Partial Fractions
Solutions

(I)

(1) $\dfrac{(3x+2)}{x^2-2x-24} = \dfrac{(3x+2)}{(x-6)(x+4)}$

$$\left[\frac{(x-6)(x+4)}{1}\right]\left[\frac{(3x+2)}{(x-6)(x+4)} = \frac{A}{(x-6)} + \frac{B}{(x+4)}\right]$$

$\Rightarrow (3x+2) = A(x+4) + B(x-6)$

Let $x=-4$

$\Rightarrow 3(-4)+2 = 0 + B(-4-6)$

$\Rightarrow -10 = B(-10)$

$\Rightarrow B = 1$

Let $x=6$

$\Rightarrow 3(6)+2 = A(6+4) + 0$

$\Rightarrow 20 = A(10)$

$\Rightarrow A = 2$

so we have:

$$\boxed{\frac{2}{(x-6)} + \frac{1}{(x+4)}}$$

(2) $\dfrac{(3x-7)}{(x^2-2x-3)} = \dfrac{(3x-7)}{(x-3)(x+1)}$

so

$$\frac{(x-3)(x+1)}{1}\left[\frac{(3x-7)}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}\right]$$

$\Rightarrow 3x-7 = A(x+1) + B(x-3)$

Let $x=3$

$\Rightarrow 9-7 = A(4)+0$

$\Rightarrow 2 = A(4)$

$\Rightarrow A = 1/2$

Let $x=-1$

$\Rightarrow 3(-1)-7 = A(0) + B(-4)$

$\Rightarrow -10 = B(-4)$

$\Rightarrow B = 5/2$

so

$$\boxed{\frac{1/2}{(x-3)} + \frac{5/2}{(x+1)}}$$

③ $\dfrac{-7x+43}{3x^2+19x-14} = \dfrac{(-7x+43)}{(3x-2)(x+7)} = \dfrac{A}{(3x-2)} + \dfrac{B}{(x+7)}$

$\Rightarrow \dfrac{(3x-2)(x+7)}{1}\left[\dfrac{(-7x+43)}{(3x-2)(x+7)} = A(x+7)+B(3x-2)\right]$

$\Rightarrow \quad -7x+43 = A(x+7)+B(3x-2)$ → easier than $x = 2/3$

Let $x=-7$

$\Rightarrow 49+43 = 0+B(-23)$

$\Rightarrow 92 = B(-23)$

$\Rightarrow B = -4$ ✓

Let $x=0$

$\Rightarrow 0+43 = A(7)+(-4)(-2)$

$\Rightarrow 43 = 7A+8$

$\Rightarrow 35 = 7A$

$\Rightarrow A = 5$ ✓

so we have $\boxed{\dfrac{5}{(3x-2)} + \dfrac{-4}{(x+7)}}$

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④ $\dfrac{2x^2+4}{x^3+5x^2+4x} = \dfrac{(2x^2+4)}{(x)(x+1)(x+4)} = \dfrac{A}{x} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+4)}$

$\Rightarrow \dfrac{(x)(x+1)(x+4)}{1}\left[\dfrac{2x^2+4}{(x)(x+1)(x+4)} = \dfrac{A}{x} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+4)}\right]$

$\Rightarrow 2x^2+4 = A(x+1)(x+4)+B(x)(x+4)+C(x)(x+1)$

Let $x=0$

$\Rightarrow 4 = A(1)(4)+0+0$

$\Rightarrow A = 1$ ✓

Let $x=-1$

$\Rightarrow 2(-1)^2+4 = 0+B(-1)(3)+0$

$\Rightarrow 6 = B(-3)$

$\Rightarrow B = -2$ ✓

Let $x=-4$

$\Rightarrow 2(-4)^2+4 = 0+0+C(-4)(-3)$

$\Rightarrow 36 = C(12)$

$\Rightarrow C = 3$ ✓

so we have:

$\boxed{\dfrac{1}{x} + \dfrac{-2}{(x+1)} + \dfrac{3}{(x+4)}}$

(5) $\frac{(3x+9)}{(x-4)(x-3)^2} = \frac{A}{(x-4)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2}$

$\frac{(x-4)(x-3)^2}{1}\left[\frac{(3x+9)}{(x-4)(x-3)^2} = \frac{A}{(x-4)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2}\right]$

$\Rightarrow \quad 3x+9 = A(x-3)^2 + B(x-4)(x-3) + C(x-4)$

Let $x=4$

$\Rightarrow 12+9 = A(-1)^2 + 0 + 0$

$\Rightarrow 21 = A$ ←

Let $x=3$

$\Rightarrow 9+9 = 0+0+C(-1)$

$\Rightarrow 18 = C(-1)$

$\Rightarrow -18 = C$ ←

Let $x=0$

$\Rightarrow 0+9 = (21)(-3)^2 + B(-4)(-3) + (-18)(4)$

$\Rightarrow \quad 9 = 189 + B(12) - 72$

$\Rightarrow \quad 9 = 117 + B(12)$

$\Rightarrow \quad -108 = B(12) \Rightarrow B = -9$

So:

$$\boxed{\frac{21}{(x-4)} + \frac{-9}{(x-3)} + \frac{-18}{(x-3)^2}}$$

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(6) $\frac{2-9x}{(x-3)(2x-1)^2} = \frac{A}{(x-3)} + \frac{B}{(2x-1)} + \frac{C}{(2x-1)^2}$

$\Rightarrow \frac{(x-3)(2x-1)^2}{1}\left[\frac{2-9x}{(x-3)(2x-1)^2} = \frac{A}{(x-3)} + \frac{B}{(2x-1)} + \frac{C}{(2x-1)^2}\right]$

$\Rightarrow \quad 2-9x = A(2x-1)^2 + B(x-3)(2x-1) + C(x-3)$

Let $x=3$

$\Rightarrow 2-27 = A(5)^2 + 0 + 0$

$\Rightarrow -25 = 25A$

$\Rightarrow A = -1$ ←

Let $x = \frac{1}{2}$

$\Rightarrow 2 - \frac{9}{2} = 0 + 0 + C\left(\frac{1}{2} - 3\right)$

$\Rightarrow -\frac{5}{2} = C\left(-\frac{5}{2}\right)$

$\Rightarrow C = 1$

Let $x=0$

$\Rightarrow 2 = (-1)(-1)^2 + B(-3)(-1) + (1)(-3)$

$\Rightarrow 2 = -1 + 3B - 3$

$\Rightarrow 2 = 3B \Rightarrow B = -2/3$

So:

$$\boxed{\frac{-1}{x-3} + \frac{-2/3}{2x-1} + \frac{1}{(2x-1)^2}}$$

(7). $x^2(x+4)$ / 1 $\left[\dfrac{5x^2+3x-20}{x^2(x+4)} = \dfrac{A}{(x+4)} + \dfrac{B}{x} + \dfrac{C}{x^2}\right]$

$\Rightarrow 5x^2+3x-20 = Ax^2 + B(x)(x+4) + C(x+4)$

Let $x=0$
$\Rightarrow -20 = 0+0+C(4)$
$\Rightarrow -5 = C$ ✓

Let $x=-4$
$\Rightarrow 5(-4)^2+3(-4)-20 = A(-4)^2+0+0$
$\Rightarrow 80-12-20 = A(16)$
$\Rightarrow 48 = A(16)$
$\Rightarrow 3 = A$ ✓

Let $x=1$
$\Rightarrow 5+3-20 = (3)(1)^2+B(1)(5)+(-5)(5)$
$\Rightarrow -12 = 3+5B-25$
$\Rightarrow 10 = 5B$
$\Rightarrow B = 2$

$\therefore$ $\boxed{\dfrac{3}{(x+4)} + \dfrac{2}{x} + \dfrac{-5}{x^2}}$

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(8). $\dfrac{(3x+1)}{(x-1)(x^2+1)}$

$(x-1)(x^2+1)$ / 1 $\left[\dfrac{(3x+1)}{(x-1)(x^2+1)} = \dfrac{A}{(x-1)} + \dfrac{(Bx+C)}{x^2+1}\right]$

$\Rightarrow 3x+1 = A(x^2+1) + (Bx+C)(x-1)$
$= Ax^2+A+Bx^2-Bx+Cx-C$
$= (A+B)x^2 + (-B+C)x + (A-C)$

Equate coefficients

$\Rightarrow$ EQ I: $A+B=0$
EQ II: $-B+C=-3$
EQ III: $A-C=1$

add EQ II + EQ III
$\Rightarrow A-B=4$ EQ IIII

$\Rightarrow$ EQ I: $A+B=0$
EQ IIII $A-B=4$
ADD $2A=4$
$\Rightarrow A=2$ ✓

IN EQ I: $2+B=0$
$\Rightarrow B=-2$ ✓

IN EQ III $2-C=1$
$\Rightarrow C=1$ ✓

$\therefore$ $\boxed{\dfrac{2}{(x-1)} + \dfrac{(-2x+1)}{(x^2+1)}}$