Trigonometry Chapter 6

Section 1

IMPORTANT NOTICE: The two notations

$f(x) = \arcsin(x)$, read "arcsine" and $f(x) = \sin^{-1}(x)$, read "sine inverse" or "inverse sine" are interchangeable and may be freely subsituted for eachother. While there are some fields of study that typically employ $f(x) = \arcsin(x)$, from this point forward in the course, I will employ the $f(x) = \sin^{-1}(x)$ notation. Similarly for the other five functions $f(x) = \arccos(x)$ "arccosine" & $f(x) = \cos^{-1}(x)$, "cosine inverse", etc.

SUPER IMPORTANT NOTICE: While it IS true that the notation x^{-1} DOES mean $\frac{1}{x}$ it is VITAL to remember that the notation $\sin^{-1}(x)$ **DOES NOT** MEAN $\frac{1}{\sin(x)}$ **!**

Similarly for the other five inverse trig functions.

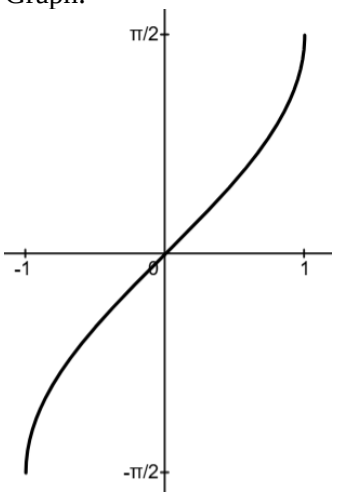
I. The Inverse Trigonometric Functions

1. $f(x) = \sin^{-1}(x)$

   Domain: $[-1,1]$

   Range: $\left[-\pi/2,\ \pi/2\right]$

Graph:

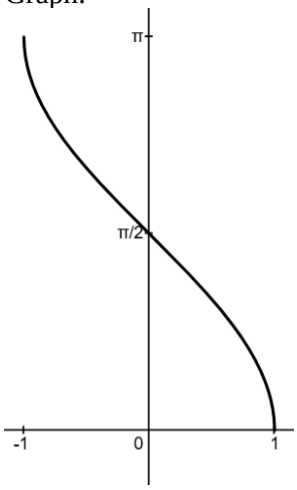


2. $f(x) = \cos^{-1}(x)$

   Domain: $[-1,1]$

   Range: $[0,\ \pi]$

Graph:

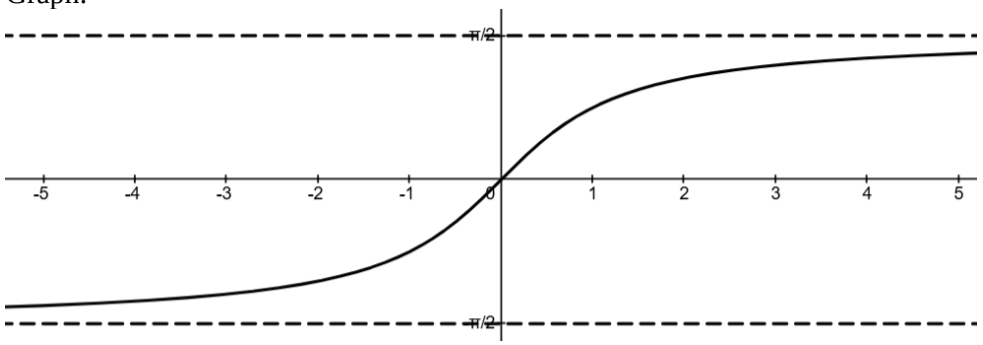


3. $f(x) = \tan^{-1}(x)$

Domain: $(-\infty, \infty)$

Range: $\left(-\pi/2, \pi/2\right)$

Graph:

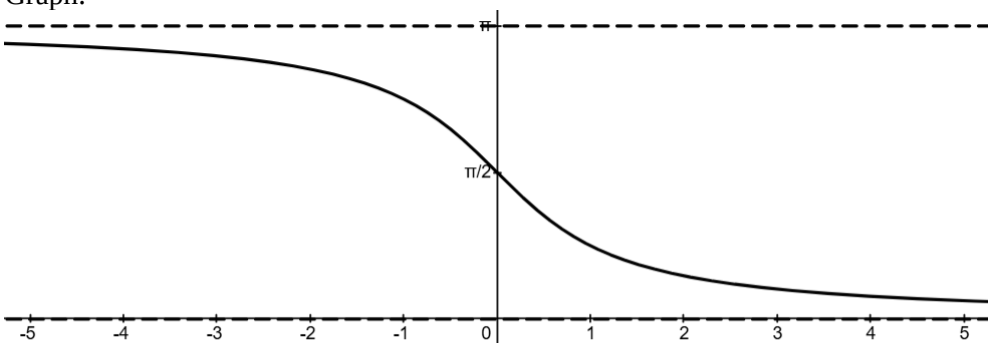


4. $f(x) = \cot^{-1}(x)$

Domain: $(-\infty, \infty)$

Range: $(0,\ \pi)$

Graph:



5. $f(x) = \sec^{-1}(x)$

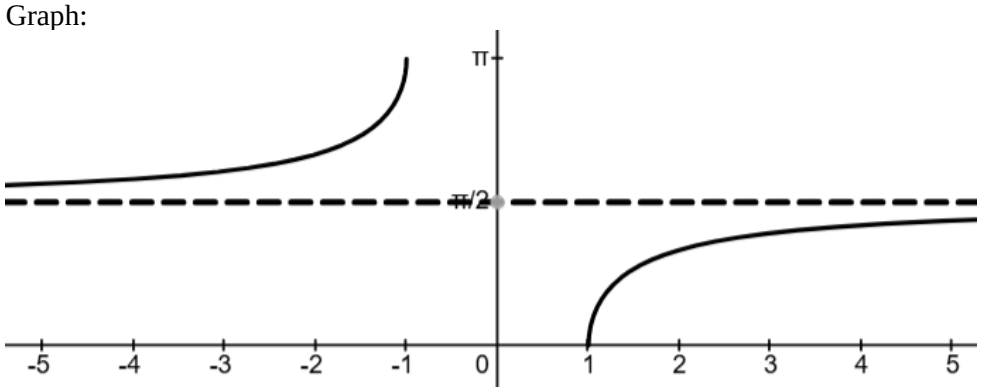
Domain:

$(-\infty, -1] \cup [1, \infty)$

Range:

$\left[0, \pi/2\right) \cup \left(\pi/2, \pi\right]$

Graph:



6. $f(x) = \csc^{-1}(x)$

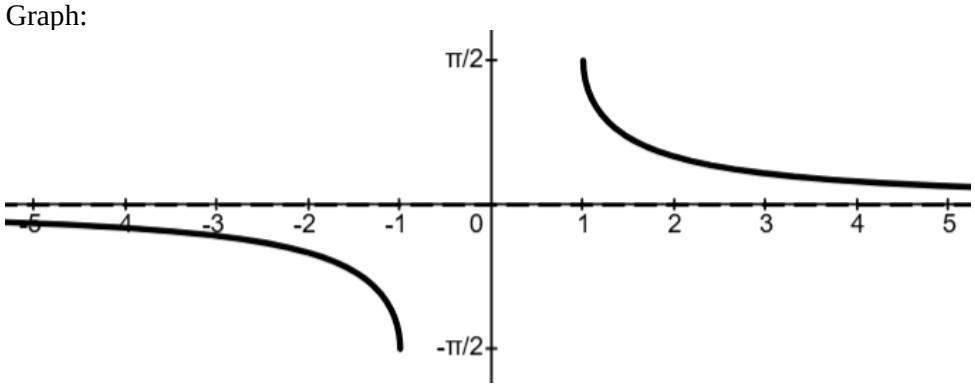
Domain:

$(-\infty, -1] \cup [1, \infty)$

Range:

$\left[-\pi/2, 0\right) \cup \left(0, \pi/2\right]$

Graph:



IMPORTANT NOTICE: THE INFORMATION LISTED ABOVE FOR
$f(x)=\sin^{-1}(x)$, $f(x)=\cos^{-1}(x)$, AND $f(x)=\tan^{-1}(x)$ MUST BE MEMORIZED.

QUIZZES WILL BE GIVEN

II. Sum and Difference Identities
Complete each Identity:

1. $\sin(A+B)=$ _____
2. $\sin(A-B)=$ _____
3. $\cos(A+B)=$ _____
4. $\cos(A-B)=$ _____

III. Double Angle Identities
Complete each Identity:

5. $\sin(2\theta)=$ _____
6. $\cos(2\theta)=$ _____
7. $=$ _____
8. $=$ _____

IIII. Half Angle Identities
Complete each Identity:

9. $\sin^2(\theta)=\frac{1}{2}[$ _____ $]$
10. $\cos^2(\theta)=\frac{1}{2}[$ _____ $]$

IMPORTANT NOTICE: THE 10 IDENTITIES LISTED ABOVE MUST BE MEMORIZED.

QUIZZES WILL BE GIVEN

V. Use a calculator to approximate each value. Express your answer in decimal **degrees** rounded to three decimal places.
Example:

a) $\theta=\sin^{-1}(0.56843262)$ b) $\alpha=\sec^{-1}(3.77141808)$ c) $x=\tan^{-1}(1.31921947)$

Solution:

a) Use the $\boxed{\sin^{-1}}$ key on your calculator. Be sure it is in degree mode.

$$\theta=\sin^{-1}(0.56843262)$$
$$=34.64099975...$$
$$\approx \boxed{34.641^\circ}$$

b) $\alpha = \sec^{-1}(3.77141808)$

$\alpha = \sec^{-1}(3.77141808)$

$\Rightarrow \sec(\alpha) = 3.77141808$

$\Rightarrow \cos(\alpha) = \dfrac{1}{3.77141808}$

$\Rightarrow \alpha = \cos^{-1}\left(\dfrac{1}{3.77141808}\right)$

$= 74.62400013...$

$\approx \boxed{74.624°}$

Or, alternately (and more simply):

$\alpha = \sec^{-1}(3.77141808)$

$= \cos^{-1}\left(\dfrac{1}{3.77141808}\right)$

$= 74.62400013...$

$\approx \boxed{74.624°}$

c) Use the $\boxed{\tan^{-1}}$ key on your calculator. Be sure it is in degree mode.

$x = \tan^{-1}(1.31921947)$

$= 52.83699592....$

$\approx \boxed{52.837°}$

11. $\theta = \sin^{-1}(0.56843262)$

12. $\beta = \tan^{-1}(-0.607890001)$

13. $\theta = \cos^{-1}(0.63683217)$

14. $x = \csc^{-1}(-3.28713844)$

15. $\alpha = \tan^{-1}(5.30452870)$

16. $\theta = \sec^{-1}(1.09709696)$

17. $\phi = \cos^{-1}(-0.96284744)$

18. $x = \cot^{-1}(-1.79701432)$

19. $\theta = \sin^{-1}(-0.14171320)$

VI. Use a calculator to approximate each value. Express your answer in **radians** rounded to 4 decimal places.

Example:

a) $x = \sin^{-1}(0.56843262)$ b) $x = \sec^{-1}(-2.55434891)$ c) $x = \tan^{-1}(-7.22659753)$

Solution:

a) Use the $\boxed{\sin^{-1}}$ key on your calculator. Be sure it is in radian mode.

$x = \sin^{-1}(0.56843262)$

$= .6045994774...$

$\approx \boxed{.6046}$

b) $x = \sec^{-1}(-2.55434891)$

$x = \sec^{-1}(-2.55434891)$

$\Rightarrow \sec(x) = -2.55434891$

$\Rightarrow \cos(x) = \dfrac{1}{-2.55434891}$

$\Rightarrow x = \cos^{-1}\left(-\dfrac{1}{2.55434891}\right)$

$= 1.97304573...$

$\approx \boxed{1.9730}$

Or, alternately (and more simply):

$x = \sec^{-1}(-2.55434891)$

$= \cos^{-1}\left(-\dfrac{1}{2.55434891}\right)$

$= 1.97304573...$

$\approx \boxed{1.9730}$

c) Use the $\boxed{\tan^{-1}}$ key on your calculator. Be sure it is in degree mode.

$x = \tan^{-1}(-7.22659753)$

$= -1.43329185....$

$\approx \boxed{-1.4333}$

11. $x = \sin^{-1}(-.95233356)$

12. $x = \tan^{-1}(-2.22587769)$

13. $x = \csc^{-1}(1.22097218)$

14. $x = \cos^{-1}(-.45234798)$

15. $x = \sec^{-1}(5.07825403)$

16. $x = \cos^{-1}(.15171913)$

17. $x = \tan^{-1}(-2.84746474)$

18. $x = \sin^{-1}(-.17033086)$

19. $x = \cot^{-1}(-9.34530276)$