(11). $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-2)^n}{n} \Rightarrow$ center: $\boxed{c=2}$

Ratio test: $\displaystyle\lim_{n\to\infty} \left| \frac{\frac{(-1)^n (x-2)^{n+1}}{(n+1)}}{\frac{(-1)^{n-1}(x-2)^n}{n}} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n}(x-2)^{n+1}}{(n+1)} \cdot \frac{n}{(-1)^{n-1}(x-2)^n} \right|$

$\displaystyle = \lim_{n\to\infty} \left| \frac{(x-2)\cdot n}{n+1} \right|$

$\displaystyle = |x-2| \lim_{n\to\infty} \frac{n}{n+1}$

$\displaystyle \stackrel{L'H}{=} |x-2| \lim_{n\to\infty} \frac{1}{1}$

$= |x-2|$

converges if $|x-2| < 1$

$\Rightarrow -1 < x-2 < 1$

$\Rightarrow 1 < x < 3$

must check endpoints individually:

If $x=1$ we have

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(1-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(-1)^n}{n}$

$\displaystyle = \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n}$ Note this exponent is ALWAYS odd $\forall n$.

$\displaystyle = \sum_{n=1}^{\infty} \frac{-1}{n}$

$\displaystyle = -\sum_{n=1}^{\infty} \frac{1}{n} \rightarrow$ diverges (harmonic series)

$\therefore$ DOES NOT CONVERGE AT $x=1$

If $x=3$ we have

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(3-2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(1)^n}{n}$

$\displaystyle = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \rightarrow$ converges (alternating harmonic series)

$\therefore$ DOES converge at $x=3$

So Interval: $\boxed{(1, 3]}$

radius: $\boxed{1}$

(12). $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)}$ center: $\boxed{c=0}$

Ratio Test:

$$\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}x^{(2n+3)}}{(2n+3)}}{\frac{(-1)^n x^{2n+1}}{(2n+1)}}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(-1)^{n+1}} x^{(2n+3)}}{(2n+3)}\cdot\frac{(2n+1)}{\cancel{(-1)^{n+1}} x^{(2n+1)}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x^{(2n+3)-(2n+1)}(2n+1)}{(2n+3)}\right|$$

$$= \lim_{n\to\infty}\frac{|x^2|(2n+1)}{(2n+3)}$$

$$= x^2 \lim_{n\to\infty}\frac{(2n+1)}{(2n+3)}$$

$$\overset{LH}{=} x^2 \lim_{n\to\infty}\frac{2}{2}$$

$$= x^2$$

converges if $x^2 < 1$

$\Rightarrow -1 < x < 1$

must check end pts individually

If $x = -1$ we have:

$$\sum_{n=1}^{\infty}\frac{(-1)^n(-1)^{2n+1}}{(2n+1)} =$$ $\longrightarrow$ This exponent is odd $\forall n$

$$= \sum_{n=1}^{\infty}\frac{(-1)^n(-1)}{(2n+1)}$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n+1)}$$

Alternating Series Test

(i) $\frac{1}{2n+1} \Rightarrow f(x) = \frac{1}{2x+1} = (2x+1)^{-1}$

$\Rightarrow f'(x) = -(2x+1)^{-2} = \frac{-1}{(2x+1)^2}$

which is negative $\forall x$.

$\therefore f(x) \searrow \therefore \frac{1}{2n+1} \searrow \leftarrow$

(ii) $\lim_{n\to\infty} \frac{1}{2n+1} \to 0 = 0 \leftarrow$

$\therefore$ converges at $x = -1$

If $x = 1$ we have

$$\sum_{n=1}^{\infty}\frac{(-1)^n(1)^{2n+1}}{(2n+1)} =$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)}$$

which converges also by the same Alternating Series Test.

$\therefore$ Converges for $x = 1$

$\therefore$ Interval $= \boxed{[-1, 1]}$

Radius: $\boxed{1}$.

(13) Easier problem:

$$\sum_{n=1}^{\infty} \frac{(2n)x^{2n+1}}{(n-1)!(2n+1)}$$ $\boxed{\text{center} \Rightarrow C=0}$

Ratio Test:

$$\lim_{n\to\infty} \left| \frac{\frac{(2n+2)x^{(2n+3)}}{(n)!(2n+3)}}{\frac{(2n)x^{(2n+1)}}{(n-1)!(2n+1)}} \right| = \lim_{n\to\infty} \left| \frac{(2n+2)x^{(2n+3)}}{(n)!(2n+3)} \cdot \frac{(n-1)!(2n+1)}{(2n)x^{(2n+1)}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(2n+2)(x^2)(2n+1)}{(n)(2n+3)(2n)} \right|$$

$$= |x^2| \lim_{n\to\infty} \frac{4n^2+6n+2}{4n^3+6n^2}$$

$$= |x^2| \lim_{n\to\infty} \frac{\overset{0}{\cancel{4n^2/n^3}} + \overset{0}{\cancel{6n/n^3}} + \overset{0}{\cancel{2/n^3}}}{4n^3/n^3 + \overset{0}{\cancel{6n^2/n^3}}}$$

$$= |x^2| \cdot \frac{0}{1}$$

$$= 0 < 1 \quad \forall x$$

$\therefore$ converges for $(-\infty, \infty)$

Interval: $\boxed{(-\infty, \infty)}$

Radius: $\boxed{\infty}$

(14). $$\int_0^1 e^{x^2}dx = \int_0^1 \left[\sum_{n=0}^{\infty} \frac{(x^2)^n}{n!}\right]dx$$

$$= \int_0^1 \left[\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}\right]dx$$

$$= \sum_{n=0}^{\infty} \left[\int_0^1 \frac{x^{2n}}{n!}\,dx\right]$$

$$= \sum_{n=0}^{\infty} \left[\frac{x^{(2n+1)}}{(2n+1)\cdot n!}\right]_0^1$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1^{(2n+1)}}{(2n+1)\,n!}\right) - \cancel{\left(\frac{0^{(2n+1)}}{(2n+1)\cdot n!}\right)}^{\,0}\right]$$

$$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)\,n!}$$

$$= \frac{1}{(2(0)+1)\cdot 0!} + \frac{1}{(2(1)+1)\cdot 1!} + \frac{1}{(2(2)+1)\cdot 2!} + \frac{1}{(2(3)+1)\cdot 3!} + \frac{1}{(2(4)+1)\cdot 4!} + \cdots$$

$$= \frac{1}{(1)(1)} + \frac{1}{(3)(1)} + \frac{1}{(5)(2)} + \frac{1}{(7)(6)} + \frac{1}{(9)(24)} + \frac{1}{(11)(120)} + \cdots$$

$$= \frac{1}{1} + \frac{1}{3} + \frac{1}{10} + \frac{1}{42} + \frac{1}{216} + \cdots$$

$$\approx \frac{1}{1} + \frac{1}{3} + \frac{1}{10} + \frac{1}{42} \qquad \text{"first four terms"}$$

$$= \frac{420+140+42+10}{420}$$

$$= \frac{612}{420}$$

$$= \boxed{\frac{153}{140}}$$