Trigonometry Chapter 3

Section 2

I. Vocabulary
Complete each statement:
1. The formula for ARC LENGTH on a circle is given by _____ .
2. When computing arc length on a circle, the measure of the CENTRAL ANGLE **MUST** be given
   in _____ .
3. When two circles C_1 and C_2 are turning simultaneously and they have DIFFERENT CENTERS, you may equate
   the _____ , that is, _____ = _____ .
4. When two circles C_1 and C_2 are turning simultaneously and they have the SAME CENTER, you may equate
   the _____ , that is, _____ = _____ .

II. Use the formular for arc length of a circle to answer each question.
   Example:
   a) In a circle a central angle measuring $5\pi/22$ subtends an arc measuring $29.4\,cm$. What is the radius
      of the circle?
   b) In a circle with radius $46.2\,in$ what is the measure of the central angle which will determine an arc
      measuring $98.7\,in$?
   c) In a circle with radius $46.2\,in$, a central angle of $98°$ will subtend an arc of how many inches?

   Solutions:

a)
$$s = r\theta$$
$$\Rightarrow (29.4) = r\left(5\pi/22\right)$$
$$\Rightarrow r = (29.4)\cdot\left(22/5\pi\right)$$
$$= \left(646.8/5\pi\right)$$
$$= 41.1765668...$$
$$\approx \boxed{41.2\,cm}$$

b)
$$s = r\theta$$
$$\Rightarrow 98.7 = 46.2\theta$$
$$\Rightarrow \theta = 98.7/46.2$$
$$= 2.1363636...$$
$$\approx \boxed{2.1364}$$
   Note: This result and the one for part c) is measured in radians. You may include that in your answer:
$$\boxed{2.1364\,rad}$$
   but this is not required, as radians are the default usit.

c)
$$s = r\theta$$
Since the angle is given in degrees, we must first convert it to radians.
$$180 = \pi$$
$$\Rightarrow \frac{180}{98.7} = \frac{\pi}{x}$$
$$\Rightarrow 180x = 98.7\pi$$
$$\Rightarrow x = \frac{98.7\pi}{180}$$
$$= 1.7226399...$$
$$\approx \boxed{1.7226}$$

5. A circle with radius $56.3\,ft$ has a central angle measuring 2.4687. What is the length of the arc thus determined?

6. In a circle an arc length of $7.1m$ subtends a central angle measuring $224°$. What is the radius of the circle?

7. If the radius of a circle is $143.9\,m$ and an arc length of $678.1m$ is measured off. What is the measure of the central angle that will be determined? Express your answer in both a) radians and b) degrees.

8. An arc length is length $38.4\,in$ is determined by a circle's central angle measuring $15.4°$. What is the radius of the circle?

III. Solve each problem. Round answers in radians to 4 decimal places. Round answers of all other units to 1 decimal place.

Example: MAKE AND INSERT PICS HERE

      a) Using a value of $6400\,km$ as an approximation for the radius of the radius of the earth, approximate the straight line distance between Oslo, Norway (latitude $59.914°$) and Aviano, Italy (latitude $46.036°$). Note: Aviano was the first place my daughter was stationed as a Second Lieutennant in the US Air Force.

      b) A large pulley (radius $14.8\,cm$) is connedcted by a belt to a small pulley (radius $6.3\,cm$). If the large pulley rotates three-fourths of a revolution, through how many degrees does the small one rotate?

Solution:

a) The central angle between Oslo and Aviano is

$$\angle OAC = (59.914 - 46.036)° = 13.878°.$$

Convert this to radians:

$$180 = \pi$$

$$\Rightarrow \frac{180}{13.878} = \frac{\pi}{\theta}$$

$$\Rightarrow 180\theta = (13.878)\pi$$

$$\Rightarrow \theta = \frac{(13.878)\pi}{180}$$

$$\approx .2422$$

Now use $s = r\theta$:

$$s = r\theta$$

$$= (6400)(.2422)$$

$$= 1550.08$$

$$\approx \boxed{1550.1\,km}$$

b) In the large pulley, C_1, we need the angle, θ_1, in radians. Three-fourths of a revolution is $\frac{3\pi}{2}$ radians.

So we have:

$$s_1 = r_1\theta_1$$

$$= (14.8)\left(\frac{3\pi}{2}\right)$$

$$= 69.7433569...$$

Since the centers are different we may equate the are lengths. That is $s_2 = s_1 = 69.7433569...$

So we have:

$$s_2 = r_2\theta_2$$

$$\Rightarrow 69.7433569... = 6.3\theta_2$$

$$\Rightarrow \theta_2 = \frac{69.7433569...}{6.3}$$

$$= 11.0703741...$$

Finally, convert this to degrees:

$$180 = \pi$$

$$\Rightarrow \frac{180}{\theta_2} = \frac{\pi}{11.0703741...}$$

$$\Rightarrow \pi\theta_2 = (180)(11.0703741...)$$

$$\Rightarrow \theta_2 = \frac{(180)(11.0703741...)}{\pi}$$

$$= 634.2857143...$$

$$\approx \boxed{634.3°}$$

9. Indianapolis, IN has an angle of latitude of $39.769°$. Pensacola, FL has an angle of latitude of $30.421°$. Using a value for the earth's radius of approximately $6400\,km$, approximate the distance between the two cities. SEE PICTURE

10. A smaller gear with a radius of $4.2\,cm$ drives a larger gear with a radius of $9.7\,cm$. If the smaller gear rotates through an angle of $135°$, through what angle (in degrees) does the larger gear rotate? SEE PICTURE

11. Two gears with the same center are fixed so that they rotate in unison. SEE PICTURE Each is connected to its own drive chain. Assume the larger gear has a radius of $22.5\,in$ and that $40\,in$ of chain feeds into the larger gear. If this causes the smaller gear to feed out $27\,in$ of chain, what is the radius of the smaller gear?

12. Two wheels are touching so they rotate in unison. SEE PICTURE Wheel #1 has a radius of $8.3\,in$ and rotates four complete revolutions. If Wheel #2 has a radius of $6.1\,in$, through how many degrees does Wheel #2 rotate?

13. A crank handle (radius $24.7\,cm$) is fixed with a common center to a gear (radius $17.4\,cm$). The gear turns the larger wheel of a winch (radius $35.1\,cm$) which is concentrically fixed to the smaller wheel (radius $6.9\,cm$) of the winch so that they turn together causing a weight to be lifted. SEE PICTURE If the crank is rotated through an angle of $25°$, how high is the weight lifted?

14. In a bicycle SEE PICTURE the pedals cause the drive wheel to drive a chain around the axle gear which causes the rear tire to rotate. Assume the radii are pedals $7.1\,in$, drive wheel $4.6\,in$, axle gear $2.3\,in$, rear tire $26\,in$. If the pedals are rotated one half of a revolution, how far does the bike travel?