Trigonometry Chapter 3

Section 3

I. Vocabulary

Complete each statement:

1. The EQUATION of the UNIT CIRCLE is given by ____________________ .
2. If (x, y) is any point on the unit circle, the value of $r = \sqrt{x^2 + y^2}$ is always equal to _____ .
3. If (x, y) is any point on the unit circle, the $x,\ y,\ r$ definitions of the TRIGONOMETRIC FUNCTIONS simplify to

$\sin(\theta) =$ ____________________ $\csc(\theta) =$ ____________________

$\cos(\theta) =$ ____________________ $\sec(\theta) =$ ____________________

$\tan(\theta) =$ ____________________ $\cot(\theta) =$ ____________________

4. If (x, y) is any point on the unit circle,

a) the trigonometric functions that have RESTRICTIONS are

____________________, ____________________, ____________________, ____________________

b) the RESTRICTIONS are

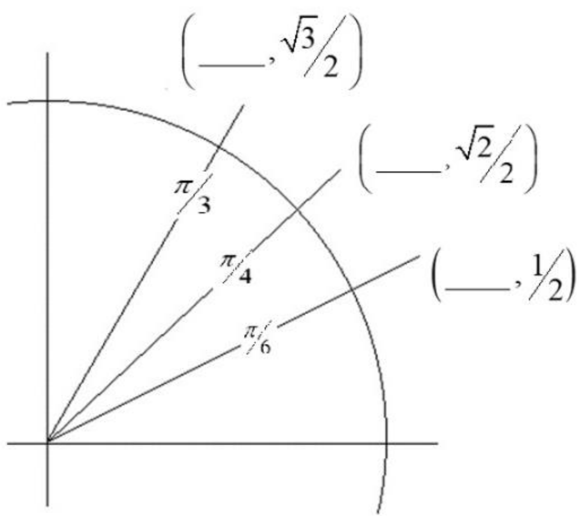
for ____________________, we require ____________________

for ____________________, we require ____________________

for ____________________, we require ____________________

for ____________________, we require ____________________

5. The special points in the UNIT FIRST QUADRANT are



6. If t is any value on the number line and we WRAP THE NUMBER LINE AROUND THE UNIT CIRCLE (as demonstrated in class),
and the value t determines the point (x, y) on the unit circle, (that is, $r = 1$),
and this point determines the angle θ in standard position measured in RADIANS,
then using the x, y, r definitions with $r = 1$ we have:

$\sin(t) = \sin(\theta) =$ _____ $\csc(t) = \csc(\theta) =$ _____

$\cos(t) = \cos(\theta) =$ _____ $\sec(t) = \sec(\theta) =$ _____

$\tan(t) = \tan(\theta) =$ _____ $\cot(t) = \cot(\theta) =$ _____

7. a) From this point forward in the course: i) the symbol $\mathbb{R}$ represents "ALL REAL NUMBERS" and
ii) when used to to construct Domains and Ranges we assume
$k \in \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

b) Using the construction in item #6 above we may define the functions with the accompanying domains and ranges:

$f(t) = \sin(t)$

$D_f : \mathbb{R}$ $R_f :$ ________

$f(t) = \csc(t)$

$D_f :$ ________ $R_f : (-\infty, -1] \cup [1, \infty)$

$f(t) = \cos(t)$

$D_f :$ ________ $R_f : [-1, 1]$

$f(t) = \sec(t)$

$D_f : \mathbb{R} - \left(\pi/2\right) \cdot k$ $R_f :$ ________

$f(t) = \tan(t)$

$D_f :$ ________ $R_f :$ ________

$f(t) = \cot(t)$

$D_f :$ ________ $R_f :$ ________

c) When considering the trig functions in this fashion, your calculator must be in RADIAN mode.

II. For each given angle i) conduct Quadrant Check and ii) determine the reference angle. Note that angles are measured in radians.

Example:

a) $t = 7\pi/6$ b) $t = -5\pi/3$ c) $t = 2.3347$ d) $t = -5.4286$

Solution:

a) $t = 7\pi/6$

i) * Quad Check

$\boxed{QIII}$

ii) * Ref Angle

$$t_R = 7\pi/6 - \pi$$
$$= 7\pi/6 - 6\pi/6$$
$$= \boxed{\pi/6}$$

b) $t = -5\pi/3$

i) * Quad Check

$\boxed{QI}$

ii) * Ref Angle

$$t_R = \boxed{\pi/3}$$

c) $t = 2.3347$

i) * Quad Check

$\pi/2 \approx 1.57$

$\pi \approx 3.14$

So $\pi/2 < t < \pi$

$\boxed{QII}$

ii) * Ref Angle

$t_R = \pi - t$

$= \pi - 2.3347$

$= .8068926...$

$\approx \boxed{.8069}$

d) $t = -5.4286$

Find a coterminal angle in one positive revolution:

$t_1 = -5.4286 + 2\pi = .8545853... \approx .8546$

i) * Quad Check

$\pi/2 \approx 1.57$

So $0 < t_1 < \pi/2$

$\boxed{QI}$

ii) * Ref Angle

$t_R \approx \boxed{.8546}$

8. $t = 5\pi/6$
9. $t = 7\pi/4$
10. $t = 7\pi/6$
11. $t = 5\pi/3$
12. $t = \pi/4$
13. $t = -11\pi/6$
14. $t = -\pi/6$
15. $t = .6647$
16. $t = 3.7261$
17. $t = -1.0973$

III. Find the EXACT VALUES of each expression. If an answer contains a radical, express it both with the radial in the numerator and the denominator.

Example:

a) $\cos\left(2\pi/3\right)$ b) $\tan\left(5\pi/4\right)$ c) $\sin\left(-7\pi/4\right)$ d) $\sec\left(5\pi/6\right)$

Solution:

a) $\cos\left(2\pi/3\right)$

* Quad Check:

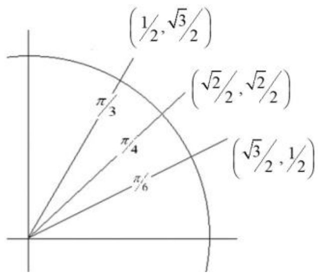
QII $\Rightarrow$ $\cos(t)$ is negative

* Ref angle:

$$t_R = \pi/3$$

* Solve:

Use the Unit First Quadrant



We have

$\cos(t_R)$ is the $x-$coordinate

So:

$$\cos\left(2\pi/3\right) = \cos\left(\left(2\pi/3\right)_R\right) = \cos\left(\pi/3\right) = \boxed{-\sqrt{3}/2} = \boxed{-3/2\sqrt{3}}$$

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b) $\tan\left(5\pi/4\right)$

* Quad Check:

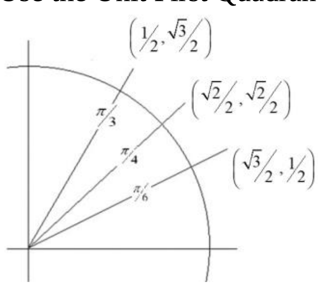
QIII $\Rightarrow$ $\tan(t)$ is positive

* Ref angle:

$$t_R = \pi/4$$

* Solve:

Use the Unit First Quadrant



We have

$$\tan(t_R) = \frac{y}{x} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = \left(\sqrt{2}/2\right)\cdot\left(2/\sqrt{2}\right) = 1$$

So:

$$\tan\left(5\pi/4\right) = \tan\left(\left(5\pi/4\right)_R\right) = \tan\left(\pi/4\right) = \boxed{1}$$

* * * * *   * * * * *

c) $\sin\left(-7\pi/4\right)$

* Quad Check:

QI $\Rightarrow$ $\sin(t)$ is positive

* Ref angle:

$$t_R = \pi/4$$

* Solve:

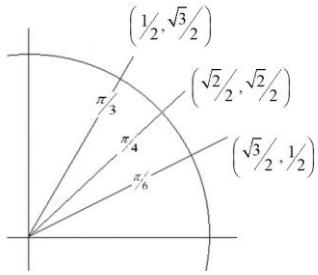
Use the Unit First Quadrant

We have

$\sin(t_R)$ is the $y-$coordinate

So:

$$\sin\left(-7\pi/4\right) = \sin\left(\left(-7\pi/4\right)_R\right) = \sin\left(\pi/4\right) = \boxed{\sqrt{2}/2} = \boxed{1/\sqrt{2}}$$



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d) $\sec\left(\frac{5\pi}{6}\right)$

* Quad Check:

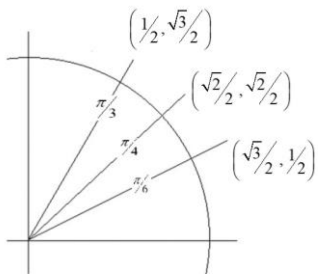
QII $\Rightarrow$ $\sec(t)$ is negative

* Ref angle:

$t_R = \frac{\pi}{6}$

* Solve:

Use the Unit First Quadrant



We have

$\sec(t_R)$ is the reciprocal of $\cos(t_R)$ and

$\cos(t_R)$ is the $x-$coordinate

So:

$$\sec\left(\frac{5\pi}{6}\right) = \frac{1}{\cos\left(\frac{5\pi}{6}\right)}$$

$$= \frac{1}{\cos\left(\left(\frac{5\pi}{6}\right)_R\right)}$$

$$= \frac{1}{\cos\left(\frac{5\pi}{6}\right)}$$

$$= \frac{1}{-\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \boxed{-\frac{2}{\sqrt{3}}} = \boxed{-\frac{2\sqrt{3}}{3}}$$

18. $\cos\left(\frac{\pi}{3}\right)$ 19. $\sin\left(\frac{\pi}{6}\right)$ 20. $\sin\left(\frac{\pi}{4}\right)$ 21. $\cos\left(\frac{\pi}{6}\right)$ 22. $\cos\left(\frac{\pi}{4}\right)$

23. $\tan\left(\frac{\pi}{6}\right)$ 24. $\tan\left(\frac{\pi}{3}\right)$ 25. $\sin\left(\frac{5\pi}{6}\right)$ 26. $\tan\left(\frac{7\pi}{6}\right)$ 27. $\cos\left(\frac{7\pi}{4}\right)$

28. $\sin\left(-\frac{\pi}{4}\right)$ 29. $\tan\left(\frac{5\pi}{4}\right)$ 30. $\sec\left(\frac{\pi}{6}\right)$ 31. $\csc\left(\frac{5\pi}{3}\right)$ 32. $\tan\left(-\frac{3\pi}{4}\right)$

IIII. Solve each equation in the indicated interval.

Example:

a) $\cos(t) = \frac{1}{2}$ in $[0, 2\pi)$

b) $\sin(x) = -\frac{\sqrt{3}}{2}$ in $[0, 2\pi)$

c) $\sec(x) = -\frac{2}{\sqrt{3}}$ in $[0, \pi)$

d) $\tan(x) = -\frac{1}{\sqrt{3}}$ in $[0, \pi)$

Solution:

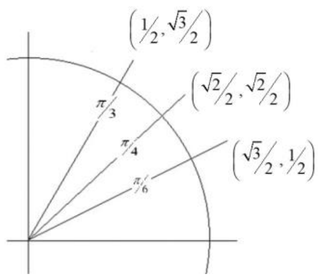
a) $\cos(t) = \frac{1}{2}$ in $[0, 2\pi)$

* Quad Check:

$\cos(t)$ is positive $\Rightarrow QI, QIIII$

* Ref angle:

Use the Unit First Quadrant



$\cos(t)$ is the FIRST coordinate

So $t_R = \frac{\pi}{3}$

* Solve:

Use the Reference Angle Families:

QI: $t = \boxed{\frac{\pi}{3}}$

QIIII: $t = \boxed{\frac{5\pi}{3}}$

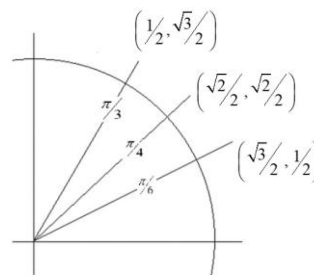
b) $\sin(x) = -\frac{\sqrt{3}}{2}$ in $[0, 2\pi)$

* Quad Check:

$\sin(x)$ is negative $\Rightarrow QIII, QIIII$

* Ref angle:

Use the Unit First Quadrant



$\sin(x)$ is the SECOND coordinate

So $x_R = \frac{\pi}{3}$

* Solve:

Use the Reference Angle Families:

QIII: $x = \boxed{\frac{4\pi}{3}}$

QIIII: $x = \boxed{\frac{5\pi}{3}}$

c) $\sec(x) = -\frac{2}{\sqrt{3}}$ in $[0, \pi)$

* Reciprocal:

$$\sec(x) = -\frac{2}{\sqrt{3}} \Rightarrow \cos(x) = -\frac{\sqrt{3}}{2}$$

* Quad Check:

$\cos(x)$ is negative $\Rightarrow QII, QIII$

Since we're in the interval $[0, \pi)$

use only QII

* Ref angle:

Use the Unit First Quadrant

d) $\tan(x) = -\frac{1}{\sqrt{3}}$ in $[0, \pi)$

* Quad Check:

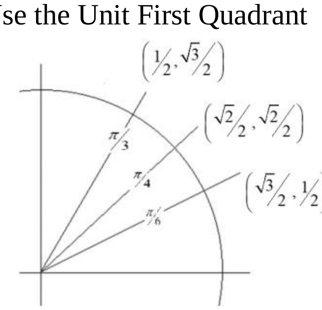
$\tan(x)$ is negative $\Rightarrow QII, QIIII$

Since we're in the interval $[0, \pi)$

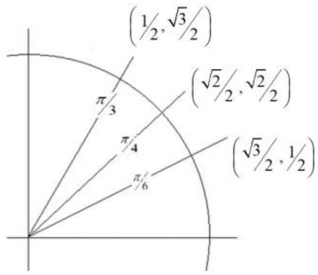
use only QII

* Ref angle:

Use the Unit First Quadrant



We want $\sqrt{3}$ in the denominator



$\cos(x)$ is the FIRST coordinate

So $x_R = \pi/6$

* Solve:

Use the Reference Angle Families:

QII: $x = \boxed{5\pi/6}$

So $x_R = \pi/6$

* Solve:

Use the Reference Angle Families:

QII: $x = \boxed{5\pi/6}$

For items #33 – #38 use the interval $[0, 2\pi)$.

33. $\cos(x) = -\sqrt{2}/2$

34. $\sin(x) = 1/2$

35. $\sin(x) = \sqrt{2}/2$

36. $\cos(x) = \sqrt{3}/2$

37. $\sec(x) = 2/\sqrt{2}$

38. $\tan(x) = -1$

For items #39 – #41 use the interval $[0, \pi)$.

39. $\csc(x) = 2$

40. $\cot(x) = -\sqrt{3}$

41. $\tan(x) = \sqrt{3}/3$