Solutions Calc II Pg 1ne

Practice exam #2wo spr 2025

(I). spring #1

$$\int_0^4 kx\,dx = 16$$

$$\Rightarrow \left[\frac{kx^2}{2}\right]_0^4 = 16$$

$$\Rightarrow \frac{k(4)^2}{2} = 16$$

$$\Rightarrow 8k = 16$$

$$\Rightarrow k = 2$$

$$\Rightarrow F_1(x) = 2x$$

spring #2wo

$$k(6) = 36$$

$$\Rightarrow k = 6$$

so

$$\Rightarrow F_2(x) = 6x$$

so we have

$\boxed{B}$ requires more work

Thus:

$$\text{(A)} \int_2^5 2x\,dx = \left[\frac{2x^2}{2}\right]_2^5$$

$$= [(5)^2 - (2^2)]$$

$$= 25 - 4$$

$$= 21$$

$$\text{(B)} \int_1^3 6x\,dx = \left[\frac{6x^2}{2}\right]_1^3$$

$$= [(3(3)^2) - (3(1^2))]$$

$$= 27 - 3$$

$$= 24$$

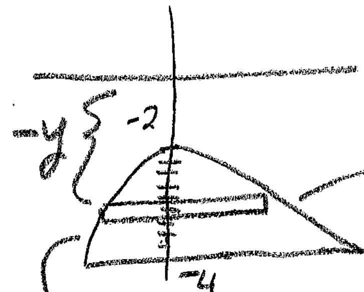
(II) $\Delta F = (\text{pressure})(\text{depth})$

$= (\text{density})(\text{area})(\text{depth})$

$= (\text{density})(\text{length})(\text{width})(\text{depth})$

$= (\text{density}(\text{Right} - \text{Left})(\text{width})(\text{depth})$

$$= \rho\left((-y-2) - \left(\left(\frac{y+4}{2}\right)^2 - 1\right)\right)(\Delta y)(-y)$$



$$y = 2\sqrt{x+1} - 4$$

$$\Rightarrow \frac{y+4}{2} = \sqrt{x+1}$$

$$\Rightarrow \left(\frac{y+4}{2}\right)^2 - 1 = x$$

$\Downarrow$ "Left"

$$y = -x - 2$$

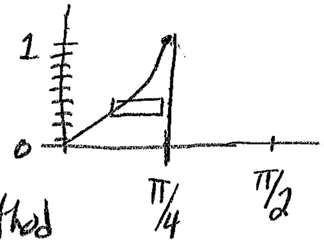
$$\Rightarrow -y - 2 = x$$

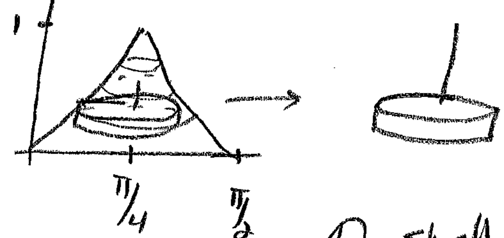
$\Downarrow$ "RIGHT"

so we have:

$$\boxed{F = \rho \int_{-4}^{-2} (-y)\left[(-y-2) - \left(\left(\frac{y+4}{2}\right)^2 - 1\right)\right] dy}$$

III A.





ⓐ use **Disk method**

↳ partition y-axis

$$V = \pi \int_a^b (\text{radius})^2\,dy$$

$$= \pi \int_0^1 (\text{Right} - \text{left})^2\,dy$$

$$\times\ \pi \int_0^1 \left(\tfrac{\pi}{4} - \tan(x)\right)^2 dy$$

NO GOOD!

⇒ we must use

$\tan(x) = y \Rightarrow x = \tan^{-1}(y)$

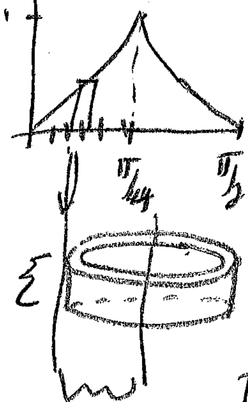
so ~~we have~~:

$$\boxed{V = \pi \int_0^1 \left(\tfrac{\pi}{4} - \tan^{-1}(y)\right)^2 dy}$$

ⓑ shell method.

↳ partition "opposite"

↳ partition x-axis



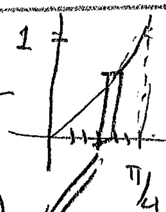
$$V = 2\pi \int_0^{\pi/4} (\text{radius})(\text{ht})\,dx$$

$$= 2\pi \int_0^{\pi/4} (\text{rt} - \text{left})(\text{ht})\,dx$$

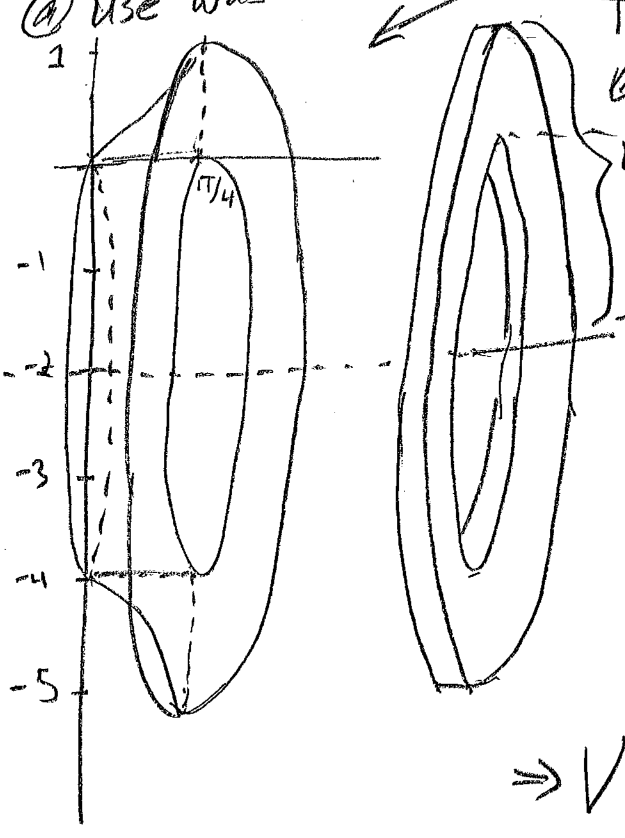
$$\boxed{= 2\pi \int_0^{\pi/4} \left(\frac{\pi}{4} - x\right)(\tan(x))\,dx}$$

B

ⓐ use washer method:



→ partition x-axis



outer radius
top − bottom
$\tan(x) - (-2)$

inner radius
top − bottom
$0 - (-2)$

so we have:

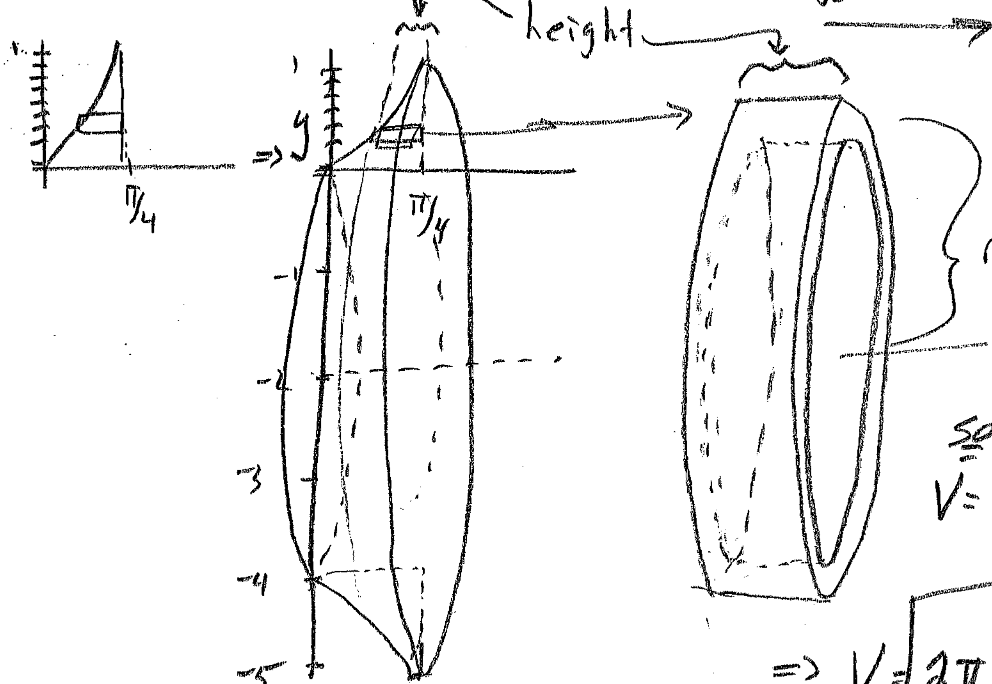
$$V = \pi \int_0^{\pi/4} \left[(\text{outer radius})^2 - (\text{inner radius})^2\right] dx$$

$$\Rightarrow V \boxed{= \pi \int_0^{\pi/2} \left[(\tan(x) - (-2))^2 - (0 - (-2))^2\right] dx}$$

Ⓑ

(b). Shell method → partition y-axis ⇒ so: $y = \tan(x)$

$\Rightarrow \tan^{-1}(y) = x$



⇒ height = right − left $= \left(\frac{\pi}{4} - \tan^{-1}(y)\right)$

radius: top − bottom $= (y - (-2))$

so

$$V = 2\pi \int_0^1 (\text{radius})(\text{height})(\text{thickness})$$

$$\Rightarrow V = \boxed{2\pi \int_0^1 (y-(-2))\left(\frac{\pi}{4} - \tan^{-1}(y)\right) dy}$$

IV $y = 1 + \sin(2x)$

$\Rightarrow y' = \cos(2x)\cdot(2)$

$= 2\cos(2x)$

so we have

$$S = \boxed{\int_0^{\pi} \sqrt{1 + [2\cos(2x)]^2}\, dx}$$

Arc Length letter

⇒

V $y = 1 + x^{2/3}$

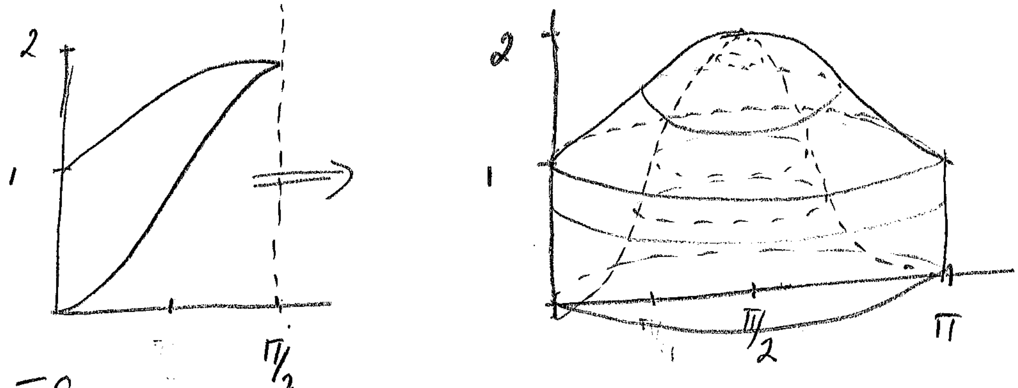
$\Rightarrow y' = \frac{2}{3}x^{-1/3}$

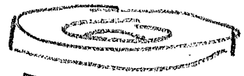
must break into two integrals at $x = 0$

so

$$S = \boxed{\int_{-1}^{0} \sqrt{1 + \left[\tfrac{2}{3}x^{-1/3}\right]^2}\, dx + \int_0^8 \sqrt{1 + \left[\tfrac{2}{3}x^{-1/3}\right]^2}\, dx}$$

VI. (A)



Note: If we use washers:  ⇒ this requires

① partitioning the y-axis

② all functions must be functions of y

③ The partitions pass over a cusp.

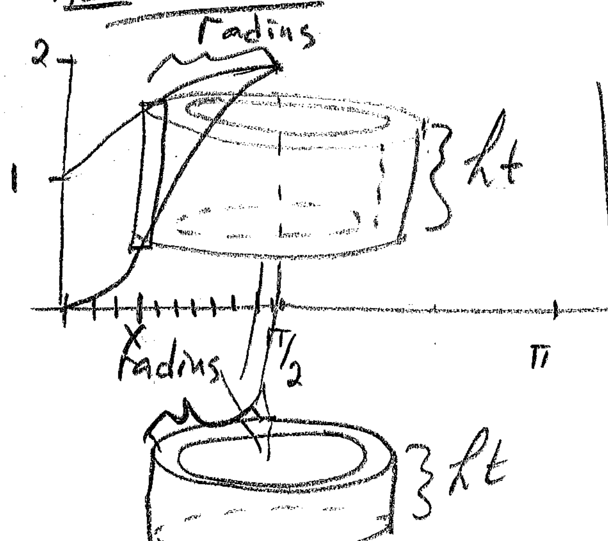
④ Must use 2 integrals ⇒ one from $\int_0^1 \cdots$

⇒ one from $\int_1^2 \cdots$

Instead I'll use

SHELL METHOD ⇒ partition x-axis from 0 to $\pi/2$

So



So

$$V = 2\pi \int_0^{\pi/2} (\text{radius})\underbrace{(\text{height})}_{(\text{top} - \text{bot})}(\text{thickness})$$

$$(\text{rt} - \text{left})$$

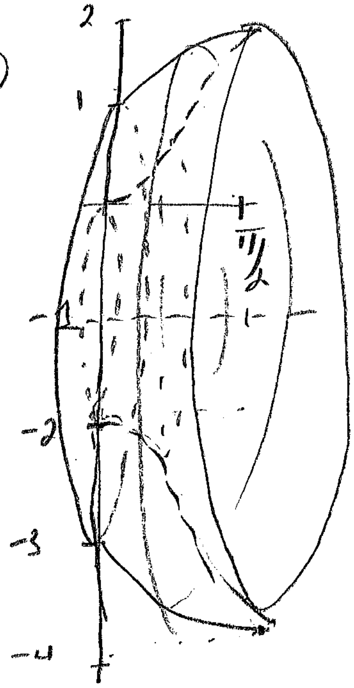
so we have

$$V = 2\pi \int_0^{\pi/2} \left(\frac{\pi}{2} - x\right)\Big((1+\sin(x)) - (1-\cos(2x))\Big)\,dx$$

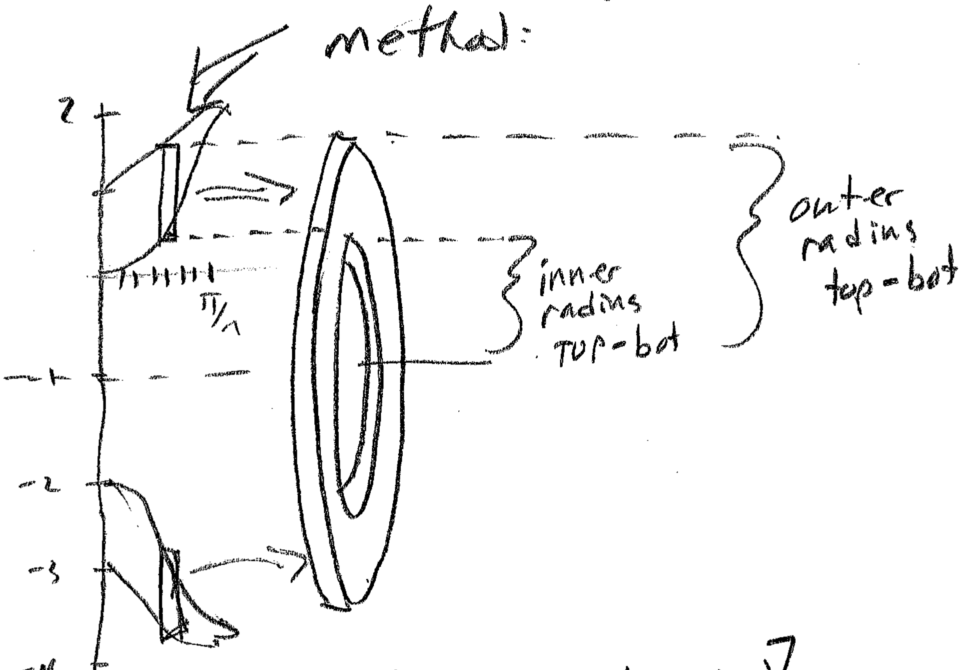
Better

$$\boxed{V = 2\pi \int_0^{\pi/2} \left(\frac{\pi}{2} - x\right)\Big[(1+\sin(x)) - (1-\cos(2x))\Big]\,dx}$$

VI B



This time if we partition the y-axis (shell method) we encounter the issues of cusp, we must use functions of y, etc. so it is easier to partition the x-axis, using the <u>washer</u> method:



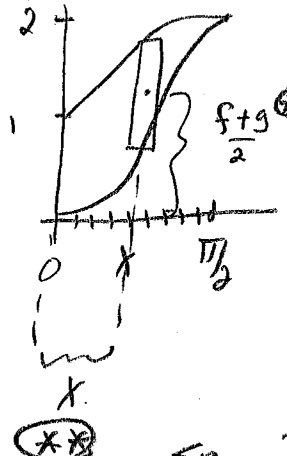
we have $\text{inner radius} = \text{top} - \text{bot} = \left[(1-\cos(2x)) - (-1)\right]$

$\text{outer radius} = \text{top} - \text{bot} = \left[(1+\sin(x)) - (-1)\right]$

so we have

$$V = \boxed{\pi \int_0^{\pi/4} \left[\left[(1-\cos(2x)) - (-1)\right]^2 - \left[(1+\sin(x)) - (-1)\right]^2 \right] dx}$$

VII



$M_x = (\text{density})(\text{distance to } x\text{-axis})^{⊛}(\text{length})(\text{width})$

$M_y = (\text{density})(\text{distance to } y\text{-axis})_{⊛⊛}(\text{length})(\text{width})$

$m = (\text{density})(\text{length})(\text{width})$

so

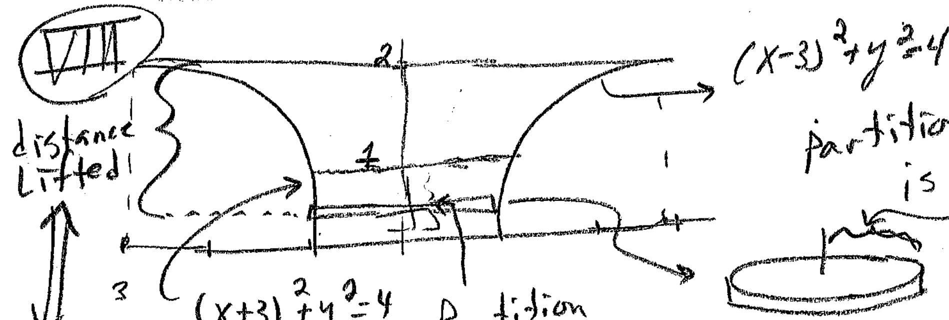
$$\boxed{M_x = \rho \int_0^{\pi/2} \left[\frac{(1+\sin(x)) + (1-\cos(2x))}{2}\right]\left[(1+\sin(x)) - (1-\cos(2x))\right] dx}$$

$$M_y = \boxed{\rho \int_0^{\pi/2} [x]\left[(1+\sin(x)) - (1-\cos(2x))\right] dx}$$

$$m = \boxed{\rho \int_0^{\pi/2} \left[(1+\sin(x)) - (1-\cos(2x))\right] dx}$$

Finally note that $\boxed{\bar{x} = \frac{M_y}{m} \qquad \bar{y} = \frac{M_x}{m}}$

VIII



partition the y-axis where there is fluid.

need functions of y

$(x-3)^2 = 4-y^2$

$\Rightarrow x = \sqrt{4-y^2}+3$

$W = (\text{density})(\text{volume})(\text{distance lifted})$

so $W = \rho \int_0^1 \pi (\text{radius})^2 (\text{distance lifted})(\text{thickness})$

$$= \boxed{\rho \int_0^1 \pi\left(\sqrt{4-x^2}+3\right)^2 (2-y)\, dy}$$

IX

This was worked out in class Monday.