$\Downarrow$ Cont.

Pg 5ive

$$\int \frac{\csc^2(3t)}{\cot(3t)}\,dt =$$

we know

$\sin^2(3t)+\cos^2(3t)=1$

so $1+\cot^2(3t)=\csc^2(3t)$

so we have:

$$\int \frac{(1+\cot^2(3t))}{\cot(3t)}\,dt =$$

$$=\int\left[\frac{1}{\cot(3t)}+\frac{\cot^2(3t)}{\cot(3t)}\right]dt$$

$$=\int\left[\tan(3t)+\cot(3t)\right]dt$$

$$=\boxed{\frac{1}{3}\ln|\sec(3t)|+\frac{1}{3}\ln|\sin(3t)|+C}$$

or $-\frac{1}{3}\ln|\csc(3t)|$

valid option

(33) $\int \frac{2}{(e^{-x}+1)}\,dx \Rightarrow$ do a bunch of algebra.

$$\frac{2}{e^{-x}+1}=\frac{2}{\frac{1}{e^x}+1}=\frac{2}{\frac{1}{e^x}+\frac{e^x}{e^x}}=\frac{2}{\frac{1+e^x}{e^x}}=\frac{2e^x}{1+e^x}$$

so $\int\frac{2}{e^{-x}+1}\,dx=\int\frac{2e^x}{(1+e^x)}\,dx = 2\int\frac{1}{u}\,du = 2\ln|u|+C$

$=\boxed{2\ln|1+e^x|+C}$

u-sub

Let $u=1+e^x$

$\Rightarrow du=e^x dx$

$\Rightarrow 2du=2e^x dx$

(34) $\int \frac{4}{3-e^x}\,dx$ same technique

$$= \int \left(\frac{4}{3-\frac{1}{e^{-x}}}\right) dx = \int \left(\frac{4}{\frac{3e^{-x}}{e^{-x}} - \frac{1}{e^{-x}}}\right) dx = \int \left(\frac{4}{\frac{(3e^{-x}-1)}{e^{-x}}}\right) dx = \int \frac{4e^{-x}}{3e^{-x}-1}\,dx$$

u-sub

Let $u = 3e^{-x}-1$

$\Rightarrow du = -3e^{-x}\,dx$

$\Rightarrow -\frac{1}{3}du = e^{-x}\,dx$

$\Rightarrow -\frac{4}{3}du = 4e^{-x}\,dx$

so we have

$$= -\frac{4}{3}\int \frac{1}{u}\,du = -\frac{4}{3}\ln|u| + C$$

$$= \boxed{-\frac{4}{3}\ln\left|3e^{-x}-1\right| + C}$$

(35) $\int \frac{\ln(x^2)}{x}\,dx = \int \frac{2\ln(x)}{x}\,dx = 2\int u\,du = \frac{2u^2}{2} + C$

$$= \boxed{(\ln(x))^2 + C}$$

u-sub:

Let $u = \ln(x)$

$\Rightarrow du = \frac{1}{x}\,dx$

$\Rightarrow 2du = \frac{2}{x}\,dx$

(36) $\int (\tan(x))\left[\ln(\cos(x))\right] dx =$

u-sub

Let $u = \ln(\cos(x))$

$\Rightarrow du = \frac{1}{\cos(x)} \cdot (-\sin(x))\,dx$

$\Rightarrow -du = \frac{\sin(x)}{\cos(x)}\,dx$

$= \tan(x)\,dx$

$= -\int u\,du$

$= -\frac{u^2}{2} + C$

$$= \boxed{-\frac{1}{2}\left[\ln(\cos(x))\right]^2 + C}$$

(37) $\int \frac{1+\cos(\alpha)}{\sin(\alpha)}\,d\alpha =$ split the numerator

$$= \int \frac{1}{\sin(\alpha)}\,d\alpha + \int \frac{\cos(\alpha)}{\sin(\alpha)}\,d\alpha$$

$$= \int \csc(\alpha)\,d\alpha + \int \cot(\alpha)\,d\alpha$$

$$= \boxed{-\ln|\csc(\alpha)+\cot(\alpha)| - \ln|\csc(x)| + C}$$

(38) $\int \frac{1}{\cos(\theta)-1}\,d\theta$ Do some trigonometry:

we have: $\frac{1}{\cos(\theta)-1} = \frac{1}{(\cos(\theta)-1)} \cdot \frac{(\cos(\theta)+1)}{(\cos(\theta)+1)} = \frac{\cos(\theta)+1}{\cos^2(\theta)-1} = \frac{\cos(\theta)-1}{\sin^2(\theta)}$

$= \frac{\cos(\theta)}{\sin^2(\theta)} - \frac{1}{\sin^2(\theta)} = \cot(\theta)\cdot\csc(\theta) - \csc^2(\theta)$

so $\int \frac{1}{\cos(\theta)-1}\,d\theta = \int [\cot(\theta)\csc(\theta) - \csc^2(\theta)]\,d\theta$

$= -\csc(\theta) - (-\cot(\theta)) + C = \boxed{-\csc(\theta) + \cot(\theta) + C}$

(39). $\int \frac{-1}{\sqrt{1-(4t+1)^2}}\,dt$ $\sin^{-1}(u)$ form.

$a = 1$ $u = 4t+1$

$\Rightarrow du = 4\,dt$

$\Rightarrow \frac{1}{4}du = dt$

$= -\frac{1}{4}\int \frac{1}{\sqrt{a^2-u^2}}\,du = -\frac{1}{4}\sin^{-1}\left(\frac{u}{a}\right) + C$

$= \boxed{-\frac{1}{4}\sin^{-1}\left(\frac{4t+1}{1}\right) + C}$

(40). $\int \frac{1}{25+4x^2}\,dx = \int \frac{1}{25+(2x)^2}\,dx$ $\tan^{-1}(u)$ form.

$a = 5$ $u = 2x$

$\Rightarrow du = 2\,dx$

$\Rightarrow \frac{1}{2}du = dx$

$= \frac{1}{2}\int \frac{1}{a^2+u^2}\,du$

$= \frac{1}{2}\cdot\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

$= \frac{1}{2}\cdot\frac{1}{5}\tan^{-1}\left(\frac{2x}{5}\right) + C$

$= \boxed{\frac{1}{10}\tan^{-1}\left(\frac{2x}{5}\right) + C}$

(41). $\int \frac{\tan\left(\frac{2}{t}\right)}{t^2}\,dt = \int \tan(2t^{-1})(t^{-2})\,dt$

u-sub
Let $u = 2t^{-1}$
$\Rightarrow du = -2t^{-2}\,dt$
$\Rightarrow -\frac{1}{2}du = t^{-2}\,dt$ ✓

$= -\frac{1}{2}\int \tan(u)\,du$

$= -\frac{1}{2}\left[\ln|\sec(u)|\right] + C$

$= \boxed{-\frac{1}{2}\ln|\sec(2t^{-1})| + C}$

(42). $\int \frac{e^{(-1/t^3)}}{t^4}\,dt = \int e^{(-t^{-3})}\cdot t^{-4}\,dt$

u-sub
Let $u = -t^{-3}$
$\Rightarrow du = 3t^{-4}\,dt$
$\Rightarrow \frac{1}{3}du = t^{-4}\,dt$ ✓

$= \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{(-t^{-3})} + C}$

(43) $\int \frac{6}{z\sqrt{9z^2-25}}\,dz$ $\quad \sec^{-1}(u)$ form:

$a = 5 \quad u = 3z \Longrightarrow \frac{1}{3}u = z$ ✓
$\Rightarrow du = 3dz$
$\Rightarrow \frac{1}{3}du = dz$ ✓

so

$\int \frac{6}{z\sqrt{9z^2-25}}\,dz = \int \frac{6}{(\frac{1}{3}u)\sqrt{u^2-a^2}}\left(\frac{1}{3}dz\right)$

$= 6\int \frac{1}{u\sqrt{u^2-a^2}}\,du$

$= 6\left(\frac{1}{a}\right)\sec^{-1}\left(\frac{|u|}{a}\right) + C$

$= \boxed{\frac{6}{5}\sec^{-1}\left(\frac{|3z|}{5}\right) + C}$