Trigonometry Chapter 2

Section 2

I. Vocabulary

Complete each statement:

1. A REFERENCE ANGLE is always an ____________________ angle and is always has ____________________ measure.
2. For non–____________________ angles $\theta, \beta, \phi, x,$ etc, the symbols for the REFERENCE ANGLES are ____________________.
3. For any non–____________________ angle θ, the θ_R is the ____________________ angle formed by the ____________________ and the ____________________ side of θ.
4. The only angles which do not have a REFERENCE ANGLE are ____________________ angles.
5. Two angles in standard position are COTERMINAL ANGLES if they have the same ____________________ side but different amounts of rotation.
6. If two angles, α and β, are ____________________ then α_R ______ β_R and

$$\sin(\alpha) = \_\_\_\_\_\_ \qquad \_\_\_\_\_\_ = \csc(\beta)$$
$$\_\_\_\_\_\_ = \cos(\beta) \qquad \sec(\alpha) = \_\_\_\_\_\_$$
$$\_\_\_\_\_\_ = \tan(\beta) \qquad \_\_\_\_\_\_ = \cot(\beta)$$

7. The values of any trig function of an angle and its ____________________ ANGLE are equal except for possibly the SIGNUM.
8. If an angle θ is given then to determine the value of θ_R:
   - In QI, the reference angle ____________________ the angle.
   - In QII, $\theta_R = (\_\_\_\_\_\_\_\_)^\circ$
   - In QIII, $\theta_R = (\_\_\_\_\_\_\_\_)^\circ$
   - In QIIII, $\theta_R = (\_\_\_\_\_\_\_\_)^\circ$
9. If the value of θ_R is given then to determine the value of θ:
   - In QI, the reference angle ____________________ the angle.
   - In QII, $\theta = (\_\_\_\_ \ \_\_\_ \theta_R)^\circ$
   - In QIII, $\theta = (\_\_\_\_ \ \_\_\_ \theta_R)^\circ$
   - In QIIII, $\theta = (\_\_\_\_ \ \_\_\_ \theta_R)^\circ$
10. For any angles, $\theta, \beta, \phi, x,$ etc, we represent the FAMILY OF ALL ANGLES COTERMINAL TO θ by:

$$\theta \pm \_\_\_\_^\circ \cdot n, \quad \beta \pm \_\_\_\_^\circ \cdot n, \quad \phi \,\_\_\_ \ \_\_\_\_^\circ \cdot n, \quad x \,\_\_\_ \ \_\_\_\_^\circ \cdot \_\_\_, \quad \text{etc. for } n \in \{0, 1, 2, 3, \ldots\}$$

11. The REFERENCE ANGLE FAMILY for a reference angle, θ_R is given by:
   - In QII, $\theta = (\_\_\_\_ \ \_\_\_ \theta_R)^\circ$
   - In QI, the reference angle ____________________ the angle.
   - In QIII, $\theta = (\_\_\_\_ \ \_\_\_ \theta_R)^\circ$
   - In QIIII, $\theta = (\_\_\_\_ \ \_\_\_ \theta_R)^\circ$

12. The PICTURES of the REFERENCE ANGLE FAMILY for a reference angle, θ_R are:

QII QI

QIII QIIII

II. Find the reference angle for each given angle.

Example:

a) $\alpha = 220°$

b) $\delta = -110°$

c) $\theta = 660°$

Solution:

a) Note that α is a QIII angle.
So we have:
$\alpha_R = 220° - 180° = \boxed{40°}$

b) Note that δ is not in the interval $0° < \delta < 360°$ so first find a coterminal angle in that interval:
$\delta_1 = (-110 + 360)°$
$= 250°$
Thus, δ is a QIII angle.
So we have:
$\delta_R = (250 - 180)°$
$= \boxed{70°}$

c) Note that θ is not in the interval $0° < \theta < 360°$ so first find a coterminal angle in that interval:
$\theta_1 = (660 - 360)°$
$= 300°$
Thus, θ is a QIIII angle.
So we have:
$\theta_R = (360 - 300)°$
$= \boxed{60°}$

13. $\theta = 316°$

14. $\phi = 122°$

15. $\beta = 45°$

16. $\theta = 256°$

17. $\alpha = 14°$

18. $\alpha = 281°$

19. $\phi = 207°$

20. $\theta = 168°$

21. $\theta = -33°$

22. $\theta = -195°$

23. $\delta = 432°$

24. $\beta = 570°$

25. $\lambda = 368°$

26. $\alpha = -60°$

27. $\phi = 835°$

28. $\theta = -594°$

III. For each given expression, determine:

a) the SIGNUM

b) the reference angle

c) the EXACT value of the function (Do NOT express as a decimal approximation.) Express results with radicals in the numerators.

Example:

i) $\cos(60°)$

ii) $\tan(210°)$

Solution:

i) Note that this is a QI angle.
So we have:

a) signum is $\boxed{\textit{negative}}$

b) in QI the reference angle is the angle, so $\theta_R = \boxed{60°}$

c) $\cos(60°) = \boxed{-\frac{1}{2}}$

ii) This is a QIII angle.
So we have:

a) signum is $\boxed{\textit{positive}}$

b) in QIII the reference angle is:
$\theta_R = (210-180)° = \boxed{30°}$

c) $\tan(210°) = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

Example:

iii) $\csc(315°)$

Solution:

iii) This is a QIIII angle. So we have:

a) signum is $\boxed{\textit{negative}}$

b) in QIIII the reference angle is:
$\theta_R = (360-315)° = \boxed{45°}$

c) $\csc(315°)$ is the reciprocal of $\sin(315°)$ so we have:

$$\csc(315°) = \frac{1}{\sin(315°)} = \frac{1}{\sin(45°)} = -\frac{1}{\left(\sqrt{2}/2\right)}$$

$$= -1 \cdot \frac{2}{\sqrt{2}} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$$

28. $\sin(30°)$
29. $\tan(45°)$
30. $\sec(45°)$
31. $\cot(60°)$
32. $\cos(240°)$
33. $\csc(150°)$
34. $\tan(300°)$
35. $\sin(135°)$
36. $\csc(330°)$
37. $\cos(225°)$
38. $\cot(30°)$
39. $\sec(150°)$
40. $\cos(60°)$
41. $\sin(45°)$
42. $\tan(30°)$
43. $\cos(30°)$

Example:

$\sec(855°)$

Solution: First find a coterminal angle in the interval $0° < \theta < 360°$ then proceed as before:

$\theta = (855° - 720°) = 135°$

This is a QII angle. So we have:

a) signum is $\boxed{\textit{negative}}$

b) in QII the reference angle is: $\theta_R = (180-135)° = \boxed{45°}$

c) $\sec(855°)$ is the reciprocal of $\cos(855°)$ so we have:

$$\sec(855°) = \frac{1}{\cos(855°)} = \frac{1}{\cos(45°)} = -\frac{1}{\left(\sqrt{2}/2\right)}$$

$$= -1 \cdot \frac{2}{\sqrt{2}} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$$

44. $\cot(750°)$ 45. $\cos(945°)$ 46 . $\sin(-330°)$ 47. $\tan(-45°)$

IIII. Solve each equation in the interval $(0°, 360°)$.

II. Find the reference angle for each given angle.

Example:

a) $\sin(\beta) = -\frac{\sqrt{3}}{2}$

b) $\cot(\theta) = 1$

Solution:

a) $\sin(\beta) = -\frac{\sqrt{3}}{2}$

* Quadrant Check:
From the diagram

S	A
T	C

we have $\sin(\beta)$ is negative in QIII & QIIII

* Reference Angle:
From the diagram

INSERT REF ANG DIAG HERE

we have $\beta_R = 60°$

* Solve:

QIII: $\beta = (180 + 60)° = \boxed{240°}$

QIIII: $\beta = (360 - 60)° = \boxed{300°}$

b) $\cot(\theta) = 1$

* Reciprocal of both sides:

$\cot(\theta) = 1 \Rightarrow \tan(\theta) = 1/1 \Rightarrow \tan(\theta) = 1$

* Quadrant Check:
From the diagram

S	A
T	C

we have $\tan(\theta)$ is positive in QI & QIII

* Reference Angle:
From the diagram

INSERT REF ANG DIAG HERE

we have $\theta_R = 45°$

* Solve:

QI: $\theta = \boxed{45°}$

QIII: $\theta = (180 + 45)° = \boxed{225°}$

48. $\sin(\theta) = \frac{1}{2}$ 49. $\tan(\theta) = -\frac{\sqrt{3}}{3}$ 50. $\cos(\alpha) = \frac{\sqrt{2}}{2}$ 51. $\sec(\theta) = -2$

52. $\cot(\phi) = \frac{1}{\sqrt{3}}$ 53. $\sin(\theta) = -\frac{\sqrt{3}}{2}$ 54. $\tan(\beta) = -1$ 55. $\csc(\phi) = -\frac{2}{\sqrt{2}}$

48. $\sin(\theta) = \pm\frac{1}{2}$ 49. $\cos(\gamma) = \frac{\sqrt{3}}{2}$ 50. $\sin(\alpha) = -\frac{\sqrt{2}}{2}$ 51. $\tan(\theta) = \pm\sqrt{3}$

V. Find the GENERAL SOLUTION for each equation.

Example:

$$\cos(\gamma) = -\frac{1}{2}$$

Solution:

a) $\cos(\gamma) = -\frac{1}{2}$

* Quadrant Check:
  we have $\cos(\gamma)$ is negative in QII & QIII
* Reference Angle:
  we have $\gamma_R = 60°$
* Solve:
  QII: $\gamma = (180 - 60)° \pm 360° \cdot n = \boxed{120° \pm 360n°}$
  QIII: $\gamma = (180 + 60)° \pm 360° \cdot n = \boxed{240° \pm 360n°}$

52. $\sin(\theta) = -\frac{\sqrt{3}}{2}$

53. $\cos(\theta) = \frac{1}{2}$