PRACTICE FOR SOLVING EQUATIONS

For each section there are three examples worked out as illustrations, then the practice problems follow.

Note that a few of these items are solved in an interval OTHER THAN $[0, 2\pi)$.

II. Equations Involving Quadrantal Angles

Solve each equation over the indicated interval

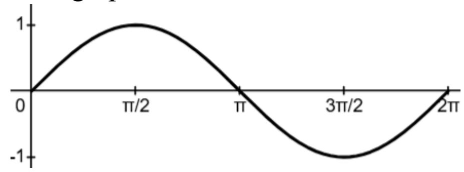
Example:

a) $\sin(x) = -1 \quad [0, 2\pi)$ b) $\tan(x) = 0 \quad [0, 2\pi)$ c) $\cos(x) = 1 \quad (\pi, 2\pi)$

Solution:

a) $\sin(x) = -1 \quad [0, 2\pi)$

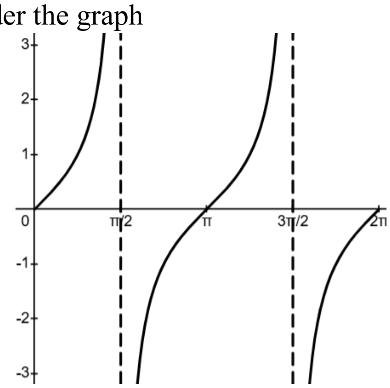
Consider the graph



The graph obtains a trough in the interval $[0, 2\pi)$ at $\boxed{x = 3\pi/2}$

b) $\tan(x) = 0 \quad [0, 2\pi)$

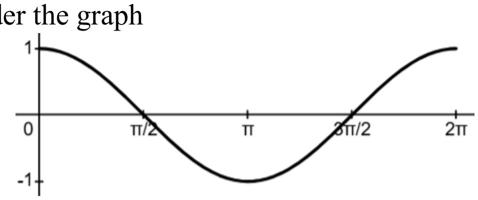
Consider the graph



The graph crosses the x-axis in the interval $[0, 2\pi)$ at $\boxed{x = 0}$ & $\boxed{x = \pi}$

c) $\cos(x) = 1 \quad (\pi, 2\pi)$

Consider the graph



In the interval $(\pi, 2\pi)$ the graph does not achieve a peak, so we have $\boxed{\textit{No Solution}}$.

5. $\sin(x) = 0 \quad [0, 2\pi)$
6. $\sin(x) = 1 \quad [0, 2\pi)$
7. $\sin(x) = -1 \quad [0, 2\pi)$
8. $\cos(x) = 1 \quad [0, 2\pi)$
9. $\cos(x) = -1 \quad [0, 2\pi)$
10. $\cos(x) = 0 \quad [0, 2\pi)$
11. $\tan(x) = 0 \quad [0, 2\pi)$
12. $\cos(x) = 0 \quad [0, \pi]$
13. $\sin(x) = 1 \quad (0, \pi)$
14. $\sin(x) = 0 \quad [-2\pi, 2\pi]$
15. $\tan(x) = 0 \quad \left(\pi/2, 3\pi/2\right)$
16. $\cos(x) = -1 \quad [-\pi, \pi]$

Example:

a) $-2\sin(x) - 2 = 0 \quad [0, 2\pi)$ b) $\sin(x)\tan(x) - \tan(x) = 0 \quad [0, 2\pi)$

Solution:

a) Isolate:

Linear in sine

* Isolate:

$$-2\sin(x) - 2 = 0$$
$$\Rightarrow -2\sin(x) = 2$$
$$\Rightarrow \sin(x) = 2/-2$$
$$\Rightarrow \sin(x) = -1$$

*Reference the graph:

$$\Rightarrow \boxed{x = 3\pi/2}$$

b) Functions are multiplied:

*Factor:

$$\sin(x)\tan(x) - \tan(x) = 0$$
$$\Rightarrow \tan(x)\left(\sin(x) - 1\right) = 0$$
$$\Rightarrow \tan(x) = 0 \quad or \quad \sin(x) - 1 = 0$$
$$\Rightarrow \tan(x) = 0 \quad or \quad \sin(x) = 1$$

*Reference the graphs:

$$\Rightarrow \boxed{x = 0}; \boxed{x = \pi} \quad or \quad \boxed{x = \pi/2}$$

17. $\sin(x) - 2 = -1 \quad [0, 2\pi)$
18. $5\tan(x) - 6 = -6 \quad [0, 2\pi)$
19. $\sec(x) + 1 = 0 \quad [0, 2\pi)$
20. $\sin(x)\cos(x) - \cos(x) = 0 \quad [0, 2\pi)$
21. $\csc(x)\cot(x) = \cot(x) \quad [0, 2\pi)$

III. Equations Involving Special Angles

Example:

a) $2\sin(x)\cos(x) + \cos(x) = 0 \quad [0, 2\pi)$ b) $\left(2\cos(x) + 1\right)\left(\tan(x) - 1\right) = 0 \quad [0, 2\pi)$

Solution:

a) Functions are multiplied, so Factor:

$$2\sin(x)\cos(x) + \cos(x) = 0$$
$$\Rightarrow \cos(x)\left(2\sin(x) + 1\right) = 0$$
$$\Rightarrow \cos(x) = 0 \quad or \quad 2\sin(x) + 1 = 0$$
$$\Rightarrow x = \pi/2; \quad x = 3\pi/2$$

* Isolate:

$$\sin(x) = -1/2$$

* Quadrant Check:

Sine is negative in QIII, QIIII

* Reference Angle:

$x_R = \pi/6$

* Solve:

$x = 7\pi/6 \quad or \quad x = 11\pi/6$

So we have:

$$\boxed{\pi/2,\ 3\pi/2,\ 7\pi/6,\ 11\pi/6}$$

Solution:

a) Functions are multiplied, so Factor:

$$2\sin(x)\cos(x) + \cos(x) = 0$$

$$\Rightarrow \cos(x)\left(2\sin(x) + 1\right) = 0$$

$$\Rightarrow \cos(x) = 0 \qquad or \qquad 2\sin(x) + 1 = 0$$

$$\Rightarrow x = \pi/2; \quad x = 3\pi/2$$

* Isolate:

$\sin(x) = -1/2$

* Quadrant Check:
  Sine is negative in QIII, QIIII
* Reference Angle:

$x_R = \pi/6$

* Solve:

$x = 7\pi/6 \quad or \quad x = 11\pi/6$

So we have:

$$\boxed{\pi/2,\ 3\pi/2,\ 7\pi/6,\ 11\pi/6}$$

22. $2\cos(x) + 3 = 4 \quad [0, 2\pi)$

23. $5\tan(x) - 1 = 4 \quad [0, 2\pi)$

24. $2\sec(x) + 1 = \sec(x) + 3 \quad [0, 2\pi)$

25. $(\tan(x) + 1)\left(\sqrt{3}\tan(x) - 1\right) = 0 \quad [0, 2\pi)$

26. $\sin^2(x)\cos(x) = \cos(x) \quad [0, 2\pi)$

27. $2\sin(x)\cos(x) = 0 \quad [0, 2\pi)$

28. $2\cos^2(x) + \cos(x) - 1 = 0 \quad [0, 2\pi)$

29. $\sec^2(x)\tan(x) = 2\tan(x) \quad [0, 2\pi)$

IIII. Equations Involving "Other" Angles

Example:

a) $5\sin(x) + 14 = 10 \quad [0, 2\pi)$

b) $21\cos^2(x) + 11\cos(x) - 2 = 0 \quad [0, 2\pi)$

Solution:

a) $5\sin(x)+14=10 \quad [0,2\pi)$

* Isolate:

$5\sin(x)+14=10$

$\Rightarrow 5\sin(x)=-4$

$\Rightarrow \sin(x)=-\frac{4}{5}$

$\Rightarrow \sin(x)=-0.8$

* Quadrant Check:
  Sine is negative in QIII, QIIII
* Reference Angle:

$x_R=\sin^{-1}(.8)$

$=.92729521...$

* Solve:

$QIII$	*or* $QIIII$
$x=\pi+.92729521...$	$x=2\pi-.92729521...$
$=4.06888787...$	$=5.35589008...$
$\approx \boxed{4.0689}$	$\approx \boxed{5.3559}$

b) $21\cos^2(x)+11\cos(x)-2=0 \quad [0,2\pi)$

* Factor:

$(7\sin(x)-1)(3\sin(x)+2)=0$

$\Rightarrow 7\sin(x)-1=0$ *or* $\Rightarrow 3\sin(x)+2=0$

* Isolate:	*Isolate:
$\Rightarrow \sin(x)=\frac{1}{7}$	$\Rightarrow \sin(x)=-\frac{2}{3}$
$\Rightarrow x=\sin^{-1}(.14285714...)$	$\Rightarrow x=\sin^{-1}(.66666666...)$
* Quadrant Check: Sine is positive in QI, QII	* Quadrant Check: Sine is negative in QIII, QIIII
* Reference Angle:	* Reference Angle:
$x_R=\sin^{-1}(.14285714...)$	$x_R=\sin^{-1}(.66666666...)$
$=.14334756...$	$=.72972765...$

* Solve:

Case I		Case II	
QI	QII	$QIII$	$QIIII$
$x=.14334756...$	$x=\pi-.14334756...$	$x=\pi+.72972765...$	$x=2\pi-.72972765...$
$\approx \boxed{.1433}$	$=2.99824508...$	$=3.87132031...$	$=5.55345765...$
	$\approx \boxed{2.9982}$	$\approx \boxed{3.8713}$	$\approx \boxed{5.5535}$

30. $5\tan(x)+17=1 \quad [0,2\pi)$

31. $-13\sin(x)-2=4 \quad [0,2\pi)$

32. $4\sec(x)+9=0 \quad [0,2\pi)$

33. $\left(\tan(x)+4\right)\left(\sqrt{7}\cot(x)-1\right)=0 \quad [0,2\pi)$

34. $3\sin^2(x)-13\sin(x)+4=0 \quad [0,2\pi)$

35. $\csc^2(x)+\csc(x)-20=0 \quad [0,2\pi)$

V. Equations solved using either Difference of Squares or $\pm$ Square Root method:

Example:

a) $\sec^2(x)-2=0 \quad [0,2\pi)$ b) $4\sin^2(x)-9=0 \quad [0,2\pi)$

Solution:

a) $\sec^2(x)-2=0 \quad [0,2\pi)$

Difference of Squares method	$\pm$ Square Root method
$\Rightarrow \left(\sec(x)+\sqrt{2}\right)\left(\sec(x)-\sqrt{2}\right)=0$ $\Rightarrow \sec(x)+\sqrt{2}=0 \quad or \quad \sec(x)-\sqrt{2}=0$ $\Rightarrow \sec(x)=-\sqrt{2} \quad or \quad \sec(x)=\sqrt{2}$ *Reciprocals: $\Rightarrow \cos(x)=-\frac{1}{\sqrt{2}} \quad or \quad \cos(x)=\frac{1}{\sqrt{2}}$ *Quadrant Check: QII, QIII *or* QI, QIIII Thus, all four Quadrants	$\Rightarrow \sec^2(x)=2$ $\Rightarrow \sec(x)=\pm\sqrt{2}$ *Reciprocals: $\Rightarrow \cos(x)=\pm\frac{1}{\sqrt{2}}$ *Quadrant Check: All four Quadrants

From this point onward the solution is the same in both methods.

*Reference Angle:

$x_R=\pi/4$

*Solve:

$x=\boxed{\pi/4}, \quad x=\boxed{3\pi/4}, \quad x=\boxed{5\pi/4}, \quad x=\boxed{7\pi/4}$

b) $4\sin^2(x)-9=0 \quad [0,2\pi)$

Difference of Squares method	$\pm$ Square Root method
$\Rightarrow 4\sin^2(x)-9=0$ $\Rightarrow 2\sin(x)+3=0 \quad or \quad 2\sin(x)-3=0$ $\Rightarrow \sin(x)=-\frac{3}{2} \quad or \quad \sin(x)=\frac{3}{2}$	$\Rightarrow 4\sin^2(x)-9=0$ $\Rightarrow \sin^2(x)=\frac{9}{4}$ $\Rightarrow \sin(x)=\pm\sqrt{\frac{9}{4}}$ $\Rightarrow \sin(x)=\pm\frac{3}{2}$

From this point onward the solution is the same in both methods.

*Reference angle:

Since ${}^{3}\!/_{2}$ is greater than one and outside the range of the sine function, there is no Reference angle

Hence there is $\boxed{\textit{No Solution}}$

36. $\tan^2(x)-1=0 \quad [0,2\pi)$

37. $\cos^2(x)-1=0 \quad [0,2\pi)$

38. $4\sin^2(x)-3=0 \quad [0,2\pi)$

39. $5\cos^2(x)-2=0 \quad [0,2\pi)$