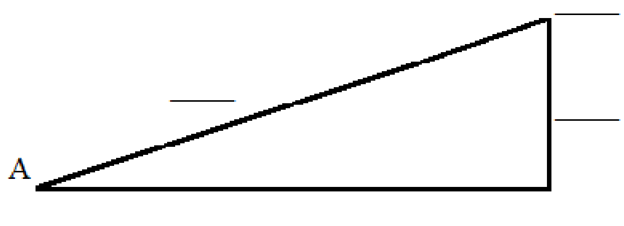
Trigonometry Chapter 2

Section 1

I. Vocabulary

Complete each statement:

1. In any RIGHT TRIANGLE the LONGEST side is always the ____________________.
2. In any RIGHT TRIANGLE the HYPOTENUSE is always opposite the ____________________ angle
   And the other two sides are called the ____________________.
3. In any RIGHT TRIANGLE which is labeled ΔABC, UNLESS THERE IS A COMPELLING REASON OTHERWISE, the vertex determined by the ____________________ will always be labeled with the letter C.
4. The mnemonic device "SOHCAHTOA" stands for:
   ____________________ is ____________________ over ____________________
   ____________________ is ____________________ over ____________________
   ____________________ is ____________________ over ____________________.
5. If A is one of the acute angles in right ΔABC, write the terms ADJACENT SIDE, OPPOSITE SIDE, and HYPOTENUSE in the correct blank. (Each will be used exactly once.)
   a) the ____________________ is the side opposite the right angle,
   b) the ____________________ is one of the sides of angle A,
   c) the ____________________ is not one of the sides of angle A.
6. In right triangle ΔABC having C as its right angle and $\angle A$ given, the correct labeling of the sides and angles is:



7. The COFUNCTIONS of each of the six trig functions are:

   The cofunction of $\sin(\theta)$ is ____________________.
   The cofunction of $\cos(\theta)$ is ____________________.
   The cofunction of $\tan(\theta)$ is ____________________.
   The cofunction of $\csc(\theta)$ is ____________________.
   The cofunction of $\sec(\theta)$ is ____________________.
   The cofunction of $\cot(\theta)$ is ____________________.

8. The rule "COFUNCTIONS OF COMPLEMENTS ARE EQUAL" implies:

$$\sin(\theta) = \cos(\_\_\_ - \theta) \qquad \csc(\theta) = \_\_\_(\_\_\_ - \theta)$$
$$\_\_\_(\theta) = \sin(90^\circ - \theta) \qquad \_\_\_(\theta) = \csc(\_\_\_ - \theta)$$
$$\tan(\theta) = \_\_\_(90^\circ - \theta) \qquad \cot(\theta) = \_\_\_(\_\_\_ - \theta)$$

9. In a $30-60-90$ triangle:
   The length of the shorter leg is ____________________ the hypotenuse.
   The length of the longer leg is ____________________ times the hypotshorter leg.
10. In a $45-45-90$ triangle:
   The length of each of the legs is the hypotenuse divided by ____________________.
   The length of the hypotenuse is ____________________ times either of the legs.

II. For each triangle, find values or, if they are variables, expressions for

$\sin(A) \quad \cos(A) \quad \tan(A) \quad \sin(B) \quad \cos(B) \quad \tan(B)$

Express your answers fractions. (NOT decimal values if your answer is a number)

Example:

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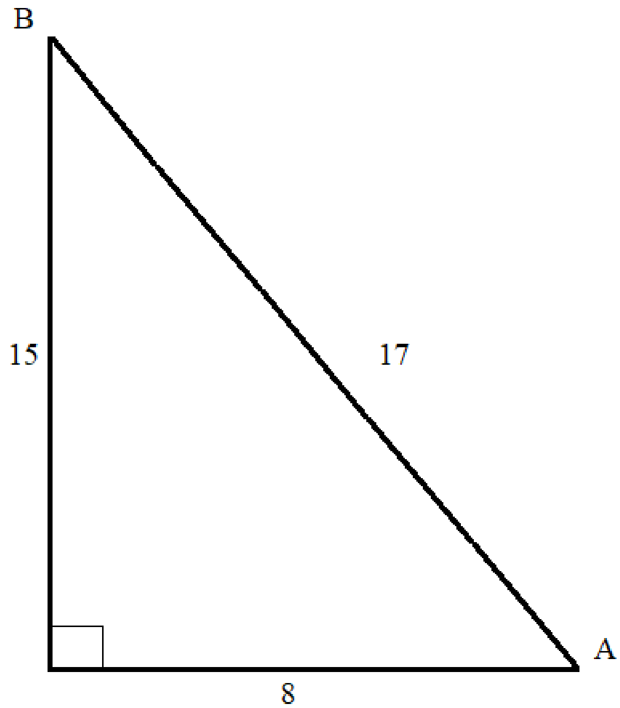


$\sin(A) =$ ___________

$\cos(A) =$ ___________

$\tan(A) =$ ___________

$\sin(B) =$ ___________

$\cos(B) =$ ___________

$\tan(B) =$ ___________

Solution: Note that with respect to $\angle A$, $adj = 8, \quad opp = 15, \quad hyp = 17$

with respect to $\angle B$, $adj = 15, \quad opp = 8, \quad hyp = 17$

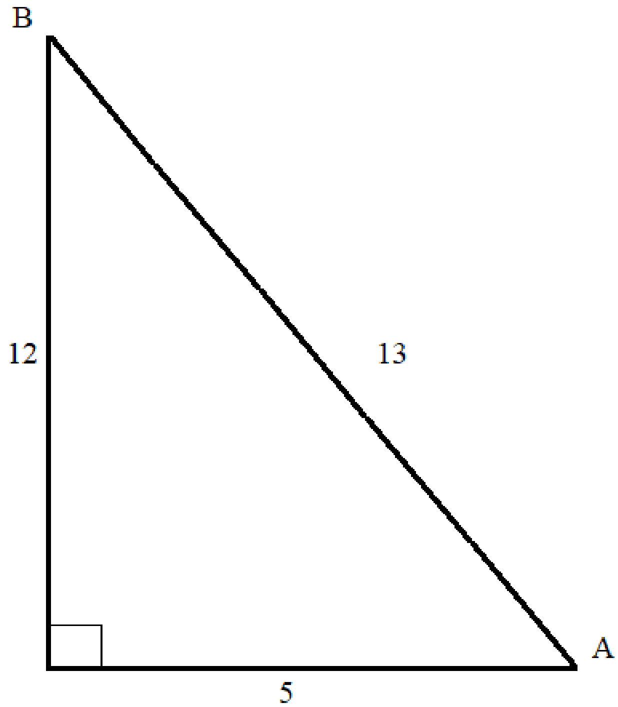
So we have:

$$\sin(A) = \frac{opp}{hyp} = \boxed{\frac{15}{17}} \qquad \sin(B) = \frac{opp}{hyp} = \boxed{\frac{8}{17}}$$

$$\cos(A) = \frac{adj}{hyp} = \boxed{\frac{8}{17}} \qquad \cos(B) = \frac{adj}{hyp} = \boxed{\frac{15}{17}}$$

$$\tan(A) = \frac{opp}{adj} = \boxed{\frac{15}{8}} \qquad \tan(B) = \frac{opp}{adj} = \boxed{\frac{8}{15}}$$

11.



$\sin(A) =$ ____________

$\cos(A) =$ ____________

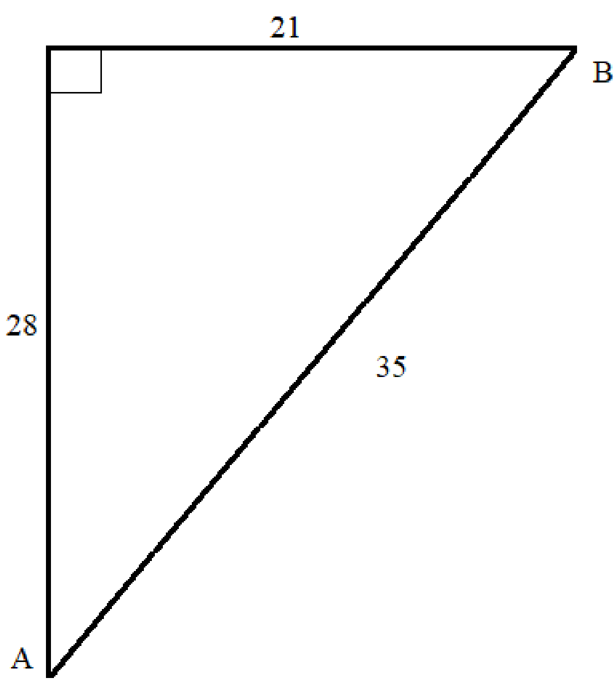
$\tan(A) =$ ____________

$\sin(B) =$ ____________

$\cos(B) =$ ____________

$\tan(B) =$ ____________

12.



$\sin(A) =$ ____________

$\cos(A) =$ ____________

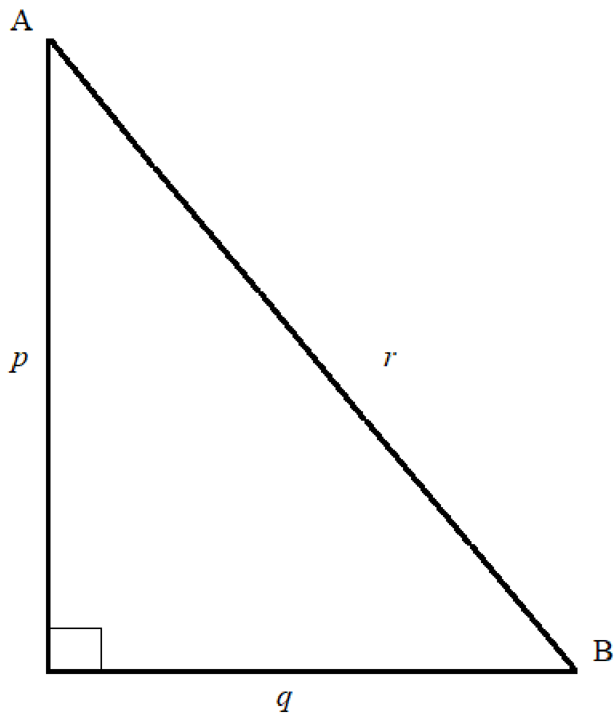
$\tan(A) =$ ____________

$\sin(B) =$ ____________

$\cos(B) =$ ____________

$\tan(B) =$ ____________

13.



$\sin(A) =$ __________

$\cos(A) =$ __________

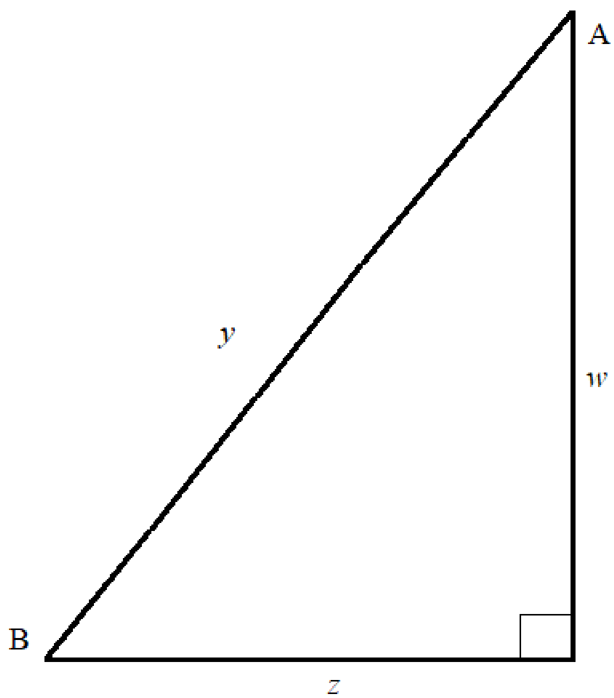
$\tan(A) =$ __________

$\sin(B) =$ __________

$\cos(B) =$ __________

$\tan(B) =$ __________

14.



$\sin(A) =$ __________

$\cos(A) =$ __________

$\tan(A) =$ __________

$\sin(B) =$ __________

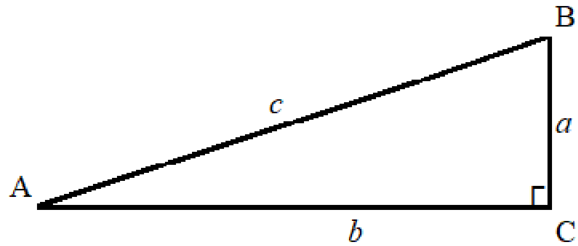
$\cos(B) =$ __________

$\tan(B) =$ __________

III. Given right triangle ΔABC having C as its right angle.

a) Use the Pythagorean Theorem to find exact value of the length of the unknown side.
If the value of a length contains a radical, express it in simplified form.

b) Find the exact values of the six trig functions for $\angle B$.
If the value of a trig function contains a radical, express it with the radical in the numerator and in simplified form.



Example:

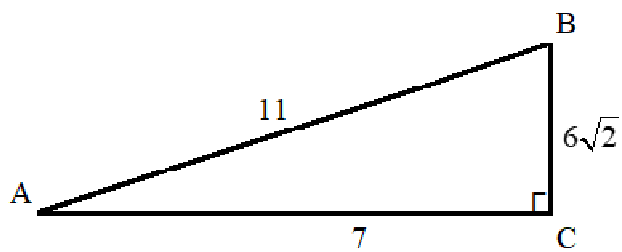
$b = 7, \quad c = 15$

Solution:

a)

$$a^2 + b^2 = c^2$$
$$\Rightarrow a^2 = c^2 - b^2$$
$$\Rightarrow a = \sqrt{c^2 - b^2}$$
$$= \sqrt{(11)^2 - (7)^2}$$
$$= \sqrt{121 - 49}$$
$$= \sqrt{72}$$
$$= \sqrt{36}\sqrt{2}$$
$$= \boxed{6\sqrt{2}}$$

So we have:



b)

$$\sin(B) = \boxed{\frac{7}{11}}$$

$$\cos(B) = \boxed{\frac{6\sqrt{2}}{11}}$$

$$\tan(B) = \frac{7}{6\sqrt{2}}$$
$$= \frac{7}{6\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \boxed{\frac{7\sqrt{2}}{12}}$$

$$\csc(B) = \boxed{\frac{11}{7}}$$

$$\sec(B) = \frac{11}{6\sqrt{2}}$$
$$= \frac{11}{6\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \boxed{\frac{11\sqrt{2}}{12}}$$

$$\cot(B) = \boxed{\frac{6\sqrt{2}}{7}}$$

15. $b=7, \quad c=15$

16. $a=8, \quad b=6$

17. $a=3, \quad c=10$

18. $a=3, \quad c=6$

19. $b=4, \quad c=6\sqrt{5}$

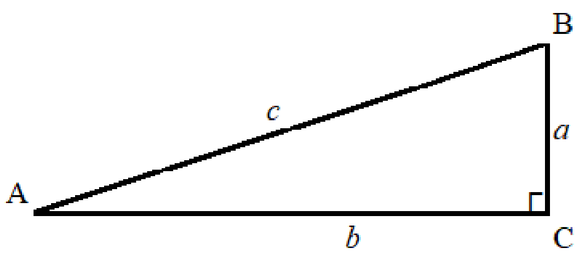
20. $a=\sqrt{7}, \quad b=\sqrt{12}$

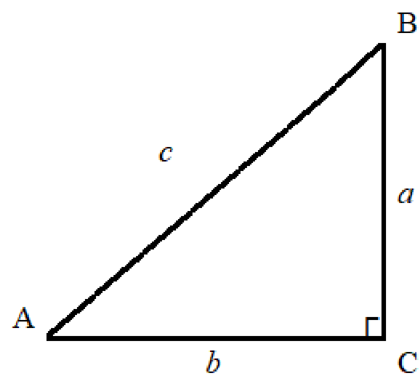
IIII. Suppose ΔABC is either a

$30-60-90$ triangle with $\angle A=30°, \angle B=60°, \angle C=90°$ and $a=1, b=\sqrt{3}, c=2$

or

$45-45-90$ triangle with $\angle A=45°, \angle B=45°, \angle C=90°$ and $a=1, b=1, c=\sqrt{2}$





Find the exact value of each expression. If a result contains a radical, express it with the radical in the numerator.

Example:

a) $\csc(60°)=$

b) $\cos(45°)=$

Solution:

$$\csc(60°)=\frac{hyp}{opp}=\frac{2}{\sqrt{3}}=\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\boxed{\frac{2\sqrt{3}}{3}}$$

$$\cos(45°)=\frac{adj}{hyp}=\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\boxed{\frac{\sqrt{2}}{2}}$$

21. $\csc(60°)=$

22. $\tan(30°)=$

23. $\sin(45°)=$

24. $\cot(60°)=$

25. $\sec(45°)=$

26. $\tan(60°)=$

27. $\cot(30°)=$

28. $\cos(60°)=$

29. $\sin(30°)=$

30. $\csc(30°)=$

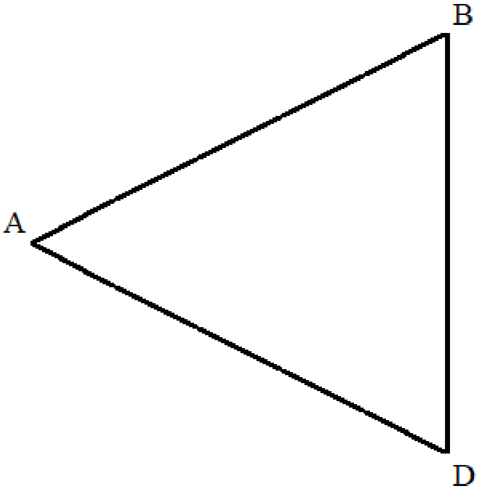
31. $\tan(45°)=$

32. $\cos(30°)=$

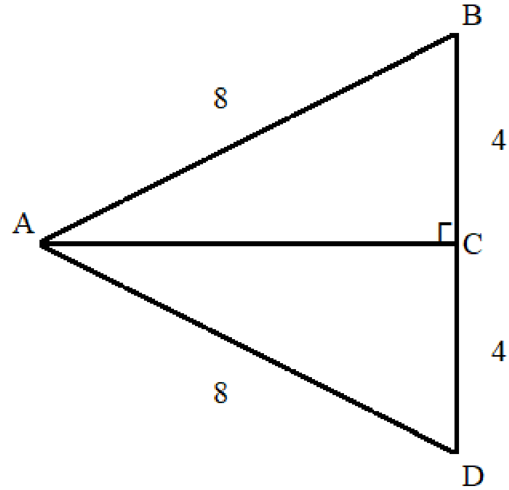
V. Work through the following proceedure.

33.

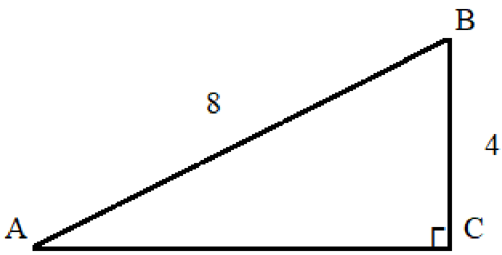
A. Consider an equilateral triangle with sides length, say, 8. Note all the angles in ΔABD measure 60°.



B. Now, drop a bisector of $\angle A$ to side BD intersecting at point C. Note that $AC \perp BD$ and that AC is Half the length of BD, hence $AC = (\frac{1}{2})BD = 4$.



C. Now, drop off the bottom triangle to create a $30-60-90$ triangle with hypotenuse $= 8$ and short leg $= 4$



D. Finally, use Pythagoras to compute AC:

$$\begin{aligned} AC &= \sqrt{(8)^2 - (4)^2} \\ &= \sqrt{64-16} \\ &= \sqrt{48} \\ &= \sqrt{16}\sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

Thus: hypotenuse $= 8$, short leg $= 4$, long leg $= 4\sqrt{3}$

34. Now rework the entire process starting with a hypotenuse length a) 10, b) 1, c) x.

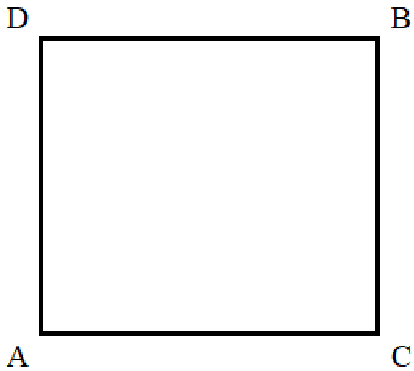
35. Based on these results, complete the following.

In any $30-60-90$ triangle the short leg is always __________ times the hypotenuse and the long leg is always ________ times the short leg.

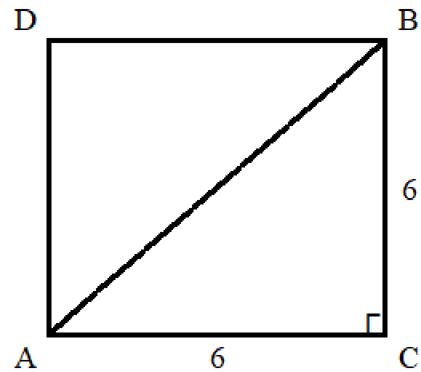
VI. Work through the following proceedure.

36.

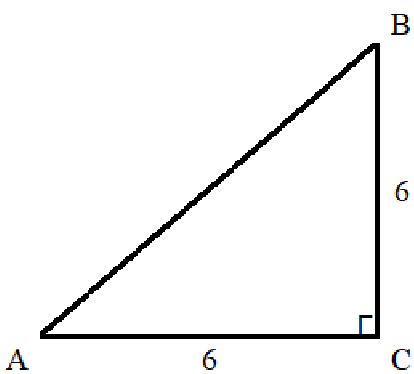
A. Consider a square with sides length, say, 6. Note all the angles in $\square ADBC$ measure $90°$.



B. Now, draw diagonal AB. Note that $AC \perp BC$, AB bisects $\angle CAD$, and both AC and BC are length 6.



C. Now, drop off the upper triangle to create a $45-45-90$ triangle (isisceles) with both legs $=6$.



D. Finally, use Pythagoras to compute AB:

$$\begin{aligned} AB &= \sqrt{(6)^2+(6)^2} \\ &= \sqrt{36+36} \\ &= \sqrt{72} \\ &= \sqrt{36}\sqrt{2} \\ &= 6\sqrt{2} \end{aligned}$$

Thus: hypotenuse $=6\sqrt{2}$, both legs $=6$.

37. Now rework the entire process starting with a leg length a) 13, b) 1, c) x.

38. Basede on these results, complete the following.

In any $45-45-90$ triangle the hypotenuse is always ___________ times either leg and the legs are always _________ (because the triangle is _____________).

VII. For each item use the given information to find the length of the remaining two sides of the triangle *a, b,* or *c*. If the triangle is $45-45-90$, assume the right angle is at vertex C. If it is $30-60-90$, assume the right angle is at C, the $60°$ is at B .and the $30°$ is at A. Use exact values and express with radicals in the numerator.

Example: a) $30-60-90$ and short leg $=14$ b) $45-45-90$ and hypotenuse $=5$

Solution:

a)

$$\text{Hypotenuse } = 2\times(\text{short leg}) = \boxed{28}$$

$$\text{Long leg } = \sqrt{3}\times(\text{short leg}) = \boxed{14\sqrt{3}}$$

b)

$$\text{both legs } = \frac{1}{\sqrt{2}}\times(\text{hypotenuse})$$

$$= \frac{1}{\sqrt{2}}\cdot 5 = \frac{5}{\sqrt{2}} = \boxed{\frac{5\sqrt{2}}{2}}$$

39. $30-60-90$ and hypotenuse $=8$

40. $30-60-90$ and long leg $=7\sqrt{3}$

41. $45-45-90$ and leg $=19$

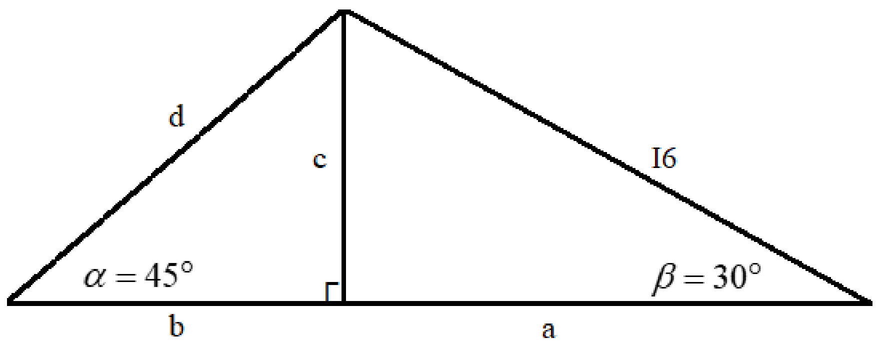
42. $45-45-90$ and hypotenuse $=22\sqrt{2}$

43. $45-45-90$ and hypotenuse $=7$

44. $30-60-90$ and short leg $=3\sqrt{3}$

VIII. Each triangle pictured is either a $30-60-90$ or a $45-45-90$ triangle. Use the given information about angles and lengths to determine the lengths of the other indicated lengths. Leave your answers in radical form with radicals in the numerator.

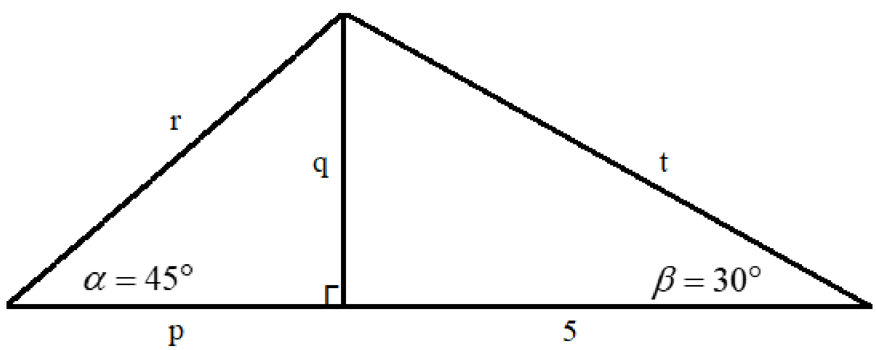
Example:



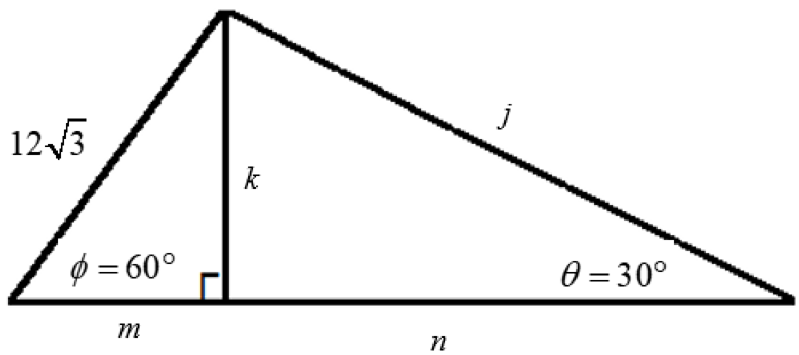
Solution:

$$c=8,\quad a=8\sqrt{3},\quad b=8,\quad d=8\sqrt{2}$$

45.

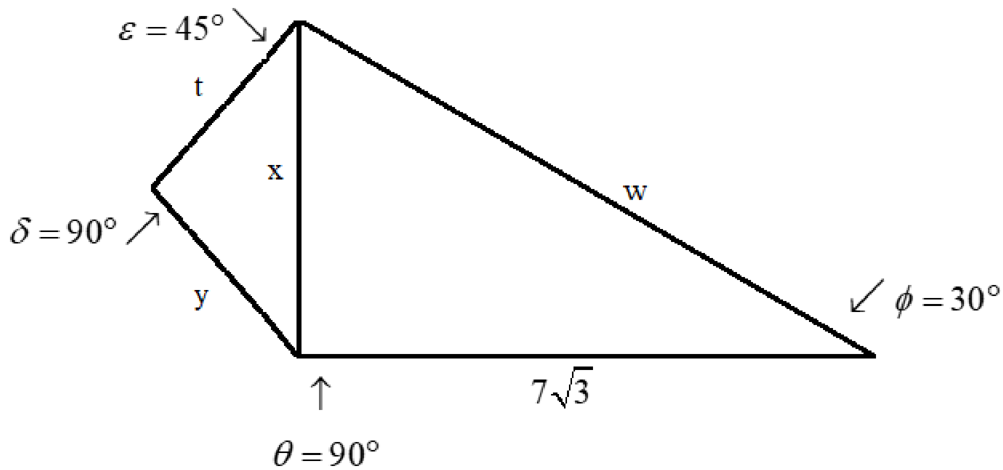


46.



$12\sqrt{3}$
k
j
$\phi = 60°$
$\theta = 30°$
m
n

47.



$\varepsilon = 45°$
t
x
w
$\delta = 90°$
y
$\phi = 30°$
$7\sqrt{3}$
$\theta = 90°$