Related Rates Problems

1. A spherical bubble is being inflated with air such that the volume of air in the bubble is increasing at the rate of 156.4 cm3/min. How fast is the radius of the bubble increasing when the radius is 7.1 cm?

2. The radius of a circle is increasing at the rate of 4 in/sec. Find the following:
   a) The rate at which the area of the circle is increasing when the radius is 20 in.
   b) The rate at which the area of the circle is increasing when the circumference is 30π.
   c) The rate at which the area of the circle is increasing when the circumference is 24.
   d) The rate at which the area of the circle is increasing when the area is 36π.

3. An explosion has occurred at the LMF Methane Company in Little Rock, Arkansas and the radius of the hemispherical shock wave is expanding at the rate of 20 meters per second. Find the rate of change of
(a) the surface area of the hemisphere (the curved portion only) and
(b) the volume of the hemisphere
when the circumference of the hemisphere is 50π meters.

4. Gravel is falling off a conveyer belt onto a conical pile. The coarseness of the gravel is such that the height of the pile is always three-fourths of its radius. If the height of the pile is increasing at the rate of 3.2 ft/min, at what rate is the volume of the pile increasing when the radius is 16??

5. Sand is falling off a conveyer belt onto a conical pile at the rate of 60 ft3/min. The coarseness of the sand is such that the radius of the pile is always 2 times the height. Find the rate at which the height of the pile is increasing when the radius is 6ft.

6. Sand is falling off a conveyer belt onto a conical pile at the rate of 25.2 cubic meters per minute. The coarseness of the sand is such that the diameter of the pile is always 2.4 times the height. Assuming the height is increasing at a constant rate of 1.2 m/min, find the rate at which the radius of the pile is increasing when the diameter is 7.2 meters.

7. A 25-foot ladder is leaning against the side of a building. The base of the ladder is sliding away from the building at the rate of 3 ft/sec. At what rate is the top of the ladder sliding down the wall when the base of the ladder is 15 ft from the base of the building?

8. An 18-foot ladder is leaning against a house and its base is sliding away from the house at the rate of 2.2 ft/sec. What is the rate of change in degrees per second of the angle formed by the ladder and the ground when the top of the ladder is 12.4 ft above the ground?

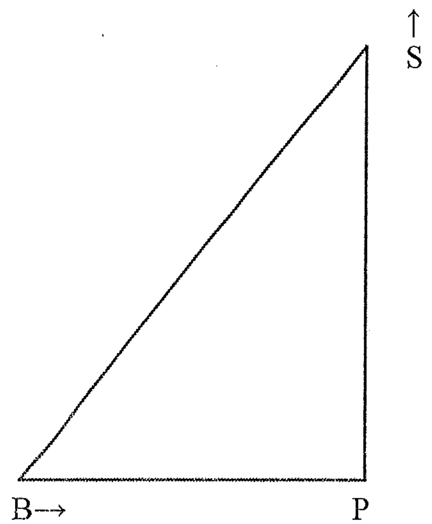
9. A 15-foot ladder is leaning against a house and the base is sliding away from the house at the rate of 2 ft/sec. how fast is the top of the ladder sliding down the wall when the top of the ladder is
   a) 14 ft high?
   b) 12 ft high?
   c) 6 ft high?
   d) 1 ft high?

12. Batman is driving in the Batmobile toward point P while Superman is flying vertically upward from point P. See figure.

a) If we suppose Batman is driving at the rate of 39.3 m/sec and that the diagonal distance between Batman and Superman is increasing at a constant rate of 46.8 m/sec, how fast is Superman flying when Batman is 120 meters from P and Superman is 50 meters high.

b) Let us now suppose that Batman is driving at 44.4 m/sec and Superman is flying at 66.6 m/s. What is the rate of change of Batman's angle of elevation (measured in degrees per second) to Superman when Batman is 80 meters from P and Superman is 100meters above point P.

Figure:



13. When Alice went into wonderland she had in her pocket a small wooden cylinder and when she shrank in size the cylinder shrank along with her in such a way that the height of the cylinder was always 3 times the radius. Find the following:

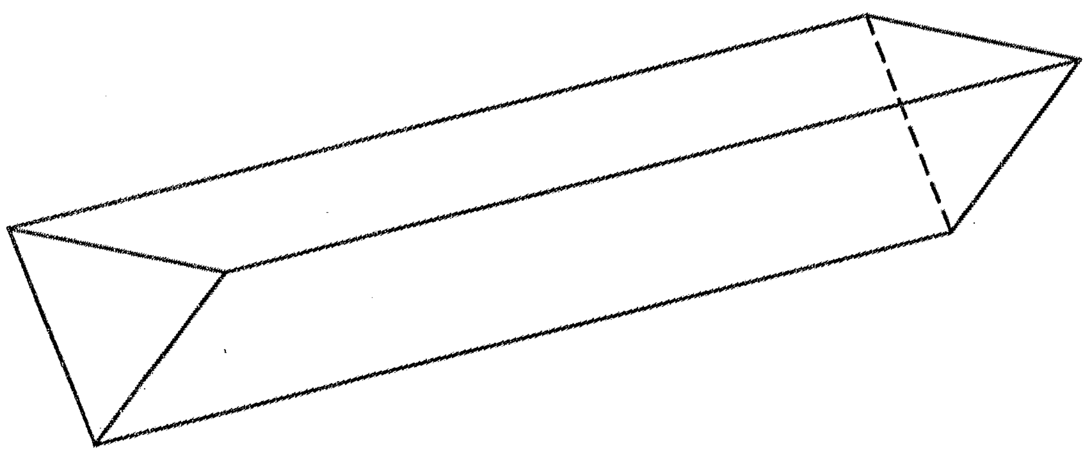
a) Assume the cylinder shrank such that the radius is decreasing at 1 cm/sec, find the rate of change of the volume when the height is 9 cm.

b) Assume the cylinder shrank such that the height is decreasing at the rate of 2 cm/sec. Find the rate of change of the volume when the diameter of the cylinder is 5 cm.

10. Clark is putting lights up on his house using his retractable ladder (a ladder which can change its length).
   a) While he is stapling lights to his eaves, the ladder starts retracting (getting shorter) at the rate of 1 ft/sec. At the same time, the base of the ladder starts sliding away from the wall at 4 ft/sec. How fast is the top of the ladder sliding down the wall when the base of the ladder is 12 feet from the wall and the ladder is 13 feet long?
   b) Having returned from the emergency room, Clark again gets on the ladder, however this time, the ladder starts retracting at the rate of 2 ft/sec and the base of the ladder starts sliding away from the wall at 3 ft/sec. How fast is the top of the ladder sliding down the wall when the top of the ladder is 12 ft high and the ladder is 20 ft long?
   c) When the paramedics finish their work, Clark climbs back on the ladder. This time the base of the ladder starts sliding away from the wall at 5 ft/sec and the top is sliding down the wall at 6 ft/sec. What is the rate of change of the angle formed by the ladder and the house (in degrees per sec) when the bottom of the ladder is 12 ft from the wall and the top of the ladder is 5 ft high.

11. A water trough is 40 inches long and has ends which are isosceles triangles having their heights equal to their bases. Both height and base are 8 inches. See figure.
   a) Assume water is being poured into the trough at the rate of 94.1in3/min. At what rate is the depth of the water increasing when the depth is 3.8 inches?
   b) Assume the water is being poured into the trough at the rate of 126.4 in3/min and that there are holes in the bottom of the trough which cause it to leak water at the rate of 42.8 in3/min. At what rate is the depth of the water changing when the water is 6.8 inches deep?

Figure:



Related Rates Solutions

① Find: $\frac{dr}{dt}$

Given: $\frac{dV}{dt} = 156.4$

$r = 7.1$

START WITH

$V = \frac{4}{3}\pi r^3$

differential count:

$\frac{dV}{dt}$ ✓ $\frac{dr}{dt}$ ✓ good

so we have

$\frac{dV}{dt} = \frac{4}{3}\pi(3)r^2\frac{dr}{dt}$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$\Rightarrow 156.4 = 4\pi(7.1)^2\frac{dr}{dt}$

$\Rightarrow 156.4 = 201.64\pi \cdot \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \boxed{\frac{156.4}{201.64\pi}} \approx .2468 9\ldots$

$\approx .2147$

so $\boxed{\frac{dr}{dt} \approx .247 \text{ cm/min}}$

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② ⓐ Find: $\frac{dA}{dt}$

Given: $\frac{dr}{dt} = 4 \text{ in/sec}$

$r = 20$

START WITH

$A = \pi r^2$

Differential count

$\frac{dA}{dt}$ ✓ $\frac{dr}{dt}$ ✓ good

so $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\Rightarrow \frac{dA}{dt} = 2\pi(20)(4)$

$= 160\pi$

so $\frac{dA}{dt} = \boxed{160\pi \text{ in}^2/\text{sec}}$

OR

$\approx \boxed{502.655 \text{ in}^2/\text{sec}}$

2b Find: $\frac{dA}{dt}$

Given: $\frac{dr}{dt} = 4$

$C = 30\pi$

START WITH

$A = \pi r^2$ → The start equation must relate the variables whose rates of change are in the problem.

Diff. cont

$\frac{dA}{dt}$ ✓ $\frac{dr}{dt}$ ✓

so $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(?)(4)$

we need a value for r

Go back to $C = 30\pi$

since $C = 2\pi r$

$\Rightarrow 30\pi = 2\pi r$

$\Rightarrow 15 = r$

so we have

$\frac{dA}{dt} = 2\pi(15)(4)$

$= 120\pi$

so

$\frac{dA}{dt} = \boxed{120\pi \frac{in^2}{sec}}$

OR

$\frac{dA}{dt} \approx \boxed{376.991 \ in^2/sec}$

2c Find $\frac{dA}{dt}$

Given: $\frac{dr}{dt} = 4$

$C = 24$

START WITH

$A = \pi r^2$

Differential Count

$\frac{dA}{dt}$ ✓ $\frac{dr}{dt}$ ✓ good ✓

$\Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\Rightarrow \frac{dA}{dt} = 2\pi(?)(4)$

Need a Value for r

$C = 2\pi r \Rightarrow 24 = 2\pi r$

$\Rightarrow \frac{24}{2\pi} = r$

$\Rightarrow \frac{12}{\pi} = r$ ✓

so $\frac{dA}{dt} = 2\pi\left(\frac{12}{\pi}\right)(4)$

$= \frac{96\pi}{\pi}$

$= 96$

so

$\boxed{\frac{dA}{dt} = 96 \ in^2/sec}$

2d. Find $\frac{dA}{dt} =$

Given $A = 36\pi$

$\frac{dr}{dt} = 4$

START WITH

$A = \pi r^2$

Differential Count

$\frac{dA}{dt}$ ✓ $\frac{dr}{dt}$ ✓ good ✓

so $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(?)(4)$

Need a Value for r

$A = \pi r^2 \Rightarrow 36\pi = \pi r^2$

$\Rightarrow \frac{36\pi}{\pi} = r^2$

$\Rightarrow 36 = r^2$

CONT.

2d) cont. $\Downarrow$ $6 = r$

so $\frac{dA}{dt} = 2\pi(6)(4)$

$= 48\pi$

so

$\frac{dA}{dt} = \boxed{48\pi \ \text{in}^2/\text{sec}}$

$\approx \boxed{150.796 \ \text{in}^2/\text{sec}}$

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3. a. Find: $\frac{dS}{dt}$

Given: $\frac{dr}{dt} = 20 \ \text{m/s}$

$C = 50\pi$

START WITH

$S = 2\pi r^2 \Leftrightarrow$ surface area of the curved portion of a hemisphere

Differential check:

$\frac{dS}{dt}$ ✓ $\frac{dr}{dt}$ ✓ good ✓

so we have

$\frac{dS}{dt} = 4\pi r \frac{dr}{dt}$

$= 4\pi(?)(20)$

Need a value for r

we have $C = 2\pi r$

$\Rightarrow 50\pi = 2\pi r$

$\Rightarrow r = \frac{50\pi}{2\pi}$

$= 25$

so:

$\frac{dS}{dt} = 4\pi(25)(20)$

$= 2000\pi$

so $\frac{dS}{dt} = \boxed{2000\pi \ \text{m}^2/\text{s}}$

$\approx \boxed{6283.185 \ \text{m}^2/\text{s}}$

(3 b) Find $\frac{dV}{dt}$

Given $\frac{dr}{dt} = 20$

$C = 50\pi$

START WITH

$V = \frac{2}{3}\pi r^3 \Rightarrow$ Note: Volume of a sphere is $\frac{4}{3}\pi r^3$

Volume of a hemisphere is $\frac{2}{3}\pi r^3$

Differential count:

$\frac{dV}{dt}$ ✓ $\frac{dr}{dt}$ ✓ good

$\Rightarrow \frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$

$= 2\pi(r)^2(20)$

Need a value for r

$C = 2\pi r \Rightarrow 50\pi = 2\pi r$

$\Rightarrow \frac{50\pi}{2\pi} = r$

$\Rightarrow r = 25$

so $\frac{dV}{dt} = 2\pi(25)^2(20)$

$= 25000\pi$

so

$$\frac{dV}{dt} = \boxed{\begin{array}{l} 25000\pi \ m^3/s \\ \approx 78539.816 \ m^3/s \end{array}}$$

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(4). Find: $\frac{dV}{dt}$

given $h = \frac{3}{4}r$

$\frac{dh}{dt} = 3.2$ ft/min

$r = 16$

start:

$V = \frac{1}{3}\pi r^2 h \Leftarrow$ Volume of a cone

Differential count

$\frac{dV}{dt}$ ✓ $\frac{dr}{dt}$ NO $\frac{dh}{dt}$ ✓

$\Rightarrow$ must eliminate r from the start equation.

we have $h = \frac{3}{4}r \Rightarrow \frac{4}{3}h = r$

so $V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h$

CONT:

④ CONT. we have $V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h$

$$= \frac{1}{3}\pi \cdot \frac{16}{9} h^3$$

$$= \frac{16\pi}{27} h^3$$

Differential Count:

$\frac{dV}{dt}$ ✓ $\frac{dh}{dt}$ ✓ good ✓

so $\frac{dV}{dt} = \cancel{3} \cdot \frac{16\pi}{\cancel{27}_{9}} h^2 \frac{dh}{dt}$

$$= \frac{16\pi}{9}(h)^2(3.2)$$

Need a value for h

$h = \frac{3}{4} r$ and $r = 16 \Rightarrow h = \frac{3}{4}(16)$
$= 12$

so $\frac{dV}{dt} = \frac{16\pi}{9}(12)^2(3.2)$

$$= \frac{7372.8\pi}{9}$$

$$= 819.2\pi$$

so $\frac{dV}{dt} = \boxed{819.2\pi \text{ ft/min}}$

$\approx \boxed{2573.593 \text{ ft/min}}$

5. Find $\frac{dh}{dt}$:

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given: $\frac{dV}{dt} = 25.2 \, m^3/min$

$d = 2.4h$

$\hookrightarrow 2r = 2.4h$

$\frac{dh}{dt} = 1.2 \, m/min$

$d = 7.2m$

$\hookrightarrow 7.2 = 2.4h$

$\Rightarrow \frac{7.2}{2.4} = \frac{1}{1}h$

$\Rightarrow 3 = h$

START

$V = \frac{1}{3}\pi r^2 h$

Differential count:

$\frac{dV}{dt}$ ✓ $\frac{dr}{dt}$ NO $\frac{dh}{dt}$ ✓

MUST ELIMINATE r

$2r = 2.4h \Rightarrow r = \frac{2.4}{2}h$

$= 1.2h$

So $V = \frac{1}{3}\pi(1.2h)^2 h$

$\Rightarrow V = .48\pi h^3$

Differential count

$\frac{dV}{dt}$ ✓ $\frac{dh}{dt}$ ✓ good ✓

So $\frac{dV}{dt} = (3)(.48\pi)h^2\frac{dh}{dt}$

$= 1.44\pi h^2 \frac{dh}{dt}$

$\Rightarrow (25.2) = 1.44\pi(h)^2\frac{dh}{dt}$

Need h $d = 2.4h$

and $d = 7.2$

$\Rightarrow 7.2 = 2.4h$

$\Rightarrow h = 3$

So $(25.2) = (1.44\pi)(3)^2\frac{dh}{dt}$

$\Rightarrow 25.2 = 4.32\pi\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{25.2}{4.32\pi}$

So $\frac{dh}{dt} = \boxed{\frac{25.2}{4.32\pi} \, m/min}$

$\approx \boxed{1.857 \, m/min}$

⑥ Find: $\frac{dr}{dt}$

Given: $\frac{dV}{dt} = 25.2 \, m^3/min$

$d = 2.4h$

$\frac{dh}{dt} = 1.2$

$d = 7.2$

START:

$V = \frac{1}{3}\pi r^2 h$

Differential Count

$\frac{dV}{dt}$ ✓ $\frac{dr}{dt}$ ✓ $\frac{dh}{dt}$ ✓ good ✓

so $$\frac{dV}{dt} = \left(\frac{1}{3}\pi\right)\underbrace{\left[r^2 \cdot \frac{dh}{dt} + h\,2r\,\frac{dr}{dt}\right]}_{\text{product rule}}$$

constant multiplier

$$\Rightarrow 25.2 = \left(\frac{1}{3}\pi\right)\left[r^2(1.2) + 2(h)(r)\frac{dr}{dt}\right]$$

Need values for r and h

$$\left.\begin{array}{l} d = 2.4h \\ d = 7.2 \end{array}\right\} \Rightarrow 7.2 = 2.4h \Rightarrow \frac{7.2}{2.4} = h \Rightarrow h = 3 ✓$$

$$\left.\begin{array}{l} d = 2r \\ d = 7.2 \end{array}\right\} \Rightarrow 7.2 = 2r \Rightarrow 3.6 = r ✓$$

so we have:

$$25.2 = \left(\frac{\pi}{3}\right)\left[(3.6)^2(1.2) + 2(3)(3.6)\frac{dr}{dt}\right]$$

$$= \frac{\pi}{3}\left[15.552 + 21.6\,\frac{dr}{dt}\right]$$

$$= 5.184\pi + 7.2\pi\,\frac{dr}{dt}$$

$$\Rightarrow \frac{25.2 - 5.184\pi}{7.2\pi} = \frac{dr}{dt}$$

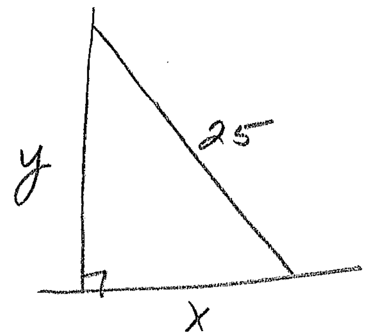
so $$\frac{dr}{dt} = \boxed{\frac{25.2 - 5.184\pi}{7.2\pi} \, m/min}$$

$$\approx \boxed{.394 \, m/min}$$

#7. Find $\frac{dy}{dt}$:

Given: $\frac{dx}{dt} = 3\ ft/sec$

$x = 15$



Start with

$x^2 + y^2 = 25^2$

Differential Count

$\frac{dx}{dt}$ ✓ $\frac{dy}{dt}$ ✓ good ✓

So $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

Need a value for y

⇒ go back:

$(15)^2 + y^2 = 25^2$

$\Rightarrow 225 + y^2 = 625$

$\Rightarrow y^2 = 400$

$\Rightarrow y = \pm\sqrt{400} = \pm 20$

choose $y = 20$ ✓

So we have

$\Rightarrow 2(15)(3) + 2(20)\frac{dy}{dt} = 0$

$\Rightarrow 90 + 40\frac{dy}{dt} = 0$

$\Rightarrow \frac{dy}{dt} = -\frac{90}{40} = -\frac{9}{4}$

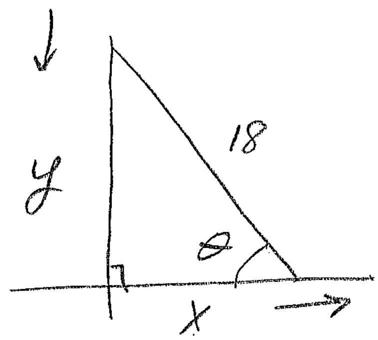
So

$$\boxed{\frac{dy}{dt} = -\frac{9}{4}\ ft/sec}$$

(8) Find: $\frac{d\theta}{dt} =$ deg/sec

Given: $\frac{dx}{dt} = 2.2$ ft/sec

$y = 12.4$ ft



start with ⇒ need an equation that involves θ & x because those are the differentials involved in the problem:

so $\cos(\theta) = \frac{x}{18}$

$\Rightarrow 18\cos(\theta) = x$

(*) Differential count:

$\frac{d\theta}{dt}$ — $\frac{dx}{dt}$ — good ✓

so $(-18)\sin(\theta)\frac{d\theta}{dt} = \frac{dx}{dt}$

Need a value for θ, or $\sin(\theta)$

we know $y = 12.4$ and hypotenuse is 18 so: $\sin(\theta) = \frac{y}{18}$

$\Rightarrow \sin(\theta) = \frac{12.4}{18}$

so we now have

$\Rightarrow (-18)\left(\frac{12.4}{18}\right)\frac{d\theta}{dt} = 2.2$

$\Rightarrow -12.4\frac{d\theta}{dt} = 2.2$

$\Rightarrow \frac{d\theta}{dt} = \frac{2.2}{-12.4}$ ⟶ cont.

⇓ cont.

We have $\frac{d\theta}{dt} = \frac{2.2}{-12.4}$

$= -.177419\ldots$ rad/sec

Note this is in RADIANS per SECOND since the calculus requires angles to be measured in radians. We are asked, however, to express the result in degrees/second. so:

$$\frac{\pi}{-.177419\ldots} = \frac{180}{x}$$

cross-multiply & solve for x:

$$\Rightarrow \pi \cdot x = 180(-.177419\ldots)$$

$$\Rightarrow x = \frac{180(-.177419\ldots)}{\pi}$$

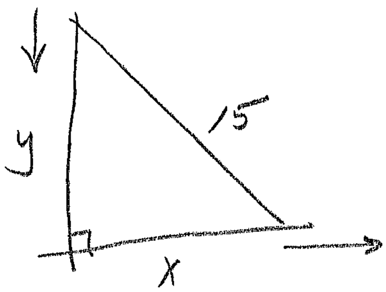
$$= 10.16538\ldots \text{ deg/sec}$$

$$\approx \boxed{10.2^\circ \text{deg/sec}}$$

9. Find: $\frac{dy}{dx}$

given $\frac{dx}{dt} = 2$ ft/sec

a) $y = 14$
b) $y = 12$
c) $y = 6$
d) $y = 1$



start: $x^2 + y^2 = 15^2$

* Differential count

$\frac{dx}{dt}$ ✓ $\frac{dy}{dt}$ ✓ good ✓

$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

Note:

ⓐ $y = 14 \Rightarrow x^2 + (14)^2 = (15)^2$
$\Rightarrow x^2 = 29$
$\Rightarrow x = \sqrt{29}$ ✓

Similarly:

ⓑ $y = 12 \Rightarrow x = 9$ ✓

ⓒ $y = 6 \Rightarrow x = \sqrt{189}$ ✓

ⓓ $y = 1 \Rightarrow x = \sqrt{224}$ ✓

so ⓐ $2(\sqrt{29})(2) + 2(14)\frac{dy}{dt} = 0$

$\Rightarrow -\frac{4\sqrt{29}}{28} = \frac{dy}{dt}$

$\Rightarrow \frac{dy}{dt} = \boxed{-\frac{4\sqrt{29}}{28}} \approx \boxed{-.769 \text{ ft/sec}}$

ⓑ $2(9)(2) + 2(12)\frac{dy}{dt} = 0$

$\Rightarrow \frac{dy}{dt} = -\frac{36}{24} = \boxed{-\frac{3}{2} \text{ ft/sec}}$

$= \boxed{-1.5 \text{ ft/sec}}$

ⓓ $2(\sqrt{224})(2) + 2(1)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \boxed{-\frac{4\sqrt{224}}{2} \text{ ft/sec}}$

$\approx \boxed{29.933 \text{ ft/sec}}$

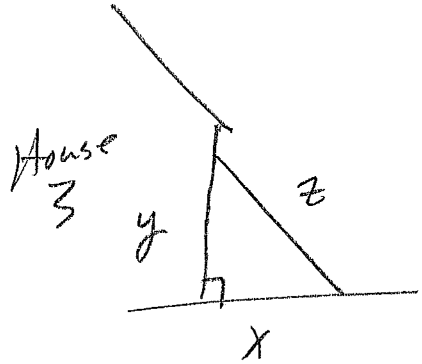
ⓒ $2(\sqrt{189})(2) + 2(6)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \boxed{-\frac{4\sqrt{189}}{12} \text{ ft/sec}}$

$\approx \boxed{-4.583 \text{ ft/sec}}$

10.

(a) Find $\frac{dy}{dt}$ ft/sec



given $\frac{dz}{dt} = -1$

Note! Negative!

$\frac{dx}{dt} = 4$

$x = 12$

$z = 13$

START: $x^2 + y^2 = z^2$

Differential count:

$\frac{dx}{dt}$ ✓ $\frac{dy}{dt}$ ✓ $\frac{dz}{dt}$ ✓ good

So $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$

$\Rightarrow (12)(4) + (y)\frac{dy}{dt} = (13)(-1)$

Need a value for y.

Go Back:

$x^2 + y^2 = z^2$

$\Rightarrow (12)^2 + y^2 = (13)^2$

$\Rightarrow y^2 = 25$

$\Rightarrow y = 5$

So we have:

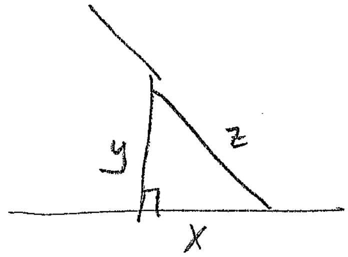
$(12)(4) + (5)\frac{dy}{dt} = (13)(-1)$

$\Rightarrow 5\frac{dy}{dt} = -13 - 48$

$\Rightarrow \frac{dy}{dt} = \boxed{\frac{-61}{5} \text{ ft/sec}}$

$= \boxed{-12.2 \text{ ft/sec}}$

(b)



Find: $\frac{dy}{dt}$

Given $\frac{dz}{dt} = -2$ ft/sec

$\frac{dx}{dt} = 3$ ft/sec

$y = 12$ ft

$z = 20$ ft

START:

$x^2+y^2=z^2$

diff count:

$\frac{dx}{dt}$ ✓ $\frac{dy}{dt}$ ✓ $\frac{dz}{dt}$ ✓ good ✓

so $2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

$\Rightarrow x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}$

so Need a value for x:

go back! $x^2+y^2=z^2$

$\Rightarrow x^2+(12)^2=(20)^2$

$\Rightarrow x=16$

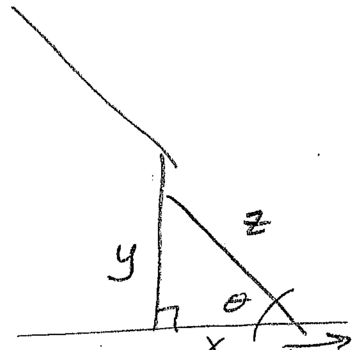
so $(16)(3)+(12)\frac{dy}{dt}=(20)(-2)$

$\Rightarrow 12\frac{dy}{dt}=-40-48$

$\Rightarrow \frac{dy}{dt}=\boxed{\frac{-88}{12}\ \frac{ft}{sec}}$

$\approx \boxed{7.333 \text{ ft/sec}}$

(c)



Find $\frac{d\theta}{dt}$ in deg/sec

Given:

$\frac{dx}{dt}=5$ ft/sec

$\frac{dy}{dt}=-6$ ft/sec

$x=12$

$y=5$

START:

$\tan(\theta)=\frac{y}{x}$

$\Rightarrow x\cdot\tan(\theta)=y$

Differential count:

$\frac{dx}{dt}$ ✓ $\frac{d\theta}{dt}$ ✓ $\frac{dy}{dt}$ ✓ good ✓

$\Rightarrow x\sec^2(\theta)\frac{d\theta}{dt}+\tan(\theta)\frac{dx}{dt}=\frac{dy}{dt}$

Need the hypotenuse to find a value for $\sec(\theta)$

$x=12\ \ y=5 \Rightarrow z=13$

so $\sec(\theta)=\frac{z}{x}=\frac{13}{12}$

$\tan(\theta)=\frac{y}{x}=\frac{5}{12}$

so $(12)\left(\frac{13}{12}\right)^2\frac{d\theta}{dt}+\frac{5}{12}(5)=(-6)$

CONT.

10b Find $\frac{dh}{dt} =$ in/min

given $\frac{dV}{dt} = 126.4 - 42.8$

$= 83.6$ in³/min

$h = 6.8$

again, we have

$$V = 20h^2$$

⇒ ① Differential Count

$\frac{dV}{dt}$ ✓ $\frac{dh}{dt}$ ✓ good

so $V = 20h^2$

$$\Rightarrow \frac{dV}{dt} = 40h\frac{dh}{dt}$$

so $83.6 = 40(6.8)\frac{dh}{dt}$

$$\Rightarrow \frac{dh}{dt} = \frac{83.6}{272} \approx .307 \text{ in/min}$$

$$\Rightarrow \boxed{\frac{dh}{dt} = .307 \text{ in/min}}$$

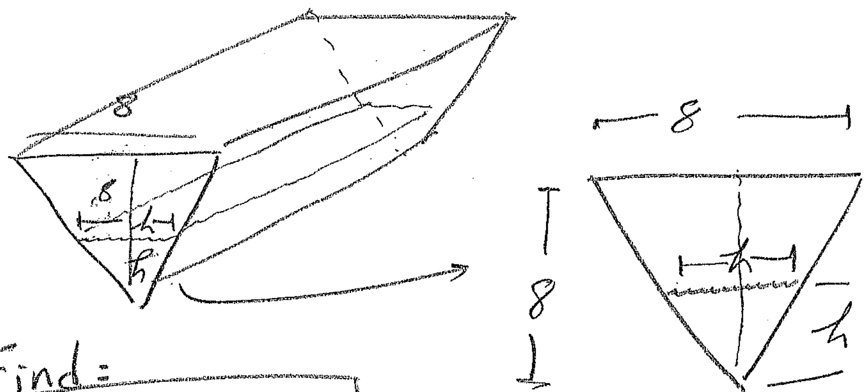
#10 part c ⇓cont⇓

$$\Rightarrow \frac{13}{12}\frac{d\theta}{dt} = -6 - \frac{25}{12}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{-6-\frac{25}{12}}{13/12} = \frac{-97}{12}\cdot\frac{12}{13} \approx -7.46$$

---

(11).



(a). Find:

$$\boxed{\frac{dh}{dt} = \quad in/min}$$

given

$$\frac{dV}{dt} = 94.1 \ in^3/min$$

$$h = 3.8$$

START:

$$V = (\text{AREA OF END})(\text{length})$$

$$= \tfrac{1}{2}h\cdot h\cdot 40$$

$$\Rightarrow V = 20h^2$$

⊛ differential count

$\frac{dV}{dt}$ ✓ $\frac{dh}{dt}$ ✓ good ✓

so we have

$$V = 20h^2$$

$$\Rightarrow \frac{dV}{dt} = 40h\frac{dh}{dt}$$

$$\Rightarrow 94.1 = 40(3.8)\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{94.1}{152} \approx .619 \frac{in}{min}$$

$$\boxed{\frac{dh}{dt} \approx .619 \ in/min}$$