(7)

$$\frac{x^2(x+4)}{1}\left[\frac{5x^2+3x-20}{x^2(x+4)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x+4)}\right]$$

$$\Rightarrow 5x^2+3x+20 = A(x)(x+4)+B(x+4)+C(x^2)$$

Let $x=0$

$\Rightarrow 20 = 0+B(0+4)+0$

$\Rightarrow B = 5$ ←

Let $x=-4$

$5(-4)^2+3(-4)+20 = 0+0+C(-4)^2$

$80-12+20 = C(16)$

$88 = C(16)$

$\frac{88}{16} = C = \frac{11}{2}$

Let $x=1$

$\Rightarrow 5+3+20 = A(1)(5)+(5)(5)+(11/2)(1)$

$\Rightarrow 28 = A(5)+25+11/2$

$\Rightarrow 3 = A(5)+11/2 \Rightarrow -\frac{5}{2} = A(5)$

$\Rightarrow -1/2 = A$

So we have:

$$\boxed{\frac{-\frac{1}{2}}{x}+\frac{5}{x^2}+\frac{11/2}{(x+4)}}$$

(8.)

$$\frac{(x-1)(x^2+1)}{1}\left[\frac{(3x+1)}{(x-1)(x^2+1)} = \frac{A}{(x-1)}+\frac{(Bx+C)}{(x^2+1)}\right]$$

$$\Rightarrow 3x+1 = A(x^2+1)+(Bx+C)(x-1)$$

$$= Ax^2+A+Bx^2-Bx+Cx-C$$

$$= (A+B)x^2+(-B+C)x+(A-C)$$

EQUATE COEFFICIENTS:

EQ I: $A+B = 0$

EQ II: $-B+C = 3$

EQ III: $A-C = 1$

ADD EQ I + EQ II: $A+C = 3$ EQ IV

ADD EQ III: $A-C = 1$

ADD EQ IV + EQ III: $2A = 4$

$\Rightarrow A = 2$ ←

in EQ I: $2+B = 0 \Rightarrow B = -2$ ←

in EQ III: $2-C = 1 \Rightarrow C = 1$ ←

So

$$\boxed{\frac{2}{(x-1)}+\frac{(-2x+1)}{(x^2+1)}}$$

(11.)

$$\frac{(x^2+4)^2}{1}\left[\frac{x^2+2x+3}{(x^2+4)^2}=\frac{(Ax+B)}{(x^2+4)}+\frac{(Cx+D)}{(x^2+4)^2}\right]$$

$$\Rightarrow x^2+2x+3=(Ax+B)(x^2+4)+(Cx+D)$$

$$=Ax^3+4Ax+Bx^2+4B+Cx+D$$

$$=Ax^3+Bx^2+(4A+C)x+(4B+D)$$

EQUATE COEFFICIENTS

$\Rightarrow$ EQ:

$$A = 0 \Rightarrow A=0$$

$$B = 1 \Rightarrow B=1$$

$$4A+C = 2 \Rightarrow 0+C=2 \Rightarrow C=2$$

$$4B+D = 3 \Rightarrow 4+D=3 \Rightarrow D=-1$$

SO:

$$\boxed{\frac{1}{x^2+4}+\frac{(2x-1)}{(x^2+4)^2}}$$

(12.)

$$\frac{x(x^2+1)^2}{1}\left[\frac{-10x^4+x^3-19x^2+x-10}{x(x^2+1)^2}=\frac{A}{x}+\frac{(Bx+C)}{(x^2+1)}+\frac{(Dx+E)}{(x^2+1)^2}\right]$$

$$\Rightarrow -10x^4+x^3-19x^2+x-10=A(x^2+1)^2+\overbrace{(Bx+C)}^{(Bx^2+Cx)}(x)(x^2+1)+(Dx+E)(x)$$

$$=A(x^4+2x^2+1)+Bx^4+Bx^2+Cx^3+Cx+Dx^2+Ex$$

$$Ax^4+2Ax^2+A$$

$$=(A+B)x^4+Cx^3+(2A+B+D)x^2+(C+E)x+(A)$$

EQUATE COEFFICIENTS:

$$\Rightarrow A+B = -10$$

$$C = 1$$

$$2A+B+D = -19$$

$$C+E = 1$$

$$A = -10$$

SO: $A=-10$

$\Rightarrow B=0$

$C=1$

$\Rightarrow 2(-10)+0+D=-19$

$-20+D=-19$

$D=1$

$C+E=1 \Rightarrow E=0$

SO

$$\boxed{\frac{-10}{x}+\frac{1}{(x^2+1)}+\frac{x}{(x^2+1)^2}}$$