN IDENTITIES:

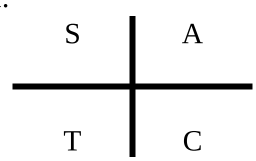
$\sin^2(\theta)+\cos^2(\theta)=$ ____________ $\quad$ $\tan^2(\theta)+1=$ ____________ $\quad$ $1+\cot^2(\theta)=$ ____________

$1-\sin^2(\theta)=$ ____________ $\quad$ $\sec^2(\theta)-1=$ ____________ $\quad$ $\csc^2(\theta)-1=$ ____________

9. Label the figure with ALL the trig functions that are POSITIVE in the various quadrants.



10. The standard memory device for the diagram:



is ____________________ and this diagram indicates the quadrants in which the various trig functions are ____________________.

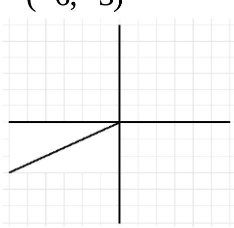
II. Assume θ is a standard position angle and that the terminal side of θ passes through the given point. State the Quadrant in which the angle lies. If it does not lie in one of the quadrants, write: "None because it is Quadrantal".

Example:

(a) $(-6,-3)$ (b) $(4,-5)$ (c) $(0,3)$ (d) $(3,3)$

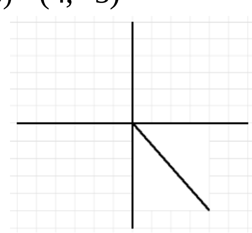
Solution:

(a) $(-6,-3)$



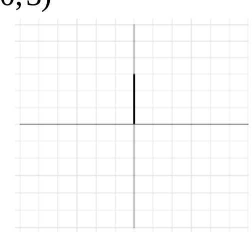
$\boxed{QIII}$

(b) $(4,-5)$



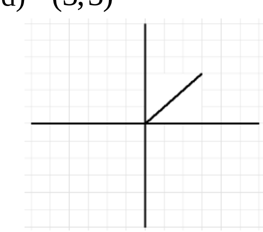
$\boxed{QIIII}$

(c) $(0,3)$



$\boxed{\text{None because it is Quadrantal}}$

(d) $(3,3)$



$\boxed{QI}$

11. $(4,0)$ 12. $(-1,8)$ 13. $(1,9)$ 14. $(8,-2)$ 15. $(0,-5)$

16. $(-7,-10)$ 17. $(5,2)$ 18. $(-6,14)$ 19. $(-6,-2)$ 20. $(7,-7)$

III. Find the value of the indicated trig function using the given information and the appropriate reciprocal identity. If a value contains a radical, express the value in two forms: with the radical in the numerator and also with the radical in the denominator. You should also write out the reciprocal identity you use. If the given value is a decimal, express your result as a deciomal rounded to two decimal places.

Example:

Find $\cos(\theta)$; Given $\sec(\theta)=\frac{8}{3}$

Solution: $\cos(\theta)=\boxed{-\frac{3}{8}}$ Identity: $\boxed{\cos(\theta)=\frac{1}{\sec(\theta)}}$ Note: $\cos(\theta)=\frac{1}{\sec(\theta)}=\frac{1}{-8/3}=1\cdot\left(-3/8\right)=\boxed{-\frac{3}{8}}$

Find $\tan(\beta)$; Given $\cot(\beta)=\frac{2}{\sqrt{7}}$

Solution: $\tan(\beta)=\boxed{\frac{\sqrt{7}}{2}}=\boxed{\frac{7}{2\sqrt{7}}}$ Identity: $\boxed{\tan(\beta)=\frac{1}{\cot(\beta)}}$ Note: $\frac{\sqrt{7}}{2}=\frac{\sqrt{7}\cdot\sqrt{7}}{2\cdot\sqrt{7}}=\boxed{\frac{7}{2\sqrt{7}}}$

21. Find $\csc(\phi)$; Given $\sin(\phi)=\frac{6}{11}$

22. Find $\cot(\theta)$; Given $\tan(\theta)=7$

23. Find $\cos(\alpha)$; Given $\sec(\alpha)=-\frac{\sqrt{13}}{3}$

24. Find $\sin(\gamma)$; Given $\csc(\gamma)=\sqrt{5}$

25. Find $\cos(\alpha)$; Given $\sec(\alpha)=6.5$

26. Find $\tan(\beta)$; Given $\cot(\beta)=-4.2$

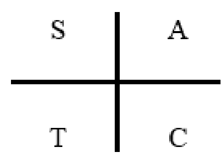
27. Find $\sin(\theta)$; Given $\csc(\theta)=\frac{2}{\sqrt{15}}$

28. Find $\csc(\phi)$; Given $\sin(\phi)=.47$

IIII. State the signum of each of the trig functions for the given standard position angle.

Example: $\phi=142°$

Solution: $\phi=142°$ is a QII angle so using



we have:

$\sin(\phi)$ is Positive	$\csc(\phi)$ is Positive
$\cos(\phi)$ is Negative	$\sec(\phi)$ is Negative
$\tan(\phi)$ is Negative	$\cot(\phi)$ is Negative

29. $\theta=251°$ 30. $\theta=18°$ 31. $\theta=330°$ 32. $\theta=-207°$

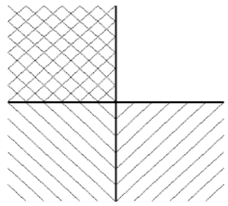
33. $\theta=194°$ 34. $\theta=518°$ 35. $\theta=-64°$ 36. $\theta=1010°$

V. Identify the quadrant that satisfies the given conditions for the given angle.

Example: $\sec(\alpha) < 0, \quad \tan(\alpha) < 0$

Solution: $\sec(\alpha) < 0 \Rightarrow$ QII or QIII ($\sec(\alpha)$ has the same signum as $\cos(\alpha)$)

$\tan(\alpha) < 0 \Rightarrow$ QII or QIIII So we have:



Thus, both take effect in $\boxed{\text{QII}}$

37. $\cos(\theta) < 0, \quad \tan(\theta) > 0$

38. $\csc(\gamma) > 0, \quad \cos(\gamma) > 0$

39. $\tan(\lambda) > 0, \quad \csc(\lambda) > 0$

40. $\sin(\phi) < 0, \quad \cos(\phi) < 0$

41. $\cot(\beta) < 0, \quad \csc(\beta) < 0$

42. $\sec(\theta) > 0, \quad \cot(\theta) > 0$

43. $\sec(\alpha) > 0, \quad \sin(\alpha) < 0$

44. $\sin(\theta) < 0, \quad \cot(\theta) > 0$

45. $\cot(\phi) < 0, \quad \sec(\phi) > 0$

VI. Use the Pythagorean Identities and the Reciprocal Identities to find the requested value. Write out the identity before you use it.

If a result contains a radical, express it with the radical in the numerator. If a result is in decimal form, express it rounded to two decimal places.

Example: Find $\csc(\theta)$ given that $\cot(\theta) = -\dfrac{\sqrt{11}}{3}$ and θ is in QIIII.

Solution:

$$\begin{aligned}\csc^2(\theta) &= 1 + \cot^2(\theta) \\ &= 1 + \left(-\frac{\sqrt{11}}{3}\right)^2 \\ &= 1 + \frac{\left(\sqrt{11}\right)^2}{(3)^2} \\ &= 1 + \frac{11}{9} \\ &= \frac{20}{9}\end{aligned}$$

So we have:

$$\begin{aligned}\csc(\theta) &= \pm\sqrt{\frac{20}{9}} \\ &= \pm\frac{\sqrt{20}}{\sqrt{9}} \\ &= \pm\frac{5\sqrt{2}}{3}\end{aligned}$$

Finally, since we are in QIIII we know $\csc(\theta) < 0$ so we choose

$$\csc(\theta) = \boxed{\pm\frac{5\sqrt{2}}{3}}$$

Example: Find $\cot(\alpha)$ given that $\cos(\alpha) = -.38$ and α is in QIII.

Solution: We wish to use $\cot^2(\alpha) = \csc^2(\alpha) - 1$ so first we must find $\csc^2(\alpha) = \dfrac{1}{\sin^2(\alpha)}$

To do this we use $\sin^2(\alpha) = 1 - \cos^2(\alpha)$

$$\begin{aligned} \sin^2(\alpha) &= 1 - \cos^2(\alpha) \\ &= 1 - (-.38)^2 \\ &= 1 - .1444 \\ &= .8556 \end{aligned}$$

So

$$\begin{aligned} \csc^2(\alpha) &= \frac{1}{\sin^2(\alpha)} \\ &= \frac{1}{.8556} \\ &= 1.16877... \end{aligned}$$

So we have:

$$\begin{aligned} \cot^2(\alpha) &= \csc^2(\alpha) - 1 \\ &= (1.16877...) - 1 \\ &= .16887... \end{aligned}$$

So now:

$$\begin{aligned} \cot(\alpha) &= \pm\sqrt{.16877...} \\ &= \pm.410816... \end{aligned}$$

Finally, since we are in QIII we know $\cot(\alpha) > 0$ so we choose

$$\begin{aligned} \cot(\alpha) &= .410816... \\ &\approx \boxed{.41} \end{aligned}$$

46. Find $\tan(\phi)$ given that $\sec(\phi) = \dfrac{15}{4}$ and ϕ is in QI.

47. Find $\cos(\theta)$ given that $\sin(\theta) = \dfrac{1}{4}$ and θ is in QII.

48. Find $\csc(\beta)$ given that $\cot(\beta) = -\dfrac{3}{\sqrt{17}}$ and β is in QIIII.

49. Find $\sec(\theta)$ given that $\cot(\theta) = \dfrac{\sqrt{37}}{2}$ and θ is in QIII.

50. Find $\cot(\alpha)$ given that $\cos(\alpha) = -\dfrac{\sqrt{7}}{5}$ and α is in QII.

51. Find $\tan(\gamma)$ given that $\sin(\gamma) = .75$ and γ is in QI.

52. Find $\sin(\theta)$ given that $\sec(\theta) = \dfrac{6}{11}$ and θ is in QIIII.

VII. Use the x, y, r definitions and the given information to find the values of the remaining five trig functions. If a result contains a single radical, express it both with the radical in the numerator and with the radical in the denominator. If a result is in decimal form, express it rounded to two decimal places.

Example: Given $\cot(\theta) = -\frac{\sqrt{13}}{3}$ and θ is in QII.

Solution: We must decide where the negative sign goes.

$\cot(\theta) = \frac{x}{y}$ and

θ in QII $\Rightarrow x < 0 \ \& \ y > 0$

$\Rightarrow x = -\sqrt{13} \ \& \ y = 3$

So we have:

$$r = \sqrt{x^2 + y^2} = \sqrt{(3)^2 + \left(-\sqrt{13}\right)^2} = \sqrt{9+13} = \sqrt{22}$$

So this gives us:

$\sin(\theta) = \frac{y}{r} = \boxed{\frac{3}{\sqrt{22}}} = \boxed{\frac{3\sqrt{22}}{22}}$ $\qquad$ $\csc(\theta) = \frac{r}{y} = \boxed{\frac{\sqrt{22}}{3}} = \boxed{\frac{22}{3\sqrt{22}}}$

$\cos(\theta) = \frac{x}{r} = \boxed{\frac{-\sqrt{13}}{\sqrt{22}}}$ $\qquad$ $\sec(\theta) = \frac{r}{x} = \boxed{\frac{\sqrt{22}}{-\sqrt{13}}}$

$\tan(\theta) = \frac{y}{x} = \boxed{-\frac{3}{\sqrt{13}}} = \boxed{-\frac{3\sqrt{13}}{13}}$ $\qquad$ $\cot(\theta) = \frac{x}{y} = \boxed{-\frac{\sqrt{13}}{3}} = \boxed{-\frac{13}{3\sqrt{13}}}$

Example: Given $\tan(\alpha) = 6$ and $\sec(\alpha) < 0$.

Solution: We have $\tan(\alpha) > 0 \ \& \sec(\alpha) < 0 \Rightarrow \alpha$ is in QIII.

$\tan(\alpha) = 6 = \frac{y}{x} = \frac{6}{1}$ and

α in QIII $\Rightarrow x < 0 \ \& \ y < 0$

$\Rightarrow x = -1 \ \& \ y = -6$

So we use:

$\tan(\alpha) = \frac{-6}{-1}$

and we have:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

So this gives us:

$\sin(\theta) = \frac{y}{r} = \boxed{-\frac{6}{\sqrt{37}}} = \boxed{-\frac{6\sqrt{37}}{37}}$ $\qquad$ $\csc(\theta) = \frac{r}{y} = \boxed{-\frac{\sqrt{37}}{6}} = \boxed{-\frac{37}{\sqrt{37}}}$

$\cos(\theta) = \frac{x}{r} = \boxed{-\frac{1}{\sqrt{37}}} = \boxed{-\frac{\sqrt{37}}{37}}$ $\qquad$ $\sec(\theta) = \frac{r}{x} = \frac{\sqrt{37}}{-1} = \boxed{-\sqrt{37}} = \boxed{-\frac{37}{\sqrt{37}}}$

$\tan(\theta) = \frac{y}{x} = \frac{6}{1} = \boxed{6}$ $\qquad$ $\cot(\theta) = \frac{x}{y} = \frac{-1}{-6} = \boxed{\frac{1}{6}}$

53. Given $\tan(\beta) = -\frac{5}{3}$ and β is in QII.

54. Given $\sin(\phi) = \frac{\sqrt{2}}{2}$ and ϕ is in QI.

55. Given $\sec(\theta) = \frac{7}{2}$ and θ is in QIIII.

56. Given $\csc(\omega) = \frac{\sqrt{15}}{4}$ and ω is in QII.

57. Given $\cos(\theta) = -.44$ and θ is in QIII.

58. Given $\cot(\gamma) = 8$ and γ is in QIII.

VII. Use the x, y, r definitions to show a derivation of each identity.

Example: Show that $\cot^2(\alpha) = \csc^2(\alpha) - 1$.

Solution: Start with the more complicated side:

$$\begin{aligned} RHS &= \csc^2(\alpha) - 1 \\ &= \left(\frac{r}{y}\right)^2 - 1 \\ &= \frac{r^2}{y^2} - \frac{y^2}{y^2} \\ &= \frac{r^2 - y^2}{y^2} \\ &= \frac{x^2}{y^2} \\ &= \left(\frac{x}{y}\right)^2 \\ &= \cot^2(\alpha) \\ &= LHS \end{aligned}$$

Important note for this section: When the value of r was generated we used $r = \sqrt{x^2 + y^2}$. However, this is equivalent to each of the following; hence they are all true and may be freely used in any setting.

$$r^2 = x^2 + y^2, \qquad r^2 - y^2 = x^2, \qquad r^2 - x^2 = y^2$$

59. Show that $\sin^2(\phi) + \cos^2(\phi) = 1$.

60. Show that $\tan^2(\theta) + 1 = \sec^2(\theta)$.

61. Show that $\cos^2(\beta) = 1 - \sin^2(\beta)$.