Trigonometry Chapter 5

Section 2

I. Vocabulary

Complete each statement:

1. An TRIGONOMETRIC IDENTITY is a statement involving trig functions which is ____________________ for all values for which the ____________________ of the trig functions is ____________________.
2. An EXPRESSION consists of ____________________ and ____________________ and does not contain an ____________________ sign.
3. An EQUATION consists of two ____________________ which are given as ____________________.
4. a) The verb for an EQUATION is to ____________________.
   b) The verb for an EXPRESSION is to ____________________.
   c) The verb for an IDENTITY is to ____________________.
5. To ____________________ an IDENTITY means to demonstrate that the two sides are ____________________.

II. Techniques/Suggestions for Verifying Identities

Complete each Identity:

6. Watch for fundamental ____________________.
7. Start with the more ____________________ side of the IDENTITY and ____________________ it until you obtain the simpler side.
8. Express all trigonometric functions in terms of ____________________ and ____________________, then ____________________ the result.
9. Watch for the chance to SPLIT THE ____________________: $\frac{a \pm b}{c} = \frac{\_\_\_}{\_\_\_} \pm \frac{\_\_\_}{\_\_\_}$
10. Watch for the chance to MULTIPLY BY THE ____________________:
    For $(1+\sin(\theta))$ multiply by ____________________
    For $(\tan(\theta)-1)$ multiply by ____________________
    For $(\cos(\theta)+1)$ multiply by ____________________ etc.
11. If you create a COMPLEX FRACTION ____________________ it.
12. If there are FRACTIONS to ADD, ____________________ them.
13. If there are FACTORS to MULTIPLY or CANCEL, ____________________ or ____________________ them.
14. When simplifying, keep in mind the other side of the identity which represents the ____________________.
15. If ____________________are of roughly the same COMPLEXITY, you may work on ____________________ individually until each side produces the same result.
16. YOU MAY NOT WORK ON BOTH SIDES OF THE IDENTITY AT THE SAME TIME.
    Write the above sentence three times:
    ____
    ____
    ____

III. Verifying Identities:

Example:

Verify each identity:

a) $\tan(\theta)+1=\sec(\theta)\left(\sin(\theta)+\cos(\theta)\right)$

b) $\dfrac{\cot(t)-\tan(t)}{\sin(t)\cos(t)}=\csc^2(\theta)-\sec^2(\theta)$

c) $\dfrac{1-\cos(x)}{\sin(x)}=\dfrac{\sin(x)}{1-\cos(x)}$

d) $\sec^4(x)-\sec^2(x)=\tan^4(x)+\tan^2(x)$

Solution:

a) $\tan(\theta)+1=\sec(\theta)\left(\sin(\theta)+\cos(\theta)\right)$

Start with Right Hand Side (*RHS*) as it is the more complicated side:

RHS

$$\begin{aligned}\sec(\theta)\left(\sin(\theta)+\cos(\theta)\right)&=\frac{1}{\cos(\theta)}\left(\sin(\theta)+\cos(\theta)\right)\\&=\frac{\sin(\theta)}{\cos(\theta)}+\frac{\cos(\theta)}{\cos(\theta)}\\&=\boxed{\tan(\theta)+1}\\&\quad\ LHS\end{aligned}$$

b) $\dfrac{\cot(t)-\tan(t)}{\sin(t)\cos(t)}=\csc^2(t)-\sec^2(t)$

Start with the LHS

$$\begin{aligned}\frac{\cot(t)-\tan(t)}{\sin(t)\cos(t)}&=\frac{\cot(t)}{\sin(t)\cos(t)}-\frac{\tan(t)}{\sin(t)\cos(t)}\\&=\cot(t)\cdot\frac{1}{\sin(t)\cos(t)}-\tan(t)\cdot\frac{1}{\sin(t)\cos(t)}\\&=\frac{\cos(t)}{\sin(t)}\cdot\frac{1}{\sin(t)\cos(t)}-\frac{\sin(t)}{\cos(t)}\cdot\frac{1}{\sin(t)\cos(t)}\\&=\frac{\cancel{\cos(t)}}{\sin(t)}\cdot\frac{1}{\sin(t)\cancel{\cos(t)}}-\frac{\cancel{\sin(t)}}{\cos(t)}\cdot\frac{1}{\cancel{\sin(t)}\cos(t)}\\&=\frac{1}{\sin^2(t)}-\frac{1}{\cos^2(t)}\\&=\boxed{\csc^2(t)-\sec^2(t)}\\&\quad\ RHS\end{aligned}$$

c) $\dfrac{1-\cos(x)}{\sin(x)}=\dfrac{\sin(x)}{1-\cos(x)}$

LHS

$$\frac{1-\cos(x)}{\sin(x)}=\frac{\left(1-\cos(x)\right)}{\sin(x)}\cdot\frac{\left(1+\cos(x)\right)}{\left(1+\cos(x)\right)}$$

$$=\frac{\left(1-\cos(x)\right)\left(1+\cos(x)\right)}{\sin(x)\left(1+\cos(x)\right)}$$

$$=\frac{\left(1-\cos^2(x)\right)}{\sin(x)\left(1+\cos(x)\right)}$$

$$=\frac{\sin^2(x)}{\sin(x)\left(1+\cos(x)\right)}$$

$$=\frac{\sin^{\not 2}(x)}{\cancel{\sin(x)}\left(1+\cos(x)\right)}$$

$$=\boxed{\frac{\sin(x)}{1+\cos(x)}}$$

RHS

d) $\sec^4(x)-\sec^2(x)=\tan^4(x)+\tan^2(x)$

Both sides are complicated, so work each side INDIVIDUALLY and obtain a common expression.

LHS

$$\sec^4(x)-\sec^2(x)=\sec^2(x)\left(\sec^2(x)-1\right)$$

$$=\boxed{\sec^2(x)\tan^2(x)}$$

RHS

$$\tan^4(x)+\tan^2(x)=\tan^2(x)\left(\tan^2(x)+1\right)$$

$$=\tan^2(x)\sec^2(x)$$

$$=\boxed{\sec^2(x)\tan^2(x)}$$

17. $\dfrac{\csc(\theta)}{\cot(\theta)}=\sec(\theta)$

18. $\dfrac{\sin(x)+\sec(x)}{\tan(x)}=\cos(x)+\csc(x)$

19. $\cos(\theta)\csc(\theta)\tan(\theta)=1$

20. $\csc^2(\theta)-\cot^2(\theta)=1$

21. $\dfrac{1-\sin^2(t)}{\cot(t)}=\sin(t)\cos(t)$

22. $\sin^2(\beta)\left(1+\cot^2(\beta)\right)=1$

23. $\frac{\cos(t)+1}{\tan^2(t)}=\frac{\cos(t)}{\sec(t)-1}$

24. $\frac{1}{1-\sin(\alpha)}+\frac{1}{1+\sin(\alpha)}=2\sec^2(\alpha)$

25. $\frac{\cot(x)+1}{\cot(x)-1}=\frac{1+\tan(x)}{1-\tan(x)}$

26. $\frac{\csc(x)+\cot(x)}{\tan(x)+\sin(x)}=\cot(x)\csc(x)$

28. $\frac{\tan(\theta)}{1+\cos(\theta)}+\frac{\sin(\theta)}{1-\cos(\theta)}=\cot(\theta)+\sec(\theta)\csc(\theta)$

29. $\sin(x)+\cos(x)=\frac{\sin(x)}{1-\cot(x)}+\frac{\cos(x)}{1-\tan(x)}$