SOLUTIONS
PRACTICE EXAM I
CALC I FALL '24

I

1. C
2. E ⇒ There are 2
   one at $x=5$
   one at $x=7$
3. D
4. B
5. C
6. D
7. C
8. E
9. C
10. A
11. B
12. E The limit is $1+\pi$
13. E
14. C

II

15. $x^2-4=0 \Rightarrow x=\pm 2$ But $x=2$ is not in the interval $x\le 1$ so $x=-2$ is a discontinuity

Must check at $x=1$

(a) $f(1) \overset{TOP}{=} \frac{-3}{(1)^2-4} = \frac{-3}{-3} = 1$

(b) $\lim_{x\to 1} f(x)$

(i) $\lim_{x\to 1^-} f(x) \overset{TOP}{=} \lim_{x\to 1^-}\left(\frac{-3}{x^2-4}\right) = \frac{-3}{(1)^2-4} = \frac{-3}{-3} = 1$

(ii) $\lim_{x\to 1^+} f(x) \overset{bot}{=} \lim_{x\to 1^+}(3-x^2) = 3-(1)^2 = 2$

since the left & right-hand limits do not agree $\lim_{x\to 1} f(x)$ D.N.E.

∴ Not continuous at $x=1$

SO THE DISCONTINUITIES are at $\boxed{x=-2,\ x=1}$

(21). we require

$$\lim_{x\to(-2)^-} f(x) = \lim_{x\to(-2)^+} f(x)$$

$$\Rightarrow \lim_{x\to(-2)^-} (x^2 - kx) = \lim_{x\to(-2)^+} (4x+2)$$

$$\Rightarrow (-2)^2 - k(-2) = 4(-2)+2$$

$$\Rightarrow 4+2k = -6$$

$$\Rightarrow \boxed{k=-5}$$

(22) $$\lim_{x\to 3}\frac{(\sqrt{x+1}-2)}{(x-3)} = \lim_{x\to 3}\frac{(\sqrt{x+1}-2)}{(x-3)}\cdot\frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3}\frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3}\frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2}$$

$$= \frac{1}{\sqrt{3+1}+2}$$

$$= \frac{1}{2+2}$$

$$= \boxed{\frac{1}{4}}$$

(23) $f(x) = \begin{cases} -\cos(x) & x \le 0 \\ \tan(x) & x > 0 \end{cases}$

Must check at $x=0$

(a) $f(0) \stackrel{top}{=} -\cos(0) = -1$

(b) $\lim_{x\to 0} f(x)$

(i) $\lim_{x\to 0^-} f(x) \stackrel{top}{=} \lim_{x\to 0^-} (-\cos(x)) = -\cos(0) = -1$

(ii) $\lim_{x\to 0^+} f(x) \stackrel{BOT}{=} \lim_{x\to 0^+} (\tan(x)) = \tan(0) = 0$

since these do not equal $\lim_{x\to 0} f(x)$ D.N.E.

$\therefore f$ is not continuous at $x=0$

f is also not continuous at $x = \pi/2$ & $x = \frac{3\pi}{2}$

But it is continuous at $x = -\pi/2$ and $x = -3\pi/2$

(do you see why?)

so Discontinuities at $\boxed{x=0 \quad x=\pi/2 \quad x=3\pi/2}$

(24) $$\lim_{x\to 4} \frac{x^2-6x+8}{x^2-x-12} = \lim_{x\to 4} \frac{\cancel{(x-4)}(x-2)}{\cancel{(x-4)}(x+3)}$$

NOTE DIRECT substitution gives "$\frac{0}{0}$"

$$= \lim_{x\to 4} \frac{(x-2)}{(x+3)}$$

$$= \frac{4-2}{4+3}$$

$$= \boxed{\frac{2}{7}}$$

(25) $\lim_{x\to 0} \frac{2\tan(3x)}{x} = \lim_{x\to 0} \frac{2\sin(3x)}{\cos(3x)\cdot x}$

$= \lim_{x\to 0} \frac{3\cdot 2\cdot \sin(3x)}{3\cdot\cos(3x)\cdot x}$

$= \lim_{x\to 0} \frac{6}{\cos(3x)} \cdot \lim_{x\to 0} \frac{\sin(3x)}{3x}$

$= \frac{6}{\cos(3(0))} \cdot 1$

$= \frac{6}{1} \cdot 1$

$= \boxed{6}$

(26) $f(x) = \begin{cases} 4x - x^2 & \text{if } x \le -1 \\ \frac{15}{x+4} & \text{if } x > -1 \end{cases}$

(a) $f(-1) \overset{\text{TOP}}{=} 4(-1) - (-1)^2 = -4 - 1 = -5$

(b) $\lim_{x\to(-1)} f(x)$

(i) $\lim_{x\to(-1)^-} f(x) \overset{\text{top}}{=} \lim_{x\to(-1)^-} (4x - x^2) = 4(-1) - (-1)^2 = -5$

(ii) $\lim_{x\to(-1)^+} f(x) \overset{\text{BOT}}{=} \lim_{x\to(-1)^+} \left(\frac{15}{x+4}\right) = \frac{15}{(-1)+4} = \frac{15}{3} = 5$

But the left- and right-hand limits do not agree so $\lim_{x\to(-1)} f(x)$ D.N.E.

so **NOT** continuous at $x = -1$

(27). $\lim_{x \to (-3)} (-5x-6) = 9$

(I) Let $\epsilon > 0$

(II) Search for δ:

$|f(x) - L| < \epsilon \Rightarrow |(-5x-6) - 9| < \epsilon$

$\Rightarrow |-5x-15| < \epsilon$

$\Rightarrow |-5(x+3)| < \epsilon$

$\Rightarrow |-5(x-(-3))| < \epsilon$

$\Rightarrow |-5|\,|x-(-3)| < \epsilon$

$\Rightarrow |x-(-3)| < \frac{\epsilon}{|-5|}$

choose $\delta = \frac{\epsilon}{|-5|}$

(III) Verify δ:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-(-3)| < \frac{\epsilon}{|-5|}$

$\Rightarrow |-5|\,|x+3| < \epsilon$

$\Rightarrow |-5(x+3)| < \epsilon$

$\Rightarrow |-5x-15| < \epsilon$

$\Rightarrow |-5x-6+6-15| < \epsilon$

$\Rightarrow |(-5x-6)-9| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$