Solutions

I

① $f(x) = \ln(\sec(x)) \Rightarrow \boxed{f'(x) = \frac{1}{\sec(x)}(\sec(x)\cdot\tan(x))}$

② $f(x) = 3x^3 e^{(2x^3)} \Rightarrow f'(x) = \boxed{(3x^3)(e^{(2x^3)}(6x^2)) + (e^{(2x^3)})(9x^2)}$

③ $f(x) = \cos(x)\ln(x^2) \Rightarrow f'(x) = \boxed{(\cos(x))(\frac{1}{x^2}(2x)) + (\ln(x^2))(-\sin(x))}$

④ $f(x) = \frac{x - e^{(5-4x)}}{1+\ln(x)} \Rightarrow f'(x) = \boxed{\frac{(1+\ln(x))(1-e^{(5-4x)}(-4)) - (x - e^{(5-4x)})(\frac{1}{x})}{(1+\ln(x))^2}}$

II

⑤ $\int(6\sqrt[3]{x} - 9x^2 - 2)\,dx = \int(6x^{1/3} - 9x^2 - 2)\,dx = \boxed{\frac{6x^{4/3}}{4/3} - \frac{9x^3}{3} - 2x + C}$

⑥ $\int\left(-\frac{1}{\sqrt[4]{x}} + \frac{2}{\sqrt[3]{x}}\right)dx = \int(-x^{-1/4} + 2x^{-1/3})\,dx = \boxed{\frac{-x^{3/4}}{3/4} + \frac{2x^{2/3}}{2/3} + C}$

⑦ $\int(8x\tan(x^2))\,dx = 4\int\tan(u)\,du$

Let $u = x^2$ $\quad = 4\ln|\sec(u)| + C$

$\Rightarrow du = 2x\,dx$

$\Rightarrow 4du = 8x\,dx$ $\quad = \boxed{4\ln|\sec(x^2)| + C}$

⑧ $\int\frac{e^{3x}}{3+e^{3x}}\,dx = \frac{1}{3}\int\frac{1}{u}\,du$

Let $u = 3 + e^{3x}$ $\quad = \frac{1}{3}\ln|u| + C$

$\Rightarrow du = e^{3x}(3)\,dx$

$\Rightarrow \frac{1}{3}du = e^{3x}\,dx$ $\quad = \boxed{\frac{1}{3}\ln|3+e^{3x}| + C}$

pg two

(9) $\int \frac{(6x+18)}{(x^2+6x)}dx = 3\int \frac{1}{u}du$

Let $u = x^2+6x$ $\quad = 3\ln|u|+C$

$\Rightarrow du = (2x+6)dx$ $\quad = \boxed{3\ln|x^2+6x|+C}$

$\Rightarrow 3du = 3(2x+6)dx$

$= (6x+18)dx$ ✓

(10) $\int \left(\frac{2\sec^2(x)}{1-\tan(x)}\right)dx = -2\int \frac{1}{u}du$

Let $u = 1-\tan(x)$ $\quad = -2\ln|u|+C$

$\Rightarrow du = -\sec^2(x)dx$ $\quad = \boxed{-2\ln|1-\tan(x)|+C}$

$\Rightarrow -2du = 2\sec^2(x)dx$ ✓

III (11) $\int_{-1}^{2}(6x^2-2x+1)dx = \left[\frac{6x^3}{3}-\frac{2x^2}{2}+x\right]_{-1}^{2}$

$= \left(2(2)^3-(2)^2+(2)\right)-\left(2(-1)^3-(-1)^2+(-1)\right)$

$= (16-4+2)-(-2-1-1)$

$= 14+4$

$= \boxed{18}$

(12) $\int_0^1 6x^2\left(\sqrt[3]{x^3-1}\right)dx = \int_0^1 6x^2(x^3-1)^{1/3}dx = 2\int_{-1}^{0}u^{1/3}du$

Let $u = x^3-1$

$\Rightarrow du = 3x^2\,dx$ $\quad = \left[\frac{2\,u^{4/3}}{4/3}\right]_{-1}^{0}$

$\Rightarrow 2du = 6x^2dx$ ✓

Limits

upper : $u = (1)^3-1 = 0$ $\quad = 2(3/4)\left[\left(0^{4/3}\right)-(-1)^{4/3}\right]$

lower : $u = (0)^3-1 = -1$

$= 3/2[0+1] = \boxed{3/2}$

(13). $\int_5^7 (x+2)(6-x)^3\,dx =$

Double Linear Scenario:

Let $u = 6-x$ $\Rightarrow du = -dx$ $\Rightarrow -du = dx$

solve for x: $u-6=-x$ $\Rightarrow -u+6=x$

create the other factor: $-u+8 = x+2$

Limits of integration

upper $u = 6-7 = -1$

lower $u = 6-5 = 1$

so we have:

$$= -\int_1^{-1} (-u+8)u^3\,du = -\int_1^{-1} (-u^4+8u^3)\,du = -\left[-\frac{u^5}{5}+\frac{8u^4}{4}\right]_1^{-1} =$$

$$= -\left[\left(-\frac{(-1)^5}{5}+2(-1)^4\right)-\left(-\frac{(1)^5}{5}+2(1)^4\right)\right]$$

$$= -\left[\left(\frac{1}{5}+2\right)-\left(-\frac{1}{5}+2\right)\right]$$

$$= -\left[\frac{1}{5}+\frac{1}{5}+2-2\right] = \boxed{-2/5}$$

(14). $\int_{-\pi}^{\pi/2} 4\cos(2x)\,dx$

"$\frac{1}{a}$ rule"

$$= \left[\frac{1}{2}\cdot 4\sin(2x)\right]_{-\pi}^{\pi/2}$$

$$= \left[2\sin(2x)\right]_{-\pi}^{\pi/2}$$

$$= (2\sin(2(\pi/2))) - (2\sin(2(-\pi)))$$

$$= (2\sin(\pi)) - (2\sin(-2\pi))$$

$$= 0-0$$

$$= \boxed{0}$$

OR

Let $u = 2x$

$\Rightarrow du = 2dx$

$\Rightarrow 2du = 4dx$

Limits:

upper: $u = 2(\pi/2) = \pi$

lower: $u = 2(-\pi) = -2\pi$

so we have

$$= 2\int_{-2\pi}^{\pi} \cos(u)\,du = 2\left[\sin(u)\right]_{-2\pi}^{\pi}$$

$$= 2\left[(\sin(\pi)) - (\sin(-2\pi))\right]$$

$$= 2[0-0]$$

$$= \boxed{0}$$