

Solutions

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① $2 \sin(x) + 8 = 7$

⊗ Isolate:
 $\Rightarrow 2 \sin(x) = -1$

$\Rightarrow \sin(x) = -\frac{1}{2}$

⊗ Quad check:

Q II Q III

⊗ Ref x:

$x_R = \pi/6$

⊗ Solve:

Q II: $x = 5\pi/6$
 Q III: $x = 7\pi/6$

② $\tan(x) - 1 = 0$

⊗ Isolate:

$\Rightarrow \tan(x) = 1$

⊗ Special angle!

⊗ Quad check:

Q I Q III

⊗ Ref x:

$x_R = \pi/4$

⊗ Solve:

Q I: $x = \pi/4$
 Q III: $x = 3\pi/4$

③ $4 \cos^2(x) + 7 = 10$

⊗ Isolate:

$\Rightarrow 4 \cos^2(x) = 3$

$\Rightarrow \cos^2(x) = \frac{3}{4}$

$\Rightarrow \cos(x) = \pm \sqrt{\frac{3}{4}}$
 $= \pm \frac{\sqrt{3}}{2}$

⊗ Quad Check:

all 4

⊗ Ref x:

$x_R = \pi/6$

⊗ Solve:

$x = \pi/6$	$x = 7\pi/6$
$x = 5\pi/6$	$x = 11\pi/6$

④ $(2 \cos(x) + 1)(\tan(x) - 1) = 0$
 set factors = 0 (Zero Product Property)

$\Rightarrow 2 \cos(x) + 1 = 0$ or $\tan(x) - 1 = 0$

$\Rightarrow$ ⊗ Isolate:

$2 \cos(x) = -1$

$\Rightarrow \cos(x) = -\frac{1}{2}$

⊗ Quad check:

Q II; Q III

⊗ Ref x:

$x_R = \pi/3$

⊗ Solve:

Q II: $x = 2\pi/3$
 Q III: $x = 4\pi/3$

⑤ $10 \cot(x) + 6.2 = 8.1$

⊗ Isolate -6.2 -6.2

$\Rightarrow 10 \cot(x) = 1.9$

$\Rightarrow \cot(x) = \frac{1.9}{10}$

⊗ Reciprocal:

$\Rightarrow \tan(x) = \frac{10}{1.9}$

⊗ Quad check:

Q I; Q III

⊗ Ref x: Not a special angle

$\hat{=}$ $x_R = \tan^{-1}\left(\frac{10}{1.9}\right)$

$\hat{=}$ $1.38303438...$

$\hat{=}$ 1.383

⊗ Solve:

Q I: $x = 1.383$

Q III: $x = \pi + 1.383 = 4.52459... \hat{=}$ 4.525

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6. $3\csc(x) - 10 = -16$

* Isolate:

$$\Rightarrow 3\csc(x) = -6$$

$$\Rightarrow \csc(x) = -2$$

* Reciprocals

$$\Rightarrow \sin(x) = -\frac{1}{2}$$

* Quad check

QIII; QIV

* Ref x:

$$x_R = \pi/6$$

* Solve:

QIII: $x = 7\pi/6$
QIV: $x = 11\pi/6$

7. $\sin(x) = 0$
Quadrantal

$$\boxed{x = 0}$$

$$\boxed{x = \pi}$$

8. $2\sin(x)\tan(x) + \tan(x) = 0$

* Factor:

$$\tan(x)(2\sin(x) + 1) = 0$$

Zero product property:

$$\tan(x) = 0$$

Quadrantal

$$\boxed{x = 0}$$

$$\boxed{x = \pi}$$

$$\text{or } 2\sin(x) + 1 = 0$$

* Isolate:

$$\Rightarrow 2\sin(x) = -1$$

$$\Rightarrow \sin(x) = -\frac{1}{2}$$

* Quad check

QIII; QIV

* Ref x:

$$x_R = \pi/6$$

* Solve

QIII: $x = 7\pi/6$
QIV: $x = 11\pi/6$

9. $4.3\sin(x) + 2.1 = 3.9$

* Isolate:

$$\Rightarrow 4.3\sin(x) = 1.8$$

$$\Rightarrow \sin(x) = \frac{1.8}{4.3}$$

* Quad check

QI; QII

* Ref x:

$$x_R = \sin^{-1}\left(\frac{1.8}{4.3}\right)$$

$$= .4319188...$$

$$= .432$$

* Solve:

QI: $x = .432$

QII: $x = \pi - .432 = 2.710$

10. $2\sin^2(x) + 3\sin(x) + 1 = 0$

* Factor

$$(2\sin(x) + 1)(\sin(x) + 1) = 0$$

Z.P.P.

$$2\sin(x) + 1 = 0 \text{ or } \sin(x) + 1 = 0$$

* Isolate:

$$\Rightarrow 2\sin(x) = -1$$

$$\Rightarrow \sin(x) = -\frac{1}{2}$$

* Quad check

QIII; QIV

* Ref x:

$$x_R = \pi/6$$

* Solve:

$$x = 7\pi/6$$

$$x = 11\pi/6$$

$$\sin(x) = -1$$

Quadrantal

$$\Rightarrow \boxed{x = 3\pi/2}$$

11. $2\sin(x)\cos(x) + \cos(x) = 0$

* Factor

$\Rightarrow (\cos(x)) (2\sin(x) + 1) = 0$
Z.P.P.

$\Rightarrow \cos(x) = 0$
Quadrantal

$x = \pi/2$

$x = 3\pi/2$

or $2\sin(x) + 1 = 0$

* Isolate:

$\Rightarrow \sin(x) = -1/2$

* Quad check

QIII QIV

* Ref x:

$x_R = \pi/6$

* Solve:

QIII: $x = 7\pi/6$

QIV: $x = 11\pi/6$

12.

$\sec^2(x) - \sec(x) - 2 = 0$

* Factor:

$(\sec(x) - 2)(\sec(x) + 1) = 0$

Z.P.P.

$\sec(x) = 2$ or

$\sec(x) + 1 = 0$

* Isolate:

$\sec(x) = 2$

* Reciprocal

$\cos(x) = 1/2$

* Quad check

QI QIV

* Ref x:

$x_R = \pi/3$

* Solve:

QI: $x = \pi/3$

QIV: $x = 5\pi/3$

pg
three

* Isolate:

$\cos(x) = -1$

* Quadrantal

$x = \pi$

13. $\sin^2(x) - 5 = -4$

* Isolate:

$\Rightarrow \sin^2(x) = 1$

$\Rightarrow \sin(x) = \pm \sqrt{1}$

$\Rightarrow \sin(x) = \pm 1$

Quadrantal

$\sin(x) = 1 \Rightarrow x = \pi/2$

$\sin(x) = -1 \Rightarrow x = 3\pi/2$

so
 $x = \pi/2 \pm 2\pi k$
 $x = 3\pi/2 \pm 2\pi k$

14. $6\cos(x) + 5 = 8$

* Isolate:

$\Rightarrow 6\cos(x) = 3$

$\Rightarrow \cos(x) = \frac{3}{6} = \frac{1}{2}$

* Quad check:

QI QIV

* Ref x:

$x_R = \pi/3$

* Solve

QI: $x = \pi/3$

QIV: $x = 5\pi/3$

so
 $x = \pi/3 \pm 2\pi k$
 $x = 5\pi/3 \pm 2\pi k$

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15. $2\cos(2x) + \sqrt{3} = 0$

⊕ Isolate:

$\Rightarrow 2\cos(2x) = -\sqrt{3}$

$\Rightarrow \cos(2x) = -\frac{\sqrt{3}}{2}$

⊕ Quadrant check:

QII; QIII

⊕ Ref 4:

$(2x)_R = \pi/6$

⊕ Solve:

$2x = 5\pi/6$

$2x = 7\pi/6$

^{use}
 $\Rightarrow 2x = 5\pi/6 \pm 2\pi k$

$2x = 7\pi/6 \pm 2\pi k$

$\Rightarrow \begin{cases} x = (\frac{1}{2})(\frac{5\pi}{6}) \pm (\frac{1}{2})(2\pi k) \\ x = (\frac{1}{2})(\frac{7\pi}{6}) \pm (\frac{1}{2})(2\pi k) \end{cases}$

$\Rightarrow \begin{cases} x = \frac{5\pi}{12} \pm \pi k \\ x = \frac{7\pi}{12} \pm \pi k \end{cases}$

$\Rightarrow \begin{cases} x = \frac{5\pi}{12} \pm \pi k \\ x = \frac{7\pi}{12} \pm \pi k \end{cases}$

$\Rightarrow \begin{cases} x = \frac{5\pi}{12} \pm \frac{12\pi k}{12} \\ x = \frac{7\pi}{12} \pm \frac{12\pi k}{12} \end{cases}$

$\Rightarrow \begin{cases} x = \frac{5\pi}{12} \pm \frac{12\pi k}{12} \\ x = \frac{7\pi}{12} \pm \frac{12\pi k}{12} \end{cases}$

If $k=0$ If $k=1$

$\begin{cases} x = \frac{5\pi}{12} \\ x = \frac{7\pi}{12} \end{cases} \begin{cases} x = \frac{17\pi}{12} \\ x = \frac{19\pi}{12} \end{cases}$

If $k=2$

~~$x = \frac{29\pi}{12}$~~
too big

16. $\tan(2x) + 1 = 0$

⊕ Isolate:

$\Rightarrow \tan(2x) = -1$

⊕ Quadrant check:

QII; QIII

$(2x)_R = \pi/4$

⊕ Solve:

QII: $2x = \frac{3\pi}{4} \pm 2\pi k$

QIII: $2x = \frac{7\pi}{4} \pm 2\pi k$

$\Rightarrow \begin{cases} x = (\frac{1}{2})(\frac{3\pi}{4}) \pm \frac{1}{2}(2\pi k) \\ x = (\frac{1}{2})(\frac{7\pi}{4}) \pm \frac{1}{2}(2\pi k) \end{cases}$

$\Rightarrow \begin{cases} x = \frac{3\pi}{8} \pm \pi k \\ x = \frac{7\pi}{8} \pm \pi k \end{cases}$

$\Rightarrow \begin{cases} x = \frac{3\pi}{8} \pm \pi k \\ x = \frac{7\pi}{8} \pm \pi k \end{cases}$

$\Rightarrow \begin{cases} x = \frac{3\pi}{8} \pm \frac{8\pi k}{8} \\ x = \frac{7\pi}{8} \pm \frac{8\pi k}{8} \end{cases}$

$\Rightarrow \begin{cases} x = \frac{3\pi}{8} \pm \frac{8\pi k}{8} \\ x = \frac{7\pi}{8} \pm \frac{8\pi k}{8} \end{cases}$

If $x=0$ If $x=1$

$\begin{matrix} x = \frac{3\pi}{8} & x = \frac{11\pi}{8} \\ x = \frac{7\pi}{8} & x = \frac{15\pi}{8} \end{matrix}$

If $x=2$

~~$x = \frac{19\pi}{8}$~~
too big