Solutions

(I) ① $\int \tan(x)\sec^3(x)\,dx = \int \sec^2(x)\cdot \underbrace{\sec(x)\tan(x)}\,dx$

Let $u = \sec(x)$

$\Rightarrow du = \sec(x)\tan(x)\,dx$

$= \int u^2\,du$

$= \frac{u^3}{3} + C$

$= \boxed{\frac{\sec^3(x)}{3} + C}$

②. $\int -8x\sin(x)\,dx$

Parts $u = -8x$ $\quad V = -\cos(x)$

$du = -8dx \quad dv = \sin(x)\,dx$

$= \overset{u}{(-8x)}\overset{v}{(-\cos(x))} - \int \overbrace{(-\cos(x))}^{v}\overbrace{(-8)\,dx}^{du}$

$= 8x\cos(x) - \int 8\cos(x)\,dx$

$= \boxed{8x\cos(x) - 8\sin(x) + C}$

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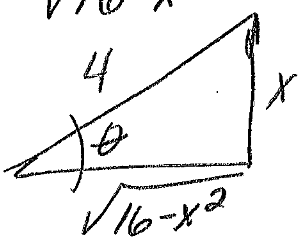
③ $\int \cos^2(3x)\,dx = \frac{1}{2}\int\left[1+\cos(6x)\right]dx$

$$= \boxed{\frac{1}{2}\left[x+\frac{1}{6}\sin(6x)\right]+C}$$

④ $\int \frac{x^3}{\sqrt{16-x^2}}\,dx = \int \frac{(4\sin(\theta))^3(\cancel{4\cos(\theta)})}{\cancel{4\cos(\theta)}}\,d\theta$



$= 64\int \sin^3(\theta)\,d\theta$

$= 64\int \sin^2(\theta)\cdot\sin(\theta)\,d\theta$

$= 64\int \left(1-\cos^2(\theta)\right)\cdot\sin(\theta)\,d\theta$

Let $u = \cos(\theta)$

$\Rightarrow du = -\sin(\theta)\,d\theta$

$\Rightarrow -du = \sin(\theta)\,d\theta$

$= -64\int (1-u^2)\,du$

$= -64\left[u-\frac{u^3}{3}\right]+C$

$= \boxed{-64\left[\cos(\theta)-\frac{\cos^3(\theta)}{3}\right]+C}$

choose

$\sin(\theta) = \frac{x}{4}$

$\Rightarrow 4\sin(\theta) = x$

$\Rightarrow 4\cos(\theta)\,d\theta = dx$

choose

$\cos(\theta) = \frac{\sqrt{16-x^2}}{4}$

$\Rightarrow 4\cos(\theta) = \sqrt{16-x^2}$

(5). $\int \frac{-2x^2-12}{x^3-x^2-6x}\,dx = \int \frac{-2(x^2+6)}{x(x^2-x-6)}\,dx$ Pg 3hree

$$= \int \frac{-2x^2-12}{x(x-3)(x+2)}\,dx$$

Par Frac Category I

$$\Rightarrow \frac{-2x^2-12}{(x)(x-3)(x+2)} = \frac{A}{x} + \frac{B}{(x-3)} + \frac{C}{(x+2)}$$

$$\Rightarrow -2x^2-12 = A(x-3)(x+2) + B(x)(x+2) + C(x)(x-3)$$

Let $x=3$

$$\Rightarrow -2(3)^2-12 = A(0) + B(3)(3+2) + C(0)$$

$$\Rightarrow -30 = B(15) \Rightarrow B=-2$$

Let $x=-2$

$$\Rightarrow -2(-2)^2-12 = A(0) + B(0) + C(-2)(-2-3)$$

$$\Rightarrow -20 = C(10) \Rightarrow C=-2$$

Let $x=0$

$$\Rightarrow -2(0)^2-12 = A(0-3)(0+2) + B(0) + C(0)$$

$$\Rightarrow -12 = A(-6) \Rightarrow A=2$$

so we have:

$$= \int \left[\frac{2}{x} - \frac{2}{(x-3)} - \frac{2}{(x+2)} \right] dx$$

$$= \boxed{2\ln|x| - 2\ln|x-3| - 2\ln|x+2| + C}$$

6. $\int \sec^4(x)\,dx = \int \sec^2(x)\,\sec^2(x)\,dx$

$= \int (\tan^2(x)+1)\cdot \sec^2(x)\,dx$

Let $u = \tan(x)$
$du = \sec^2(x)\,dx$

$= \int (u^2+1)\,du$

$= \frac{u^3}{3} + u + C$

$= \boxed{\frac{\tan^3(x)}{3} + \tan(x) + C}$

7. $\int \frac{3x^2+2x+27}{x^3+9x}\,dx = \int \frac{3x^2+2x+27}{x(x^2+9)}\,dx$

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category III

$\Rightarrow \frac{3x^2+2x+27}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$

$\Rightarrow 3x^2+2x+27 = A(x^2+9)+(Bx+C)(x)$

$\Rightarrow 3x^2+2x+27 = Ax^2+9A+Bx^2+Cx$

$= (A+B)x^2+Cx+9A$

cont.

EQUATE COEFFICIENTS

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CONT. $\Rightarrow$

$$\begin{aligned} A+B &= 3 \longrightarrow 3+B=3 \Rightarrow B=0 \\ C &= 2 \Rightarrow C=2 \\ 9A &= 27 \Rightarrow A=3 \end{aligned}$$

so we have

$$= \int \left[\frac{3}{x} + \frac{2}{x^2+9} \right] dx$$

$$= \int \frac{3}{x}\,dx + 2\int \frac{1}{x^2+9}$$

$\tan^{-1}(u)$ form, $u=x$, $a=3$

$$= \boxed{3\ln|x| + (2)\left(\tfrac{1}{3}\right)\tan\left(\tfrac{x}{3}\right) + C}$$

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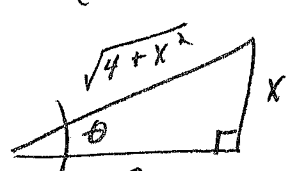
(8) $\int \frac{4x}{(4+x^2)^2}\,dx$ <u>REWRITE</u> $\int \frac{4x}{\left[\left(\sqrt{4+x^2}\right)^2\right]^2}\,dx$

$$= \int \frac{4x}{\left(\sqrt{4+x^2}\right)^4}\,dx$$



<u>Choose</u> $\tan(\theta) = \frac{x}{2}$

$\Rightarrow 2\tan(\theta) = x$

$\Rightarrow 2\sec^2(\theta)\,d\theta = dx$

<u>Choose</u> $\sec(\theta) = \frac{\sqrt{4+x^2}}{2}$

$\Rightarrow 2\sec(\theta) = \sqrt{4+x^2}$

$$= \int \frac{(4)(2\tan(\theta))2\sec^2(\theta)}{2\sec(\theta)}\,d\theta$$

$$= \int 8\tan(\theta)\sec(\theta)\,d\theta$$

$$= 8\sec(\theta) + C$$

$$= \boxed{8\left(\frac{\sqrt{4+x^2}}{2}\right) + C}$$

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⑨. $\int_3^{12} \frac{1}{\sqrt{x-3}}\,dx =$ Note the function has a vertical asymptote at $x=3$

$= \lim_{a\to 3^+}\left[\int_a^{12} \frac{1}{\sqrt{x-3}}\,dx\right] = \lim_{a\to 3^+}\left[\int_a^{12} (x-3)^{-1/2}\,dx\right]$

Let $u = \cancel{x}-3$
$du = dx$

$= \lim_{a\to 3^+}\left[\int_j^k u^{-1/2}\,dx\right]$

$= \lim_{a\to 3^+}\left[\frac{u^{1/2}}{1/2}\right]_j^k$

$= \lim_{a\to 3^+}\left[2(x-3)^{1/2}\right]_a^{12}$

$= \lim_{a\to 3^+}\left[(2(12-3)^{1/2}) - (2(a-3)^{1/2})\right]$

$= \lim_{a\to 3^+}\left[2(3) - 2(a-3)^{1/2}\right]$

$= \left[6 - 2(3-3)^{1/2}\right]$

$= 6 - 0$

$= \boxed{6}$

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(10). $\int_1^{\infty} \frac{9}{x^4}\,dx = \lim_{b\to\infty}\left[\int_1^b 9x^{-4}\,dx\right]$

$= \lim_{b\to\infty}\left[\frac{9x^{-3}}{-3}\right]_1^b$

$= \lim_{b\to\infty}\left[-3x^{-3}\right]_1^b$

$= \lim_{b\to\infty}\left[\frac{-3}{x^3}\right]_1^b$

$= \lim_{b\to\infty}\left[\left(\frac{-3}{b^3}\right)-\left(\frac{-3}{1^3}\right)\right]$

$= \lim_{b\to\infty}\left[\left(\frac{-3}{b^3}\right)+3\right]$

$= \left[\frac{-3}{\infty}\xrightarrow{}0 + 3\right]$

$= 0+3$

$= \boxed{3}$

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(11). $\int_{-\infty}^{1} 2xe^{2x}\,dx = \lim_{a\to(-\infty)}\left[\int_a^1 2xe^{2x}\,dx\right.$

parts $u = 2x$ $\quad v = \frac{1}{2}e^{2x}$

$du = 2dx$ $\quad dv = e^{2x}$

$= \lim_{a\to(-\infty)}\left[(2x)\left(\frac{1}{2}e^{2x}\right)\right]_a^1 - \int (2)\left(\frac{1}{2}e^{2x}\right)dx$

$= \lim_{a\to(-\infty)}\left[xe^{2x}\right]_a^1 - \int_a^1 e^{2x}\,dx$

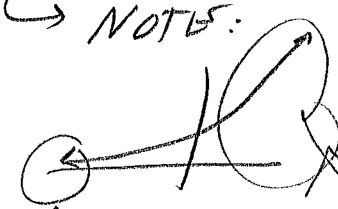
$= \lim_{a\to(-\infty)}\left[xe^{2x} - \frac{1}{2}e^{2x}\right]_a^1$

Cont.

Q. 11 cont.

$$= \lim_{a\to(-\infty)} \left[\left(1e^{2(1)} - \tfrac{1}{2}e^{2(1)}\right) - \left(ae^{2a} - \tfrac{1}{2}e^{2(a)}\right)\right]$$ pg 8ight

NOTES:



$\lim_{a\to(-\infty)} e^{2a} = 0$

$\lim_{a\to\infty} e^{2a} = \infty$

$$= \left[\left(e^2 - \tfrac{1}{2}e^2\right) - \lim_{a\to(-\infty)} \left(ae^{2a} - \tfrac{1}{2}e^{2a}\right)\right.$$

$$= \left(e^2 - \tfrac{1}{2}e^2\right) - \lim_{a\to(-\infty)} ae^{2a} + \lim_{a\to(-\infty)} \tfrac{1}{2}e^{2a}$$

$$\lim_{a\to(-\infty)} ae^{2a} \longrightarrow (-\infty)e^{2(-\infty)} \longrightarrow \frac{-\infty}{e^{2(\infty)}} \longrightarrow \frac{\infty}{\infty}$$

so

$$\lim_{a\to(-\infty)} ae^{2a} = \lim_{a\to(-\infty)} \frac{a}{e^{-2a}} \overset{L'H}{=} \lim_{a\to(-\infty)} \frac{1}{-2e^{-2a}} = \frac{1}{-2e^{-2(-\infty)}} = \frac{1}{-2e^{\infty}} = 0$$

$$\lim_{a\to(-\infty)} \tfrac{1}{2}e^{2a} = \tfrac{1}{2}e^{2(-\infty)} = \tfrac{1}{2}e^{-\infty} = \frac{1}{2e^{\infty}} = 0$$

so we have

$$= \left(e^2 - \tfrac{1}{2}e^2\right) - 0 + 0$$

$$= \boxed{e^2 - \tfrac{1}{2}e^2} \text{ or } \boxed{\tfrac{1}{2}e^2}$$

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(12) $\int_0^{\pi/2} \sec(x)\,dx$: Note $y=\sec(x)$ has a vertical asymptote at $x=\pi/2$

$$= \lim_{b\to \pi/2^-}\left[\int_0^b \sec(x)\,dx\right]$$

$$= \lim_{b\to \pi/2^-}\left[\ln|\sec(x)+\tan(x)|\right]_0^b$$

$$= \lim_{b\to \pi/2^-}\left[(\ln|\sec(b)+\tan(b)|)-(\ln|\sec(0)+\tan(0)|)\right]$$

$$= \lim_{b\to \pi/2^-}\left[(\ln|\sec(b)+\tan(b)|)-(\ln|1+0|)\right]$$

$$= \lim_{b\to \pi/2^-}\left[(\ln|\sec(b)+\tan(b)|)-(0)\right]$$

$$= \left[(\ln|\sec(\pi/2^-)+\tan(\pi/2^-)|)-(0)\right]$$

$$= \left[(\ln|\infty+\infty|)-(0)\right]$$

$$= \infty - 0$$

$$= \boxed{\infty}.$$

(13). $\lim_{x\to 0} \frac{4x}{\tan^{-1}(x)} \to \frac{0}{0}$

$\Rightarrow \lim_{x\to 0} \frac{4x}{\tan^{-1}(x)} \overset{L'H}{=} \lim_{x\to 0} \frac{4}{\frac{1}{\sqrt{x^2+1}}}$

$= \frac{4}{\frac{1}{\sqrt{0+1}}}$

$= \boxed{4}$.

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(14). $\lim_{x\to 0} \frac{e^{2x}+x-1 \to 0}{x^2-2x \to 0}$

$\underline{so}$ $\lim_{x\to 0} \frac{e^{2x}+x-1}{x^2-2x} \overset{L'H}{=} \lim_{x\to 0} \frac{2e^{2x}+1}{2x-2}$

$= \frac{2e^{2(0)}+1}{2(0)-2}$

$= \frac{2(1)+1}{-2}$

$= \boxed{-\frac{3}{2}}$

(15) $\lim_{x\to 0^+} x^{(x^2)} \to 0^0 \to$ <u>indeterminate</u>

$$\Rightarrow \ln\left[\lim_{x\to 0^+} x^{(x^2)}\right] = \lim_{x\to 0^+}\left[\ln\left(x^{(x^2)}\right)\right]$$

$$= \lim_{x\to 0^+}\left[x^2 \ln(x)\right]$$

$$= \lim_{x\to 0^+}\left[\frac{\ln(x)}{x^{-2}}\right]$$

$$= \lim_{x\to 0^+}\left[\frac{\ln(x)}{\frac{1}{x^2}}\right] \begin{matrix}\to -\infty \\ \to \infty\end{matrix}$$

$$\stackrel{L'H}{=} \lim_{x\to 0^+}\left[\frac{\frac{1}{x}}{-2x^{-3}}\right]$$

$$= \lim_{x\to 0^+}\left[\frac{1/x}{\frac{-2}{x^3}}\right]$$

$$= \lim_{x\to 0^+}\left[\frac{1}{x}\cdot\frac{x^3}{-2}\right]$$

$$= \lim_{x\to 0^+}\left[\frac{x^2}{-2}\right]$$

$$= 0$$

$\underline{\underline{\text{so}}}$ $\ln\left[\lim_{x\to 0^+}\left(x^{(x^2)}\right)\right] = 0$

$$\Rightarrow \lim_{x\to 0^+}\left(x^{(x^2)}\right) = e^0$$

$$= \boxed{1}$$