(44). $\int \frac{1}{(x-1)\sqrt{(4x^2-8x+3)}}\,dx =$

complete the square

$4x^2-8x+3$

$(4x^2-8x+\quad)+3-(\quad)$

$=4(x^2-2x+\quad)+3-(\quad)$

$-1 \longrightarrow 1$

$=4(x^2-2x+1)+3-4$

$=4(x-1)^2-1$

$= \int \frac{1}{(x-1)\sqrt{4(x-1)^2-1}}\,dx$

$\sec^{-1}(u)$ form:

$a=1 \qquad u=2(x-1)$

$\frac{1}{2}u = x-1$

$\frac{1}{2}du = dx$

$= \int \frac{1}{(\frac{1}{2}u)\sqrt{u^2-a^2}}\left(\frac{1}{2}\right)du$

$= \int \frac{1}{u\sqrt{u^2-a^2}}\,du$

$= \frac{1}{a}\sec^{-1}\left(\frac{|u|}{a}\right)+C$

$= \frac{1}{1}\sec^{-1}\left(\frac{|2(x-1)|}{1}\right)+C$

$= \boxed{\sec^{-1}(|2(x-1)|)+C}$

(45). $\int \frac{4}{4x^2+4x+65}\,dx$

complete the square:

$4x^2+4x+65 =$

$=(4x^2+4x+\quad)+65$

$=4(x^2+x+\quad)+65$

$1/2 \longrightarrow 1/4$

$=4(x^2+x+\frac{1}{4})+65-1$

$=4(x+\frac{1}{2})^2+64$

so we have

$\int \frac{4}{4(x+\frac{1}{2})^2+64}\,dx =$

$= \int \frac{4}{(2(x+\frac{1}{2}))^2+8^2}\,dx$

$\tan^{-1}(u)$ form

$a=8 \qquad u=2(x+\frac{1}{2})$

$\Rightarrow \quad = 2x+1$

$\Rightarrow du = 2dx$

$\Rightarrow \frac{1}{2}du = dx$

so we have:

$= \int \frac{4}{u^2+a^2}\left(\frac{1}{2}\right)du$

$= 2\int \frac{1}{u^2+a^2}\,du$

$= 2\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{u}{a}\right)+C$

$= 2\left(\frac{1}{8}\right)\tan^{-1}\left(\frac{2x+1}{8}\right)+C$

$= \boxed{\frac{1}{4}\tan^{-1}\left(\frac{2x+1}{8}\right)+C}$

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46. $\int \frac{1}{x^2-4x+9}\,dx =$

complete the square

$x^2 - 4x + (\quad) + 9$

$-2 \to 4$

$(x^2 - 4x + 4) + 9 - 4$

$(x-2)^2 + 5$

$= \int \frac{1}{(x-2)^2 + (\sqrt{5})^2}\,dx$

$\tan^{-1}(u)$ form

$a = \sqrt{5}$ $\quad u = x - 2$

$du = dx$

$= \int \frac{1}{u^2 + a^2}\,du$

$= \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C = \boxed{\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x-2}{\sqrt{5}}\right) + C}$

9. $\int (1+6x)^4 (6)\,dx$

u-sub

Let $u = 1 + 6x$

$\Rightarrow du = 6\,dx$

$\Rightarrow \frac{1}{6}\,du = dx$

$= \frac{1}{6}\int u^4\,du$

$= \frac{1}{6}\frac{u^5}{5} + C$

$= \boxed{\frac{1}{30}(1+6x)^5 + C}$

10. $\int (x^2 - 9)^3 (2x)\,dx$

u-sub

Let $u = x^2 - 9$

$\Rightarrow du = 2x\,dx$

$= \int u^3\,du$

$= \frac{1}{4}u^4 + C$

$= \boxed{\frac{1}{4}(x^2-9)^4 + C}$

11. $\int \sqrt{25 - x^2}\,(-2x)\,dx =$

$= \int (25 - x^2)^{1/2}(-2x)\,dx$

u-sub

Let $u = 25 - x^2$

$\Rightarrow du = -2x\,dx$

$= \int u^{1/2}\,du$

$= \frac{u^{3/2}}{3/2} + C = \boxed{\frac{2}{3}(25 - x^2)^{3/2} + C}$

12. $\int \sqrt[3]{3 - 4x^2}\,(-8x)\,dx =$

$= \int (3 - 4x^2)^{1/3}(-8x)\,dx$

u-sub

Let $u = 3 - 4x^2$

$\Rightarrow du = -8x\,dx$

$= \int u^{1/3}\,du$

$= \frac{u^{4/3}}{4/3} + C$

$= \boxed{\frac{3}{4}(3 - 4x^2)^{4/3} + C}$

(13). $\int x^3(x^4+3)^2\,dx$

u-sub:
Let $u = x^4+3$
$\Rightarrow du = 4x^3\,dx$
$\Rightarrow \frac{1}{4}du = x^3\,dx$

$= \frac{1}{4}\int u^2\,du$

$= \frac{1}{4}\,\frac{u^3}{3} + C$

$= \boxed{\frac{1}{12}(x^4+3)^3 + C}$

(14). $\int x^2(6-x^3)^5\,dx$

u-sub:
Let $u = 6-x^3$
$\Rightarrow du = -3x^2\,dx$
$\Rightarrow -\frac{1}{3}du = x^2\,dx$

$= -\frac{1}{3}\int u^5\,du$

$= -\frac{1}{3}\cdot\frac{u^6}{6} + C$

$= \boxed{-\frac{1}{18}(6-x^3)^6 + C}$

(15). $\int x^2(2x^3-1)^4\,dx$

u-sub
Let $u = 2x^3-1$
$\Rightarrow du = 6x^2\,dx$
$\Rightarrow \frac{1}{6}du = x^2\,dx$

$= \frac{1}{6}\int u^4\,du$

$= \frac{1}{6}\left(\frac{u^5}{5}\right) + C$

$= \boxed{\frac{1}{30}(2x^3-1)^5 + C}$

(16). $\int x(5x^2+4)^3\,dx$

u-sub
Let $u = 5x^2+4$
$\Rightarrow du = 10x\,dx$
$\Rightarrow \frac{1}{10}du = x\,dx$

$= \frac{1}{10}\int u^3\,du$

$= \frac{1}{10}\cdot\frac{u^4}{4} + C$

$= \boxed{\frac{1}{40}(5x^2+4)^4 + C}$

(17) $\int t\sqrt{t^2+2}\,dt =$

$= \int t\,(t^2+2)^{1/2}\,dt$

u-sub:
Let $u = t^2+2$
$\Rightarrow du = 2t\,dt$
$\Rightarrow \frac{1}{2}du = t\,dt$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\left(\frac{u^{3/2}}{3/2}\right) + C$

$= \boxed{\frac{1}{3}(t^2+2)^{3/2} + C}$

(18). $\int t^3(2t^4+3)^{1/2}\,dt$

u-sub
Let $u = 2t^4+3$
$\Rightarrow du = 8t^3\,dt$
$\Rightarrow \frac{1}{8}du = t^3\,dt$

$= \frac{1}{8}\int u^{1/2}\,du$

$= \frac{1}{8}\left(\frac{u^{3/2}}{3/2}\right) + C$

$= \boxed{\frac{1}{12}(2t^4+3)^{3/2} + C}$

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(19) $\int 5x\sqrt[3]{1-x^2}\,dx$

$= \int 5x(1-x^2)^{1/3}dx$

u-sub:

Let $u = 1-x^2$

$\Rightarrow du = -2x\,dx$

$\Rightarrow -\frac{1}{2}du = x\,dx$

$\Rightarrow -\frac{5}{2}du = 5x\,dx$

$= -\frac{5}{2}\int u^{1/3}\,du$

$= -\frac{5}{2}\cdot\frac{u^{4/3}}{4/3} + C$

$= \boxed{-\frac{15}{8}(1-x^2)^{4/3} + C}$

(20) $\int 6u^6\sqrt{u^7+8}\,du =$

$= \int 6u^6(u^7+8)^{1/2}\,du$

t-sub:

Let $t = u^7+8$

$\Rightarrow dt = 7u^6du$

$\Rightarrow \frac{1}{7}dt = u^6du$

$\Rightarrow \frac{6}{7}dt = 6u^6du$

$= \frac{6}{7}\int u^{1/2}du$

$= \frac{6}{7}\cdot\frac{u^{3/2}}{3/2} + C$

$= \boxed{\frac{12}{21}(u^7+8)^{3/2} + C}$

(21) $\int \frac{7x}{(1-x^2)^3}dx =$

$= \int 7x(1-x^2)^{-3}dx$

u-sub:

Let $u = 1-x^2$

$\Rightarrow du = -2x\,dx$

$\Rightarrow -\frac{7}{2}du = 7x\,dx$

$= \frac{-7}{2}\int u^{-3}du$

$= -\frac{7}{2}\cdot\frac{u^{-2}}{-2} + C$

$= \boxed{\frac{7}{4}(1-x^2)^{-2} + C}$

(22) $\int \frac{x^3}{(1+x^4)^2}dx = \int x^3(1+x^4)^{-2}dx$

u-sub:

Let $u = 1+x^4$

$\Rightarrow du = 4x^3dx$

$\Rightarrow \frac{1}{4}du = x^3dx$

$= \frac{1}{4}\int u^{-2}du$

$= \frac{1}{4}\cdot\frac{u^{-1}}{-1} + C$

$= \boxed{-\frac{1}{4}(1+x^4)^{-1} + C}$

(23) $\int \frac{x^2}{(1+x^3)^2}dx = \int x^2(1+x^3)^{-2}dx$

Let $u = 1+x^3$

$\Rightarrow du = 3x^2dx$

$\Rightarrow \frac{1}{3}du = x^2dx$

$= \frac{1}{3}\int u^{-2}du$

$= \frac{1}{3}(-1)u^{-1} + C$

$= \boxed{-\frac{1}{3}(1+x^3)^{-1} + C}$

(24) $\int \frac{6x^2}{(4x^3-9)^3}dx = \int 6x^2(4x^3-9)^{-3}dx$

u-sub:

Let $u = 4x^3-9$

$\Rightarrow du = 12x^2dx$

$\Rightarrow \frac{1}{2}du = 6x^2dx$

$= \frac{1}{2}\int u^{-3}du$

$= \left(\frac{1}{2}\right)\frac{u^{-2}}{-2} + C$

$= \boxed{-\frac{1}{4}(4x^3-9)^{-2} + C}$