Trigonometry Chapter 3

Section 1

I. Vocabulary

Complete each statement:

1. The values of the SPECIAL ANGLES measured in RADIANS are:

$0°$ = _____ radians
$90°$ = _____ radians
$180°$ = _____ radians
$270°$ = _____ radians
$360°$ = _____ radians
$30°$ = _____ radians
$45°$ = _____ radians
$60°$ = _____ radians

2. The REFERENCE ANGLE FAMILIES for the SPECIAL ANGLES in radians are:

$30° \to 30°,\ 150°,\ 210°,\ 330° \quad \leftrightarrow \quad \pi/6 \to$ ____, ____, ____, ____

$45° \to 45°,\ 135°,\ 225°,\ 315° \quad \leftrightarrow \quad \pi/4 \to$ ____, ____, ____, ____

$60° \to 60°,\ 120°,\ 240°,\ 300° \quad \leftrightarrow \quad \pi/3 \to$ ____, ____, ____, ____

3. Again, the REFERENCE ANGLE FAMILIES for the SPECIAL ANGLES in radians are:

$\pi/6 \to$ ____, ____, ____, ____
$\pi/4 \to$ ____, ____, ____, ____
$\pi/3 \to$ ____, ____, ____, ____

Note: These are so important, they are worth writing out a second time.

4. Yet again, the REFERENCE ANGLE FAMILIES for the SPECIAL ANGLES in radians are:

$\pi/6 \to$ ____, ____, ____, ____
$\pi/4 \to$ ____, ____, ____, ____
$\pi/3 \to$ ____, ____, ____, ____

Note: These are so important, they are worth writing out a third time.

5. The values of the QUANDRANTAL angles in RADIANS are:
   a) The ZERO ANGLE: _____
   b) ONE-QUARTER REVOLUTION: _____
   c) A STRAIGHT ANGLE: _____
   d) THREE-QUARTER REVOLUTION: _____
   e) ONE REVOLUTION: _____

6. Again, the values of the QUANDRANTAL angles in RADIANS are:
   a) The ZERO ANGLE: _____
   b) ONE-QUARTER REVOLUTION: _____
   c) A STRAIGHT ANGLE: _____
   d) THREE-QUARTER REVOLUTION: _____
   e) ONE REVOLUTION: _____
   Note: These are so important, they are worth writing out a second time.

7. Yey again, the values of the QUANDRANTAL angles in RADIANS are:
   a) The ZERO ANGLE: _____
   b) ONE-QUARTER REVOLUTION: _____
   c) A STRAIGHT ANGLE: _____
   d) THREE-QUARTER REVOLUTION: _____
   e) ONE REVOLUTION: _____
   Note: These are so important, they are worth writing out a third time.

8. A very rough approximation which is good to keep in mind is:
   ONE RADIAN equals approximately a little less than _____ degrees.
   Note: A slightly better value, but ***still an approximation*** is _____ degrees.
9. To CONVERT from either DEGREES to RADIANS or from RADIANS to DEGREES,
   The equation to start with is: _____ = _____.
   Note that this equation will typically generate an _____ value.

II. Convert each degree measure to radians. Note that each item is a multiple of a special angle. Express your results as EXACT VALUES in terms of π.

Example:

a) $240°$ b) $630°$ c) $-225°$

Solution:

a) $240°$ is $4\cdot 60°$ which is $4\cdot\left(\pi/3\right)$ which is $\boxed{4\pi/3}$ or $\boxed{\frac{4\pi}{3}}$

b) $630°$ is $7\cdot 90°$ which is $7\cdot\left(\pi/2\right)$ which is $\boxed{7\pi/2}$ or $\boxed{\frac{7\pi}{2}}$

c) $-225°$ is $5\cdot(-45)°$ which is $5\cdot\left(-\pi/4\right)$ which is $\boxed{-5\pi/4}$ or $\boxed{-\frac{5\pi}{4}}$

10. $30°$ 11. $270°$ 12. $315°$ 13. $150°$ 14. $45°$

15. $300°$ 16. $360°$ 17. $60°$ 18. $120°$ 19. $180°$

20. $-540°$ 21. $-210°$ 22. $900°$ 23. $-3600°$ 24. $0°$

III. Express each radian measure in degrees. Note that each item is a multiple of a special angle.

Example:

a) $\frac{\pi}{4}$ b) $\frac{7\pi}{3}$ c) $-\frac{5\pi}{2}$

Solution:

a) $\frac{\pi}{4}$ is a special angle: $\boxed{45°}$

b) $\frac{7\pi}{3}$: $\frac{\pi}{3}$ is a special angle $(60°)$ so we have $\frac{7\pi}{3} = 7 \cdot 60° = \boxed{420°}$

c) $-\frac{5\pi}{2}$: $\frac{\pi}{2}$ is a special angle $(90°)$ so we have $-\frac{5\pi}{2} = -5 \cdot 90° = \boxed{-450°}$

25. $\frac{\pi}{4}$ 26. $\frac{11\pi}{6}$ 27. $\frac{5\pi}{3}$ 28. $\frac{3\pi}{4}$ 29. $\frac{\pi}{2}$

30. π 31. $\frac{11\pi}{3}$ 32. $-\frac{7\pi}{6}$ 33. 2π 34. $\frac{3\pi}{2}$

35. 0 36. $\frac{20\pi}{3}$ 37. -3π 38. $\frac{5\pi}{4}$ 39. $-\frac{\pi}{6}$

IIII. Use the equation $180° = \pi$ to convert each given radian measure to degree measure and each given degree measure to radian measure. Express radian answers rounded to ***four*** decimal places. Express degree answers rounded to ***one*** decimal place. When using a value for π in your approximations, use the ***continued decimal value*** from your calculator – DO NOT use simply 3.14.

Example:

a) $117°$ b) $\frac{7\pi}{13}$ c) $284.6°$ d) 2.8874

Solution:

a)

$$180° = \pi$$
$$\Rightarrow \frac{180}{117} = \frac{\pi}{x}$$
$$\Rightarrow 180x = 117\pi$$
$$\Rightarrow x = \frac{117\pi}{180}$$
$$= 2.0420352...$$
$$\approx \boxed{2.0420}$$

b)

$$180° = \pi$$
$$\Rightarrow \frac{7(180)}{13} = \frac{7\pi}{13}$$
$$\Rightarrow 96.9230769... = \frac{7\pi}{13}$$

So we have:

$$\frac{7\pi}{13} \approx \boxed{96.9°}$$

OR

b) alternative solution

$$180° = \pi$$
$$\Rightarrow \frac{180}{x} = \frac{\pi}{\left(\frac{7\pi}{13}\right)}$$
$$\Rightarrow \frac{180}{x} = \left(\frac{\pi}{1}\right)\cdot\left(\frac{13}{7\pi}\right)$$
$$\Rightarrow \frac{180}{x} = \frac{13}{7}$$
$$\Rightarrow 13x = 7(180)$$
$$\Rightarrow x = \frac{7(180)}{13}$$
$$= 96.9230769...$$
$$\approx \boxed{96.9°}$$

c)

$$180° = \pi$$
$$\Rightarrow \frac{180}{248.6} = \frac{\pi}{x}$$
$$\Rightarrow 180x = 248.6\pi$$
$$\Rightarrow x = \frac{248.6\pi}{180}$$
$$= 4.9672070...$$
$$\approx \boxed{4.9672}$$

d)

$$180° = \pi$$
$$\Rightarrow \frac{180}{x} = \frac{\pi}{2.8874}$$
$$\Rightarrow \pi \cdot x = 2.8874 \cdot (180)$$
$$\Rightarrow x = \frac{2.8874 \cdot (180)}{\pi}$$
$$= 165.4358338...$$
$$\approx \boxed{165.4°}$$

40. $674°$ 41. $-217.5°$ 42. 2.6179 43. $249°$ 44. $\frac{28\pi}{19}$

45. 1.5708 46. $83°$ 47. -11.2541 48. $\frac{6\pi}{23}$ 49. 23.3258