Trigonometry Chapter 6

Section 2

I. Flow Chart for Solving Trig Equations

1. Insert the following statements in the proper locations in the flow chart.

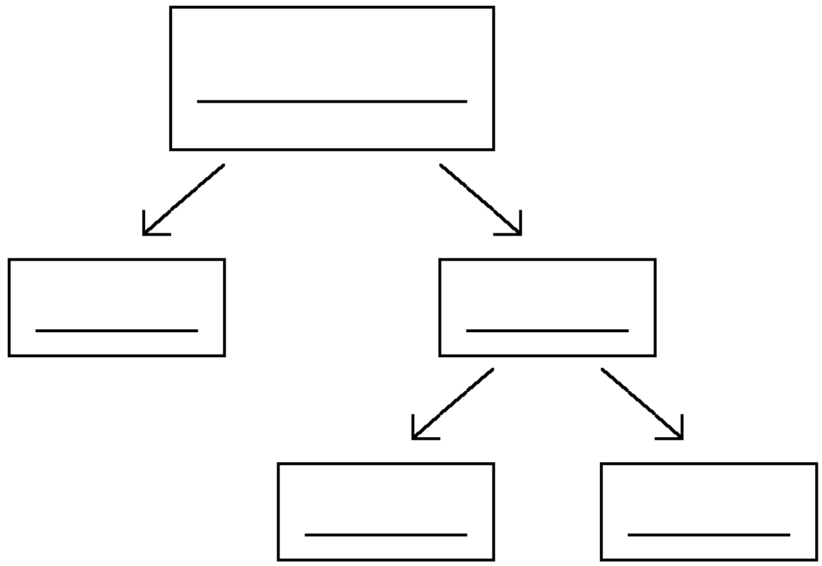
NON-QUADRANTAL ANGLE

SPECIAL ANGLE

SOLVING TRIG EQUATIONS

QUADRANTAL ANGLE

ANYTHING ELSE



2. Solving an equation involving a QUADRANTAL angle can be done by referencing the __________________ of the function.
3. Solving an equation involving a QUADRANTAL angle requires NEITHER a __________________ __________________ NOR finding a __________________ __________________.
4. Solving an equation involving a NON-QUADRANTAL angle DOES require BOTH a __________________ __________________ AND finding a __________________ __________________.

II. Equations Involving Quadrantal Angles

Solve each equation over the indicated interval

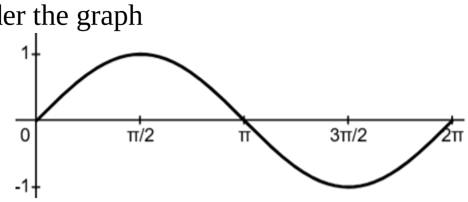
Example:

a) $\sin(x) = -1 \quad [0, 2\pi)$ b) $\tan(x) = 0 \quad [0, 2\pi)$ c) $\cos(x) = 1 \quad (\pi, 2\pi)$

Solution:

a) $\sin(x) = -1 \quad [0, 2\pi)$

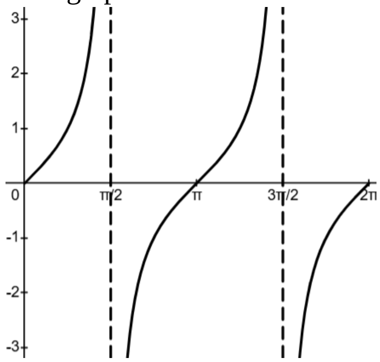
Consider the graph



The graph obtains a trough in the interval $[0, 2\pi)$ at $\boxed{x = 3\pi/2}$

b) $\tan(x)=0 \quad [0,2\pi)$

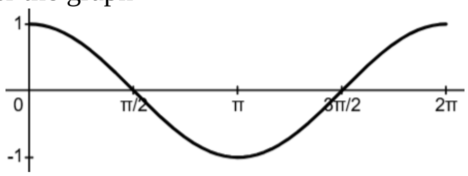
Consider the graph



The graph crosses the x-axis in the interval $[0,2\pi)$ at $\boxed{x=0}$ & $\boxed{x=\pi}$

c) $\cos(x)=1 \quad (\pi,2\pi)$

Consider the graph



In the interval $(\pi,2\pi)$ the graph does not achieve a peak, so we have $\boxed{\textit{No Solution}}$.

5. $\sin(x)=0 \quad [0,2\pi)$
6. $\sin(x)=1 \quad [0,2\pi)$
7. $\sin(x)=-1 \quad [0,2\pi)$
8. $\cos(x)=1 \quad [0,2\pi)$
9. $\cos(x)=-1 \quad [0,2\pi)$
10. $\cos(x)=0 \quad [0,2\pi)$
11. $\tan(x)=0 \quad [0,2\pi)$
12. $\cos(x)=0 \quad [0,\pi]$
13. $\sin(x)=1 \quad (0,\pi)$
14. $\sin(x)=0 \quad [-2\pi,2\pi]$
15. $\tan(x)=0 \quad \left(\pi/2, 3\pi/2\right)$
16. $\cos(x)=-1 \quad [-\pi,\pi]$

Example:

a) $-2\sin(x)-2=0 \quad [0,2\pi)$ b) $\sin(x)\tan(x)-\tan(x)=0 \quad [0,2\pi)$

Solution:

a) Isolate:

Linear in sine

* Isolate:

$$-2\sin(x)-2=0$$
$$\Rightarrow -2\sin(x)=2$$
$$\Rightarrow \sin(x)=2/-2$$
$$\Rightarrow \sin(x)=-1$$

*Reference the graph:

$$\Rightarrow \boxed{x=3\pi/2}$$

b) Functions are multiplied:

*Factor:

$$\sin(x)\tan(x)-\tan(x)=0$$
$$\Rightarrow \tan(x)\left(\sin(x)-1\right)=0$$
$$\Rightarrow \tan(x)=0 \quad or \quad \sin(x)-1=0$$
$$\Rightarrow \tan(x)=0 \quad or \quad \sin(x)=1$$

*Reference the graphs:

$$\Rightarrow \boxed{x=0};\ \boxed{x=\pi} \quad or \quad \boxed{x=\pi/2}$$

17. $\sin(x)-2=-1 \quad [0,2\pi)$ 18. $5\tan(x)-6=-6 \quad [0,2\pi)$ 19. $\sec(x)+1=0 \quad [0,2\pi)$

20. $\sin(x)\cos(x)-\cos(x)=0 \quad [0,2\pi)$ 21. $\csc(x)\cot(x)=\cot(x) \quad [0,2\pi)$

III. Equations Involving Special Angles

Example:

a) $2\sin(x)\cos(x)+\cos(x)=0 \quad [0,2\pi)$ b) $(2\cos(x)+1)(\tan(x)-1)=0 \quad [0,2\pi)$

Solution:

a) Functions are multiplied, so Factor:

$$2\sin(x)\cos(x)+\cos(x)=0$$
$$\Rightarrow \cos(x)(2\sin(x)+1)=0$$
$$\Rightarrow \cos(x)=0 \quad or \quad 2\sin(x)+1=0$$
$$\Rightarrow x=\pi/2; \quad x=3\pi/2$$

* Isolate:

$$\sin(x)=-1/2$$

* Quadrant Check:
Sine is negative in QIII, QIIII

* Reference Angle:

$$x_R=\pi/6$$

* Solve:

$$x=7\pi/6 \quad or \quad x=11\pi/6$$

So we have:

$$\boxed{\pi/2,\quad 3\pi/2,\quad 7\pi/6,\quad 11\pi/6}$$

Solution:

a) Functions are multiplied, so Factor:

$$2\sin(x)\cos(x)+\cos(x)=0$$
$$\Rightarrow \cos(x)(2\sin(x)+1)=0$$
$$\Rightarrow \cos(x)=0 \quad or \quad 2\sin(x)+1=0$$
$$\Rightarrow x=\pi/2; \quad x=3\pi/2$$

* Isolate:

$$\sin(x)=-1/2$$

* Quadrant Check:
Sine is negative in QIII, QIIII

* Reference Angle:

$$x_R=\pi/6$$

* Solve:

$$x=7\pi/6 \quad or \quad x=11\pi/6$$

So we have:

$$\boxed{\pi/2,\ 3\pi/2,\ 7\pi/6,\ 11\pi/6}$$

22. $2\cos(x)+3=4 \quad [0,2\pi)$

23. $5\tan(x)-1=4 \quad [0,2\pi)$

24. $2\sec(x)+1=\sec(x)+3 \quad [0,2\pi)$

25. $(\tan(x)+1)(\sqrt{3}\tan(x)-1)=0 \quad [0,2\pi)$

26. $\sin^2(x)\cos(x)=\cos(x) \quad [0,2\pi)$

27. $2\sin(x)\cos(x)=0 \quad [0,2\pi)$

28. $2\cos^2(x)+\cos(x)-1=0 \quad [0,2\pi)$

29. $\sec^2(x)\tan(x)=2\tan(x) \quad [0,2\pi)$

IIII. Equations Involving "Other" Angles

Example:

a) $5\sin(x)+14=10 \quad [0,2\pi)$

b) $21\cos^2(x)+11\cos(x)-2=0 \quad [0,2\pi)$

Solution:

a) $5\sin(x)+14=10 \quad [0,2\pi)$

* Isolate:

$$5\sin(x)+14=10$$
$$\Rightarrow 5\sin(x)=-4$$
$$\Rightarrow \sin(x)=-\frac{4}{5}$$
$$\Rightarrow \sin(x)=-0.8$$

* Quadrant Check:
Sine is negative in QIII, QIIII

* Reference Angle:

$$x_R=\sin^{-1}(.8)$$
$$=.92729521...$$

* Solve:

$QIII$ or $QIIII$

$$x=\pi+.92729521... \qquad x=2\pi-.92729521...$$
$$=4.06888787... \qquad =5.35589008...$$
$$\approx \boxed{4.0689} \qquad \approx \boxed{5.3559}$$

b) $21\cos^2(x)+11\cos(x)-2=0 \quad [0,2\pi)$

* Factor:

$(7\sin(x)-1)(3\sin(x)+2)=0$

$\Rightarrow 7\sin(x)-1=0$ *or* $\Rightarrow 3\sin(x)+2=0$

* Isolate:	*Isolate:
$\Rightarrow \sin(x) = 1/7$	$\Rightarrow \sin(x) = -2/3$
$\Rightarrow x = \sin^{-1}(.14285714...)$	$\Rightarrow x = \sin^{-1}(.66666666...)$
* Quadrant Check: Sine is positive in QI, QII	* Quadrant Check: Sine is negative in QIII, QIIII
* Reference Angle: $x_R = \sin^{-1}(.14285714...) = .14334756...$	* Reference Angle: $x_R = \sin^{-1}(.66666666...) = .72972765...$

* Solve:

Case I		Case II	
QI	*QII*	*QIII*	*QIIII*
$x = .14334756...$	$x = \pi - .14334756...$	$x = \pi + .72972765...$	$x = 2\pi - .72972765...$
$\approx \boxed{.1433}$	$= 2.99824508...$	$= 3.87132031...$	$= 5.55345765...$
	$\approx \boxed{2.9982}$	$\approx \boxed{3.8713}$	$\approx \boxed{5.5535}$

30. $5\tan(x)+17=1 \quad [0,2\pi)$

31. $-13\sin(x)-2=4 \quad [0,2\pi)$

32. $4\sec(x)+9=0 \quad [0,2\pi)$

33. $(\tan(x)+4)(\sqrt{7}\cot(x)-1)=0 \quad [0,2\pi)$

34. $3\sin^2(x)-13\sin(x)+4=0 \quad [0,2\pi)$

35. $\csc^2(x)+\csc(x)-20=0 \quad [0,2\pi)$

V. Equations solved using either Difference of Squares or $\pm$ Square Root method:

Example:

a) $\sec^2(x)-2=0 \quad [0,2\pi)$ b) $4\sin^2(x)-9=0 \quad [0,2\pi)$

Solution:

a) $\sec^2(x)-2=0 \quad [0,2\pi)$

Difference of Squares method	$\pm$ Square Root method
$\Rightarrow (\sec(x)+\sqrt{2})(\sec(x)-\sqrt{2})=0$	$\Rightarrow \sec^2(x)=2$
$\Rightarrow \sec(x)+\sqrt{2}=0$ *or* $\sec(x)-\sqrt{2}=0$	$\Rightarrow \sec(x)=\pm\sqrt{2}$
$\Rightarrow \sec(x)=-\sqrt{2}$ *or* $\sec(x)=\sqrt{2}$	
*Reciprocals:	*Reciprocals:
$\Rightarrow \cos(x) = -1/\sqrt{2}$ *or* $\cos(x) = 1/\sqrt{2}$	$\Rightarrow \cos(x) = \pm 1/\sqrt{2}$
*Quadrant Check: QII, QIII *or* QI, QIIII Thus, all four Quadrants	*Quadrant Check: All four Quadrants

From this point onward the solution is the same in both methods.

*Reference Angle:

$x_R = \pi/4$

*Solve:

$x = \boxed{\pi/4}, \quad x = \boxed{3\pi/4}, \quad x = \boxed{5\pi/4}, \quad x = \boxed{7\pi/4}$

b) $4\sin^2(x) - 9 = 0 \quad [0, 2\pi)$

Difference of Squares method	$\pm$ Square Root method
$\Rightarrow 4\sin^2(x) - 9 = 0$ $\Rightarrow 2\sin(x) + 3 = 0 \quad or \quad 2\sin(x) - 3 = 0$ $\Rightarrow \sin(x) = -3/2 \quad or \quad \sin(x) = 3/2$	$\Rightarrow 4\sin^2(x) - 9 = 0$ $\Rightarrow \sin^2(x) = 9/4$ $\Rightarrow \sin(x) = \pm\sqrt{9/4}$ $\Rightarrow \sin(x) = \pm 3/2$

From this point onward the solution is the same in both methods.

*Reference angle:

Since $3/2$ is greater than one and outside the range of the sine function, there is no Reference angle

Hence there is $\boxed{\textit{No Solution}}$

36. $\tan^2(x) - 1 = 0 \quad [0, 2\pi)$

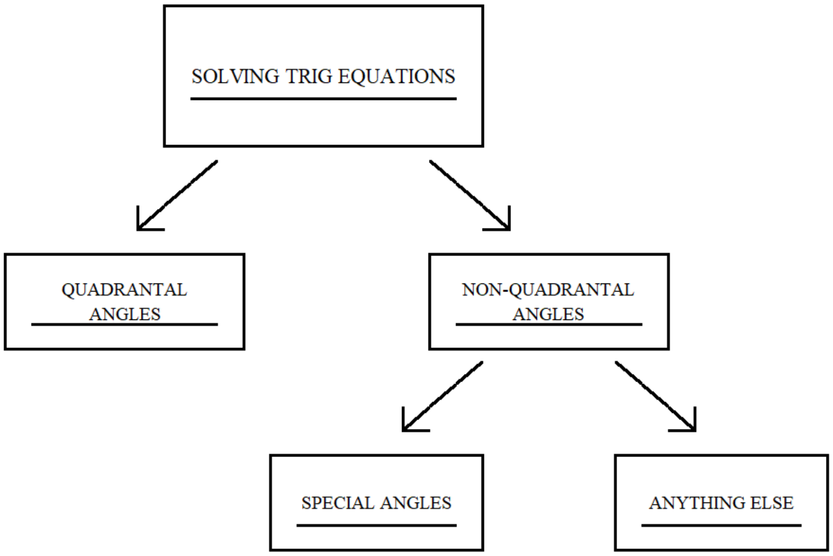
37. $\cos^2(x) - 1 = 0 \quad [0, 2\pi)$

38. $4\sin^2(x) - 3 = 0 \quad [0, 2\pi)$

39. $5\cos^2(x) - 2 = 0 \quad [0, 2\pi)$

Answers:

1.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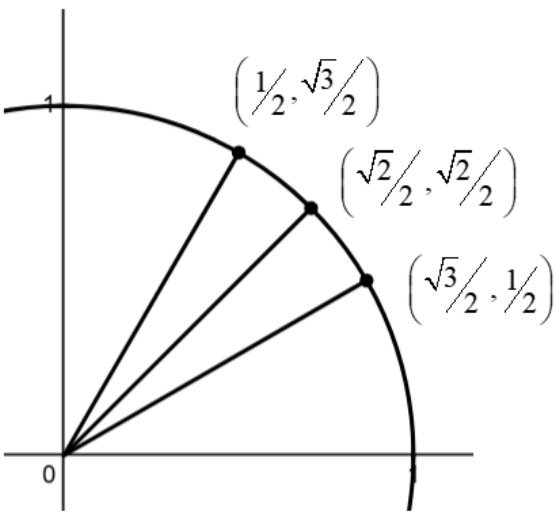
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
1
0
1