solutions

① Prove $\lim_{x \to 2} (4x-3) = 5$

Ⅰ Let $\epsilon > 0$

Ⅱ Search for δ

we want

$|f(x) - L| < \epsilon \Rightarrow |(4x-3) - 5| < \epsilon$

$\Rightarrow |4x - 3 - 5| < \epsilon$

$\Rightarrow |4x - 8| < \epsilon$

$\Rightarrow |4(x-2)| < \epsilon$

$\Rightarrow |4|\,|x-2| < \epsilon$

$\Rightarrow |x-2| < \frac{\epsilon}{|4|}$

Choose $\delta = \frac{\epsilon}{|4|}$

Ⅲ Verify δ:

$0 < |x-a| < \delta \Rightarrow |x-2| < \frac{\epsilon}{|4|}$

$\Rightarrow |4|\,|x-2| < \epsilon$

$\Rightarrow |4(x-2)| < \epsilon$

$\Rightarrow |4x-8| < \epsilon$

$\Rightarrow |4x-3+3-8| < \epsilon$

$\Rightarrow |(4x-3) - 5| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$ ✓

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② Prove $\lim_{x \to 4} (-6x+17) = -7$

Ⅰ Let $\epsilon > 0$

Ⅱ Search for δ

we want

$|f(x) - L| < \epsilon \Rightarrow |(-6x+17) - (-7)| < \epsilon$

$\Rightarrow |-6x+17+7| < \epsilon$

$\Rightarrow |-6x+24| < \epsilon$

$\Rightarrow |(-6)(x-4)| < \epsilon$

$\Rightarrow |-6|\,|x-4| < \epsilon$

$\Rightarrow |x-4| < \frac{\epsilon}{|-6|}$

Choose $\delta = \frac{\epsilon}{|-6|}$

Ⅲ Verify this δ

$0 < |x-a| < \delta \Rightarrow |x-4| < \frac{\epsilon}{|-6|}$

$\Rightarrow |-6|\,|x-4| < \epsilon$

$\Rightarrow |-6(x-4)| < \epsilon$

$\Rightarrow |-6x+24| < \epsilon$

$\Rightarrow |-6x+17-17+24| < \epsilon$

$\Rightarrow |(-6x+17)+7| < \epsilon$

$\Rightarrow |(-6x+17) - (-7)| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$ ✓

③ Prove $\lim_{x\to(-5)} (2x+13)=3$

Ⓘ Let $\epsilon>0$

Ⓘⓘ Search for δ

we want

$|f(x)-L|<\epsilon \Rightarrow |(2x+13)-3|<\epsilon$

$\Rightarrow |2x+13-3|<\epsilon$

$\Rightarrow |2x+10|<\epsilon$

$\Rightarrow |2(x+5)|<\epsilon$

$\Rightarrow |2||x-(-5)|<\epsilon$

$\Rightarrow |x-(-5)|<\frac{\epsilon}{|2|}$

Choose $\delta=\frac{\epsilon}{|2|}$

Ⓘⓘⓘ Verify this δ:

$0<|x-a|<\delta \Rightarrow |x-(-5)|<\frac{\epsilon}{|2|}$

$\Rightarrow |2||x+5|<\epsilon$

$\Rightarrow |2(x+5)|<\epsilon$

$\Rightarrow |2x+10|<\epsilon$

$\Rightarrow |2x+13-13+10|<\epsilon$

$\Rightarrow |(2x+13)-3|<\epsilon$

$\Rightarrow |f(x)-L|<\epsilon$ ✓

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④. Prove $\lim_{x\to 0} (6x-4)=-4$

Ⓘ Let $\epsilon>0$

Ⓘⓘ Search for δ

we want

$|f(x)-L|<\epsilon \Rightarrow |(6x-4)-(-4)|<\epsilon$

$\Rightarrow |6x-4+4|<\epsilon$

$\Rightarrow |6x|<\epsilon$ — alternate version

$\Rightarrow |6||x|<\epsilon$ $\quad |6||x-0|<\epsilon$

$\Rightarrow |x|<\frac{\epsilon}{|6|}$ $\quad |x-0|<\frac{\epsilon}{|6|}$

Choose $\delta=\frac{\epsilon}{|6|}$

Ⓘⓘⓘ Verify this δ:

$0<|x-a|<\delta \Rightarrow |x-0|<\frac{\epsilon}{|6|}$

$\Rightarrow |6||x|<\epsilon$

$\Rightarrow |6x|<\epsilon$

$\Rightarrow |6x-4+4|<\epsilon$

$\Rightarrow |(6x-4)+4|<\epsilon$

$\Rightarrow |(6x-4)-(-4)|<\epsilon$

$\Rightarrow |f(x)-L|<\epsilon$ ✓

⑤ Prove $\lim_{x \to (-7)} (x+3) = -4$

(I) Let $\epsilon > 0$

(II) Search for δ

we want

$$|f(x) - L| < \epsilon \Rightarrow |(x+3) - (-4)| < \epsilon$$

$$\Rightarrow |x+3+4| < \epsilon$$

$$\Rightarrow |x+7| < \epsilon$$

$$\Rightarrow |x-(-7)| < \epsilon$$

choose $\delta = \epsilon$

(III) Verify δ:

$$0 < |x - a| < \epsilon \Rightarrow |x - (-7)| < \epsilon$$

$$\Rightarrow |x+7| < \epsilon$$

$$\Rightarrow |x+3-3+7| < \epsilon$$

$$\Rightarrow |(x+3)+4| < \epsilon$$

$$\Rightarrow |(x+3)-(-4)| < \epsilon$$

$$\Rightarrow |f(x) - L| < \epsilon$$

⑥ Prove $\lim_{x \to 4} (-4x+4) = -12$

(I) Let $\epsilon > 0$

(II) Search for δ

we want

$$|f(x) - L| < \epsilon \Rightarrow |(-4x+4) - (-12)| < \epsilon$$

$$\Rightarrow |-4x+4+12| < \epsilon$$

$$\Rightarrow |-4x+16| < \epsilon$$

$$\Rightarrow |-4(x-4)| < \epsilon$$

$$\Rightarrow |-4||x-4| < \epsilon$$

$$\Rightarrow |x-4| < \frac{\epsilon}{|-4|}$$

Choose $\delta = \frac{\epsilon}{|-4|}$

(III) Verify the δ

$$0 < |x-a| < \delta \Rightarrow |x-4| < \frac{\epsilon}{|-4|}$$

$$\Rightarrow |-4||x-4| < \epsilon$$

$$\Rightarrow |-4(x-4)| < \epsilon$$

$$\Rightarrow |-4x+16| < \epsilon$$

$$\Rightarrow |+4x+4-4+16| < \epsilon$$

$$\Rightarrow |(-4x+4)+12| < \epsilon$$

$$\Rightarrow |(-4x+4)-(-12)| < \epsilon$$

$$\Rightarrow |f(x) - L| < \epsilon$$

(7). Prove $\lim_{x \to a} (mx+b) = (ma+b)$

(I) Let $\epsilon > 0$

(II) Search for δ

We want

$|f(x) - L| < \epsilon \Rightarrow |(mx+b) - (ma+b)| < \epsilon$

$\Rightarrow |mx+b-ma-b| < \epsilon$

$\Rightarrow |mx-ma+b-b| < \epsilon$

$\Rightarrow |m(x-a)| < \epsilon$

$\Rightarrow |m||x-a| < \epsilon$

$\Rightarrow |x-a| < \frac{\epsilon}{|m|}$

Choose $\delta = \frac{\epsilon}{|m|}$

(III) Verify this δ

$0 < |x-a| < \delta \Rightarrow |x-a| < \frac{\epsilon}{|m|}$

$\Rightarrow |m||x-a| < \epsilon$

$\Rightarrow |m(x-a)| < \epsilon$

$\Rightarrow |mx-ma| < \epsilon$

$\Rightarrow |mx+b-ma-b| < \epsilon$

$\Rightarrow |(mx+b)-(ma+b)| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$ ✓