Solutions

(15). $\int 14(x-5)^6\,dx$

u-sub

Let $u = x-5$

$du = dx$

$= \int 14u^6\,du$

$= \frac{14u^7}{7} + C$

$= \boxed{2(x-5)^7 + C}$

(16). $\int \frac{5}{(t+6)^2}\,dt =$

$= \int 5(t+6)^{-2}\,dt$

u-sub.

Let $u = t+6$

$du = dt$

$= \int 5u^{-2}\,du$

$= \frac{5u^{-1}}{-1} + C$

$= \boxed{-5(t+6)^{-1} + C}$

(17). $\int \frac{7}{(z-10)^7}\,dz =$

$= \int 7(z-10)^{-7}\,dz$

u-sub.

Let $u = z-10$

$du = dz$

$= \int 7(u)^{-7}\,du$

$= \frac{7u^{-6}}{-6} + C$

$= \boxed{-\tfrac{7}{6}(z-10)^{-6} + C}$

(18) $\int t^3\sqrt{t^4+1}\,dt =$

$= \int t^3(t^4+1)^{1/2}\,dt$

u-sub:

Let $u = t^4+1$

$\Rightarrow du = 4t^3\,dt$

$\Rightarrow \frac{1}{4}du = t^3\,dt$

$= \frac{1}{4}\int u^{1/2}\,du$

$= \frac{1}{4}\left[\frac{u^{3/2}}{3/2}\right] + C$

$= \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{1}{6}(t^4+1)^{3/2} + C}$

(19). $\int \left[z^2 + \frac{1}{(1-z)^6}\right] dz =$

$= \int z^2\,dz + \int \frac{1}{(1-z)^6}\,dz$

so: $\int z^2\,dz = \frac{1}{3}z^3 + C_1$

$\int \frac{1}{(1-z)^6}\,dz = \int (1-z)^{-6}\,dz$

u-sub:

Let $u = 1-z$

$\Rightarrow du = -dz$

$\Rightarrow -du = dz$

$= -\int u^{-6}\,du$

$= -\frac{u^{-5}}{-5} + C_2$

$= \frac{1}{5}(1-z)^{-5} + C_2$

so we have:

$= \boxed{\frac{1}{3}z^3 + \frac{1}{5}(1-z)^{-5} + C}$

(21) $\int \frac{t^2-3}{-t^3+9t+1}\,dt =$

$= \int (t^2-3)(-t^3+9t+1)^{-1}\,dt$

u-sub:

Let $u = -t^3+9t+1$

$\Rightarrow du = (-3t^2+9)\,dt$

$= -3(t^2-3)\,dt$

$\Rightarrow -3\,du = (t^2-3)\,dt$

$= -3\int u^{-1}\,du$

$= -3\ln|u| + C$

$= \boxed{-3\ln|-t^3+9t+1| + C}$

(22) $\int \frac{x+1}{\sqrt{3x^2+6x}}\,dx =$

$= \int (x+1)(3x^2+6x)^{-1/2}\,dx$

u-sub

Let $u = 3x^2+6x$

$\Rightarrow du = (6x+6)\,dx$

$\Rightarrow \frac{1}{6}du = (x+1)\,dx$

$= \frac{1}{6}\int u^{-1/2}\,du$

$= \frac{1}{6}\left[\frac{u^{1/2}}{-1/2}\right] + C$

$= \frac{1}{6}\cdot\left(-\frac{1}{2}\right)(u^{1/2}) + C$

$= \boxed{-\frac{1}{12}(3x^2+6x)^{1/2} + C}$

(23) $\int \frac{x^2}{(x-1)}\,dx$

The degree of the numerator is NOT SMALLER than the degree of the denominator

SO: Do the Long division:

$$\begin{array}{r|l} & x + 1 + \frac{1}{x-1} \\ x-1 & x^2 + 0x + 0 \\ & (-)\,x^2 \,(+)\, x \\ \hline & \quad x + 0 \\ & (-)\,x \,(+)\, 1 \\ \hline & \quad\quad 1 \end{array}$$

SO $\int \frac{x^2}{(x-1)}\,dx = \int \left(x+1+\frac{1}{x-1}\right)dx$

$= \boxed{\frac{x^2}{2} + x + \ln|x-1| + C}$

Note: Important property you may (and will need to) freely use:

$\int \frac{1}{(ax+b)}\,dx = \boxed{\frac{1}{a}\ln|ax+b| + C}$

This must be linear

Notice the $\frac{1}{a}$ rule

(24) $\int \frac{3x}{x+4}\,dx$

Polynomial division:

$$\begin{array}{r} 3 - \frac{12}{x+4} \\ (x+4)\overline{)\,3x+0} \\ (-)3x \mp 12 \\ \hline -12 \end{array}$$

so $\int \frac{3x}{x+4}dx = \int\left[3 - \frac{12}{x+4}\right]dx$

$= \boxed{3x + 12\ln|x+4| + C}$

(25) $\int \frac{x+2}{x+1}\,dx$

Polynomial division:

$$\begin{array}{r} 1 + \frac{1}{x+1} \\ x+1\overline{)\,x+2} \\ (-)x \mp 1 \\ \hline 1 \end{array}$$

so $\int \frac{x+2}{x+1}dx =$

$= \int\left[1 + \frac{1}{x+1}\right]dx$

$= \boxed{x + \ln|x+1| + C}$

(26). $\int\left(\frac{1}{9z-5} - \frac{1}{9z+5}\right)dz$

$= \boxed{\frac{1}{9}\ln|9z-5| - \frac{1}{9}\ln|9z+5| + C}$

(27). $\int \underbrace{(5+4x^2)^2}_{\text{multiply this}} dx = \int(25 + 40x^2 + 16x^4)dx$

$= \boxed{25x + \frac{40x^3}{3} + \frac{16x^5}{5} + C}$

(28). $\int x\underbrace{\left(3+\frac{2}{x}\right)^2}_{\text{multiply}} dx =$

scratch work

$\left(3+\frac{2}{x}\right)\left(3+\frac{2}{x}\right)$

$= 9 + \frac{6}{x} + \frac{6}{x} + \frac{4}{x^2} = 9 + \frac{12}{x} + \frac{4}{x^2}$

$= \int x\left(9 + \frac{12}{x} + \frac{4}{x^2}\right)dx$

$= \int\left(9x + 12 + \frac{4}{x}\right)dx = \boxed{\frac{9x^2}{2} + 12x + 4\ln|x| + C}$

(29). $\int x\cos(2\pi x^2)\,dx =$

u-sub

Let $u = 2\pi x^2$

$\Rightarrow du = 4\pi x\,dx$

$\Rightarrow \frac{1}{4\pi}du = x\,dx$

$= \frac{1}{4\pi}\int \cos(u)\,du$

$= \frac{1}{4\pi}\sin(u) + C$

$= \boxed{\frac{1}{4\pi}\sin(2\pi x^2) + C}$

(30). $\int \csc(\pi x)\cot(\pi x)\,dx$

this is the $\frac{1}{a}$ rule with $a = \pi$.

$= \boxed{-\frac{1}{\pi}\csc(\pi x) + C}$

(31). $\int \frac{\sin(x)}{\sqrt{\cos(x)}}\,dx =$

$= \int \sin(x)(\cos(x))^{-1/2}\,dx$

u-sub:

Let $u = \cos(x)$

$\Rightarrow du = -\sin(x)\,dx$

$\Rightarrow -du = \sin(x)\,dx$

$= -\int u^{-1/2}\,du$

$= -\frac{u^{3/2}}{\frac{3}{2}} + C$

$= \boxed{-\frac{2}{3}(\cos(x))^{3/2} + C}$

(32). $\int \frac{\csc^2(3t)}{\cot(3t)}\,dt =$

$= \int \frac{\frac{1}{\sin^2(3t)}}{\frac{\cos(3t)}{\sin(3t)}}\,dt$

$= \int \left[\frac{1}{\sin^2(3t)} \cdot \frac{\sin(3t)}{\cos(3t)}\right] dt$

$= \int \frac{1}{\sin(3t)\cdot\cos(3t)}\,dt$

Nope: Didn't work.

Try something else

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