Basic Forms from Calculus I

1. Definition of a Limit:

$$\lim_{x\to a} f(x) = L \Leftrightarrow \text{ for all } \varepsilon > 0, \text{ there exists a } \delta \text{ such that if } 0 < |x-a| < \delta \text{ then } |f(x) - L| < \varepsilon$$

2. $\lim_{x\to a} k = k$

3. $\lim_{x\to a} x = a$

4. $\lim_{x\to a} x^n = a^n$

5. $\lim_{x\to a}[kf(x)] = k\lim_{x\to a}[f(x)]$

6. $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a}[f(x)] \pm \lim_{x\to a}[g(x)]$

7. $\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a}[f(x)]\cdot\lim_{x\to a}[g(x)]$

8. $\lim_{x\to a}\left[[f(x)]^n\right] = \left[\lim_{x\to a}[f(x)]\right]^n$

9. $\lim_{x\to a}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim_{x\to a}[f(x)]}{\lim_{x\to a}[g(x)]}$ $\quad \lim_{x\to a}[g(x)] \neq 0$

10. $\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$

11. $\lim_{x\to 0}\dfrac{\sin(x)}{x} = 1$

12. $\lim_{x\to 0}\dfrac{1-\cos(x)}{x} = 0$

13. Definition of Continuity:

$f(x)$ is continuous at $x = a$ if and only if:

A. $f(a)$ is defined

B. $\lim_{x\to a} f(x)$ exists

C. $\lim_{x\to a} f(x) = f(a)$

14. Definition of the Derivative:

$$\frac{d}{dx}[f(x)] = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

15. $\frac{d}{dx}[k] = 0$

16. $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

17. $\frac{d}{dx}[k \cdot f(x)] = k\, f\,'(x)$

18. $\frac{d}{dx}[f(x) \pm g(x)] = f\,'(x) \pm g\,'(x)$

19. $\frac{d}{dx}[f(x) \cdot g(x)] = f(x)\, g\,'(x) + g(x)\, f\,'(x)$

20. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\, f\,'(x) - f(x)\, g\,'(x)}{[g(x)]^2}$

21. $\frac{d}{dx}[\sin(x)] = \cos(x)$

22. $\frac{d}{dx}[\cos(x)] = -\sin(x)$

23. $\frac{d}{dx}[\tan(x)] = \sec^2(x)$

24. $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$

25. $\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$

26. $\frac{d}{dx}[\cot(x)] = -\csc^2(x)$

27. $\frac{d}{dx}[f(g(x))] = f\,'(g(x)) \cdot g\,'(x)$

Note: For the remaining forms, we assume u and v are functions of x and u' and v' are is their derivatives.

28. $\frac{d}{dx}[f(u)] = f'(u) \cdot u'$

29. $\frac{d}{dx}\left[u^n\right] = nu^{n-1} \cdot u'$

30. $\frac{d}{dx}[k \cdot u] = k u'$

31. $\frac{d}{dx}[u \pm v] = u' \pm v'$

32. $\frac{d}{dx}[u \cdot v] = u v' + v u'$

33. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v u' - u v'}{v^2}$

34. $\frac{d}{dx}[\sin(u)] = \cos(u) \cdot u'$

35. $\frac{d}{dx}[\cos(u)] = -\sin(u) \cdot u'$

36. $\frac{d}{dx}[\tan(u)] = \sec^2(u) \cdot u'$

37. $\frac{d}{dx}[\sec(u)] = \sec(u)\tan(u) \cdot u'$

38. $\frac{d}{dx}[\csc(u)] = -\csc(u)\cot(u) \cdot u'$

39. $\frac{d}{dx}[\cot(u)] = -\csc^2(u) \cdot u'$

40. $\frac{d}{dx}\left[e^u\right] = e^u \cdot u'$

41. $\frac{d}{dx}[\ln(u)] = \frac{1}{u} \cdot u'$

42. $\frac{d}{dx}\left[\sin^{-1}(u)\right] = \frac{u'}{\sqrt{1-u^2}}$

43. $\frac{d}{dx}\left[\tan^{-1}(u)\right] = \frac{u'}{1+u^2}$

44. $\frac{d}{dx}\left[\sec^{-1}(u)\right] = \frac{u'}{|u|\sqrt{u^2-1}}$

45. $\int k\ du = k u + C$

46. $\int u^n\ du = \frac{u^{n+1}}{n+1} + C \quad n \neq -1$

47. $\int k\ f(u)\ du = k \int f(u)\ du$

48. $\int [f(u) \pm g(u)]\ du = \int f(u)\,du \pm \int g(u)\,du$

49. $\int \sin(u)\ du = -\cos(u) + C$

50. $\int \csc(u)\cot(u)\ du = -\csc(u) + C$

51. $\int \cos(u)\ du = \sin(u) + C$

52. $\int \sec(u)\tan(u)\ du = \sec(u) + C$

53. $\int \sec^2(u)\ du = \tan(u) + C$

54. $\int \csc^2(u)\ du = -\cot(u) + C$

55. $\int e^u\ du = e^u + C$

56. $\int \frac{1}{u}\ du = \ln|u| + C$

57. $\int u^{-1}\ du = \ln|u| + C$

58. $\int \tan(u)\ du = \ln|\sec(u)| + C$

59. $\int \cot(u)\,du = \ln|\sin(u)| + C$

60. $\int \sec(u)\ du = \ln|\sec(u) + \tan(u)| + C$

61. $\int f(au+b)\ du = \frac{1}{a} F(au+b) + C$

62. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$

63. $\int \frac{1}{a^2 + u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

64. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{|u|}{a}\right) + C$