SOLUTIONS PARTIAL FRACTIONS EXAMPLES.

I. ① $\dfrac{(x+14)}{x^2+2x}$

* FACTOR DENOMINATOR:

$$\Rightarrow \frac{(x+14)}{x(x+2)} = \frac{A}{x} + \frac{B}{(x+2)}$$

* Multiply thru by denominator of the L.H.S.

$$\Rightarrow \frac{x(x+2)}{1}\left[\frac{(x+14)}{x(x+2)} = \frac{A}{x} + \frac{B}{(x+2)}\right]$$

$$\Rightarrow x+14 = A(x+2) + B(x)$$

Let $x = -2$

$\Rightarrow (-2)+14 = A(0) + B(-2)$

$\Rightarrow 12 = B(-2)$

$\Rightarrow B = -6$

Let $x = 0$

$\Rightarrow (0)+14 = A((0)+2) + B(0)$

$\Rightarrow 14 = A(2)$

$\Rightarrow A = 7$

SO WE HAVE

$$\frac{x+14}{x^2+2x} = \boxed{\frac{7}{x} + \frac{-6}{(x+2)}}$$

② $\dfrac{-2x+26}{(x-3)(x+2)}$

$$\Rightarrow \frac{-2x+26}{(x-3)(x+2)} = \frac{A}{(x-3)} + \frac{B}{(x+2)}$$

* MULTIPLY THRU BY denominator of L.H.S.

$$\Rightarrow \frac{(x-3)(x+2)}{1}\left[\frac{-2x+26}{(x-3)(x+2)} = \frac{A}{(x-3)} + \frac{B}{(x+2)}\right]$$

$$\Rightarrow -2x+26 = A(x+2) + B(x-3)$$

Let $x = -2$

$\Rightarrow -2(-2)+26 = A(0) + B((-2)-3)$

$\Rightarrow 30 = B(-5)$

$\Rightarrow B = -6$

Let $x = 3$

$\Rightarrow -2(3)+26 = A((3)+2) + B(0)$

$\Rightarrow 20 = A(5)$

$\Rightarrow A = 4$

SO WE HAVE:

$$\frac{-2x+26}{(x-3)(x+2)} = \boxed{\frac{4}{(x-3)} + \frac{-6}{(x+2)}}$$

(3) $\frac{-2x+1}{x(x-1)^2}$ ← TYPE II

$\Rightarrow \frac{-2x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$

* Multiply thru by denominator of the L.H.S.

$\Rightarrow \frac{x(x-1)^2}{1}\left[\frac{-2x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}\right] \Rightarrow$

$\Rightarrow -2x+1 = A(x-1)^2 + B(x)(x-1) + C(x)$

Let $x=1$

$\Rightarrow -2(1)+1 = 0+0+C(1)$

$\Rightarrow -1 = C(1)$

$\Rightarrow C = -1$

Let $x=0$

$\Rightarrow -2(0)+1 = A(0-1)^2 + 0 + 0$

$\Rightarrow -1 = A(1)$

$\Rightarrow A = -1$

we may choose any value. I choose

Let $x=2$

$\Rightarrow -2(2)+1 = (-1)(2-1)^2 + B(2)(2-1) + (-1)(2)$

$\Rightarrow -3 = -1 + B(2) + (-2)$

$\Rightarrow -3 = -3 + B(2)$

$\Rightarrow 0 = B(2)$

$\Rightarrow B = 0$

So we have:

$\frac{-2x+1}{x(x-1)^2} = \boxed{\frac{-1}{x} + \frac{-1}{(x-1)^2}}$

NOTE: MY ANSWER ON THE ORIGINAL PAGE IS NOT CORRECT. THIS IS

(4) $\frac{3x^2+26x+46}{(x-1)(x+4)^2}$ ← TYPE II

$\Rightarrow \frac{3x^2+26x+46}{(x-1)(x+4)^2} = \frac{A}{(x-1)} + \frac{B}{(x+4)} + \frac{C}{(x+4)^2}$

* MULTIPLY THRU BY denominator of L.H.S.

$\Rightarrow \frac{(x-1)(x+4)^2}{1}\left[\frac{3x^2+26x+46}{(x-1)(x+4)^2} = \frac{A}{(x-1)} + \frac{B}{(x+4)} + \frac{C}{(x+4)^2}\right]$

$\Rightarrow 3x^2+26x+46 = A(x+4)^2 + B(x-1)(x+4) + C(x-1)$

Let $x=-4$

$\Rightarrow 3(-4)^2+26(-4)+46 = 0+0+C(-4-1)$

$\Rightarrow -10 = C(-5)$

$\Rightarrow C = 2$

Let $x=1$

$\Rightarrow 3(1)^2+26(1)+46 = A(1+4)^2+0+0$

$\Rightarrow 75 = A(25)$

$\Rightarrow A = 3$

Let $x=0$ ← YOU MAY CHOOSE ANY VALUE

$\Rightarrow 3(0)^2+26(0)+46 = (3)(0+4)^2 + B(0-1)(0+4) + (2)(0-1)$

$\Rightarrow 46 = 3(16) + B(-4) + 2(-1)$

$\Rightarrow 46 = 46 + B(-4)$

$\Rightarrow 0 = B(-4)$

$\Rightarrow B = 0$

SO WE HAVE:

$\frac{3x^2+26x+46}{(x-1)(x+4)^2} = \boxed{\frac{3}{(x-1)} + \frac{2}{(x+4)^2}}$

(5). $\frac{x^2+5}{x(x^2-5)}$ ← TYPE III

$$\Rightarrow \frac{(x^2+5)}{x(x^2-5)} = \frac{A}{x} + \frac{(Bx+C)}{(x^2-5)}$$

* multiply thru by denominator of L.H.S.

$$\Rightarrow \frac{x(x^2-5)}{1}\left[\frac{(x^2+5)}{x(x^2-5)} = \frac{A}{x} + \frac{(Bx+C)}{(x^2-5)}\right]$$

$$\Rightarrow x^2+5 = A(x^2-5) + (Bx+C)(x)$$

* Multiply out R.H.S.

$$\Rightarrow x^2+5 = Ax^2-5A+Bx^2+Cx$$

* Gather Coefficients:

$$\Rightarrow x^2+5 = (A+B)x^2+Cx-5A$$

* EQUATE COEFFICIENTS

$$\Rightarrow A+B = 1 \longrightarrow (-1)+B=1 \Rightarrow B=2$$

$$C = 0 \Rightarrow C=0$$

$$-5A = 5 \Rightarrow -5A=5 \Rightarrow A=-1$$

So we have:

$$\frac{x^2+5}{x(x^2-5)} = \boxed{\frac{-1}{x} + \frac{2x}{(x^2-5)}}$$

(6) $\dfrac{2x^2-x+15}{(x-3)(x^2+6)}$ ← TYPE III

$$\Rightarrow \frac{2x^2-x+15}{(x-3)(x^2+6)} = \frac{A}{(x-3)} + \frac{(Bx+C)}{(x^2+6)}$$

* Multiply thru by denominator of L.H.S.

$$\Rightarrow \frac{(x-3)(x^2+6)}{1}\left[\frac{2x^2-x+15}{(x-3)(x^2+6)} = \frac{A}{(x-3)} + \frac{(Bx+C)}{(x^2+6)}\right]$$

$$\Rightarrow 2x^2-x+15 = A(x^2+6) + (Bx+C)(x-3)$$

* Multiply out R.H.S. (F O I L)

$$\Rightarrow 2x^2-x+15 = Ax^2+6A+Bx^2-3Bx+Cx-3C$$

* GATHER COEFFICIENTS ON R.H.S.

$$\Rightarrow 2x^2-x+15 = (A+B)x^2+(-3B+C)x+(6A-3C)$$

* EQUATE COEFFICIENTS:

$\Rightarrow A+B$ ...

(7) $\dfrac{2x^4+4x^2+x+2}{x(x^2+1)^2}$ TYPE IIII HANG ON!

(*) Form the Partial Fractions:

$$\Rightarrow \frac{2x^4+4x^2+x+2}{x(x^2+1)^2} = \frac{A}{x} + \frac{(Bx+C)}{(x^2+1)} + \frac{(Dx+E)}{(x^2+1)^2}$$

(*) Multiply thru by denominator of L.H.S.

$$\Rightarrow \frac{[x(x^2+1)^2]}{1}\left[\frac{2x^4+4x^2+x+2}{x(x^2+1)^2} = \frac{A}{x} + \frac{(Bx+C)}{x^2+1} + \frac{(Dx+E)}{(x^2+1)^2}\right]$$

$$\Rightarrow 2x^4+4x^2+x+2 = A(x^2+1)^2 + (Bx+C)(x)(x^2+1) + (Dx+E)(x)$$

(*) Multiply out the R.H.S. (Foil, Foil)

$$\Rightarrow 2x^4+4x^2+x+2 = A(x^4+2x^2+1) + (Bx^2+Cx)(x^2+1) + (Dx+E)(x)$$

$$= Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 + Cx^3 + Cx + Dx^2 + Ex$$

(*) Gather Coefficients

$$\Rightarrow 2x^4+4x^2+x+2 = (A+B)x^4 + Cx^3 + (2A+B+D)x^2 + (C+E)x + A$$

(*) EQUATE coefficients:

$A+B = 2 \Rightarrow 2+B=2 \Rightarrow \boxed{B=0}$

$C = 0 \Rightarrow \boxed{C=0}$

$2A+B+D = 4 \Rightarrow 2(2)+0+D=4 \Rightarrow 4+D=4 \Rightarrow \boxed{D=0}$

$C+E = 1 \Rightarrow 0+E=1 \Rightarrow \boxed{E=1}$

$A = 2 \Rightarrow \boxed{A=2}$

So we have:

$$\frac{2x^4+4x^2+x+2}{x(x^2+1)^2} = \boxed{\frac{2}{x} + \frac{1}{(x^2+1)^2}}$$