25) $\int \frac{x}{\sqrt{1-x^2}}\, d = \int x(1-x^2)^{-1/2}dx$

u-sub:

Let $u = 1-x^2$

$\Rightarrow du = -2x\,dx$

$\Rightarrow -\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\int u^{-1/2}du$

$= (-\frac{1}{2})(\frac{u^{3/2}}{3/2}) + C$

$= \boxed{-\frac{1}{3}(1-x^2)^{3/2}+C}$

26). $\int \frac{x^3}{\sqrt{1+x^4}}\, dx$

$= \int x^3(1+x^4)^{-1/2}dx$

u-sub:

Let $u = (1+x^4)$

$\Rightarrow du = 4x^3dx$

$\Rightarrow \frac{1}{4}du = x^3dx$

$= \frac{1}{4}\int u^{-1/2}dx$

$= (\frac{1}{4})(\frac{u^{1/2}}{1/2}) + C$

$= \boxed{\frac{1}{2}(1+x^4)^{1/2}+C}$

61). $\int_{-1}^{1} x(x^2+1)^3 dx$

u-sub

Let $u = x^2+1$

$\Rightarrow du = 2x\,dx$

$\Rightarrow \frac{1}{2}du = x\,dx$

Limits:

upper $u = (1)^2+1 = 2$

Lower $u = (-1)^2+1 = 2$

$= \frac{1}{2}\int_2^2 u^3 du$

$= \boxed{0}$

OR

u-sub

Let $u = x^2+1$

$\Rightarrow du = 2x\,dx$

$\Rightarrow \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int_a^b u^3 du$

$= \frac{1}{2}\left[\frac{u^4}{4}\right]_a^b$

$= \frac{1}{8}\left[(x^2+1)\right]_{-1}^{1}$

$= \frac{1}{8}\left[((1)^2+1) - ((-1)^2+1)\right]$

$= \frac{1}{8}[2-2]$

$= \boxed{0}$.

62. $\int_0^1 x^3(2x^4+1)^2\,dx$

u-sub

Let $u = 2x^4+1$

$\Rightarrow du = 8x^3\,dx$

$\Rightarrow \frac{1}{8}du = x^3\,dx$

$= \frac{1}{8}\int_a^b u^2\,du$

Limits

upper $u = 2(1)^4+1 = 3$

lower $u = 2(0)^4+1 = 1$

$= \frac{1}{8}\int_1^3 u^2\,du$

$= \frac{1}{8}\left[\frac{u^3}{3}\right]_1^3$

$= \frac{1}{8}\left[\left(\frac{3^3}{3}\right) - \left(\frac{1^3}{3}\right)\right]$

$= \frac{1}{8}\left[9 - \frac{1}{3}\right]$

$= \boxed{\frac{26}{24}} = \boxed{\frac{13}{12}}$

OR

$= \frac{1}{8}\left[\frac{u^3}{3}\right]_a^b$

$= \frac{1}{8}\left[\frac{(2x^4+1)^3}{3}\right]_0^1$

$= \frac{1}{8}\left[\frac{(2(1)^4+1)^3}{3} - \frac{(2(0)^4+1)}{3}\right]$

$= \frac{1}{8}\left[9 - \frac{1}{3}\right]$

$= \boxed{\frac{26}{24}} = \boxed{\frac{13}{12}}$

63. $\int_0^2 2x^2\sqrt{x^3+1}\,dx$

$= \int_0^2 2x^2(x^3+1)^{1/2}\,dx$

u-sub

Let $u = x^3+1$

$\Rightarrow du = 3x^2\,dx$

$\Rightarrow \frac{1}{3}du = x^2\,dx$

Limits

upper $u = (2)^3+1 = 9$

Lower $u = (0)^3+1 = 1$

$= \frac{1}{3}\int_1^9 u^{1/2}\,du$

$= \frac{1}{3}\left[\frac{u^{3/2}}{3/2}\right]_1^9$

$= \frac{2}{9}\left[u^{3/2}\right]_1^9$

$= \frac{2}{9}\left[27-1\right] = \boxed{\frac{52}{9}}$

64. $\int_{-1}^0 x\sqrt{1-x^2}\,dx$

$= \int_{-1}^0 x(1-x^2)^{1/2}\,dx$

u-sub:

Let $u = 1-x^2$

$\Rightarrow du = -2x\,dx$

$\Rightarrow -\frac{1}{2}du = x\,dx$

Limits upper $u = 1-(0)^2 = 1$

lower $u = 1-(-1)^2 = 0$

$= -\frac{1}{2}\int_0^1 u^{1/2}\,du$

$= -\frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]_0^1$

$= -\frac{1}{3}\left[u^{3/2}\right]_0^1$

$= -\frac{1}{3}\left[(1)^{3/2} - (0)^{3/2}\right]$

$= -\frac{1}{3}\left[1-0\right]$

$= \boxed{-\frac{1}{3}}$

65. $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$

$= \int_0^4 (2x+1)^{-1/2}\,dx$

u-sub:

Let $u = (2x+1)$

$\Rightarrow du = 2dx$

$\Rightarrow \frac{1}{2}du = dx$

Limits upper: $u = 2(4)+1) = 9$

Lower $u = 2(0)+1 = 1$

$= \frac{1}{2}\int_1^9 \frac{1}{\sqrt{u}}\,du$

$= \frac{1}{2}\int_1^9 u^{-1/2}\,du$

$= \frac{1}{2}\left[\frac{u^{1/2}}{1/2}\right]_1^9$

$= [(9)^{1/2} - (1)^{1/2}]$

$= 3 - 1$

$= \boxed{2}$

OR $\frac{1}{2}$ rule $\int_0^4 (2x+1)^{-1/2}\,dx$

$= \frac{1}{2}\left[\frac{[2x+1]^{1/2}}{1/2}\right]_0^4$

$= \frac{1}{2}\cdot\frac{2}{1}\left[(2x+1)^{1/2}\right]_0^4$

$= [(2(4)+1)^{1/2} - (2(0)+1)^{1/2}]$

$= [9^{1/2} - 1^{1/2}]$

$= 3 - 1$

$= \boxed{2}$

66. $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx = \int_0^2 x(1+2x^2)^{-1/2}\,dx$

u-sub

Let $u = 1+2x^2$

$\Rightarrow du = 4x\,dx$

$\Rightarrow \frac{1}{4}du = x\,dx$

Limits

upper $u = 1+2(2)^2 = 9$

Lower $u = 1+2(0)^2 = 1$

$= \frac{1}{4}\int_1^9 u^{-1/2}\,du$

$= \frac{1}{4}\left[\frac{u^{1/2}}{2}\right]_1^9$

$= \frac{1}{8}[(9^{1/2}) - (1)^{1/2}]$

$= \frac{1}{8}[3-1]$

$= \boxed{\frac{1}{4}}$

67. $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx =$

$= \int_1^9 x^{-1/2}(1+x^{1/2})^{-2}\,dx$

u-sub:

Let $u = 1+x^{1/2}$

$\Rightarrow du = \frac{1}{2}x^{-1/2}\,dx$

$\Rightarrow 2du = x^{-1/2}\,dx$

Limits upper: $u = 1+(9)^{1/2} = 4$

Lower $u = 1+(1)^{1/2} = 2$

$= 2\int_2^4 u^{-2}\,du$

$= \left[\frac{2u^{-1}}{-1}\right]_2^4$

$= [(-2(4)^{-1}) - (-2(2)^{-1})]$

$= [(-\frac{2}{4}) - (-\frac{2}{2})]$

$= -\frac{1}{2} + 1$

$= \boxed{\frac{1}{2}}$

68. $\int_4^5 \frac{x}{\sqrt{2x-6}}\,dx = \int_4^5 x(2x-6)^{-1/2}\,dx$

Double Linear

Let $u = 2x-6 \Rightarrow u+6 = 2x$

$\Rightarrow du = 2dx \Rightarrow \frac{1}{2}(u+6) = x$ ✓

$\Rightarrow \frac{1}{2}du = dx$ ✓

Limits: upper $u = 2(5)-6 = 4$

lower $u = 2(4)-6 = 2$

$= (\frac{1}{2})(\frac{1}{2})\int_2^4 (u+6)u^{-1/2}\,du$

$= \frac{1}{4}\int_2^4 (u^{1/2} + 6u^{-1/2})\,du$

$= \frac{1}{4}\left[\frac{u^{3/2}}{3/2} + \frac{6u^{1/2}}{1/2}\right]_2^4$

$= \frac{1}{4}\left[\left(\frac{2}{3}(4)^{3/2} + 12(4)^{1/2}\right) - \left(\frac{2}{3}(2)^{3/2} + 12(2)^{1/2}\right)\right]$

$= \frac{1}{4}\left[\left(\frac{2}{3}(8)+24\right) - \left(\frac{2}{3}(8)^{1/2} + 12\sqrt{2}\right)\right]$

$= \frac{1}{4}\left[\left(\frac{16}{3}+24\right) - \left(\frac{4}{3}\sqrt{2} + 12\sqrt{2}\right)\right]$

$= \boxed{\frac{1}{4}\left[\frac{88}{3} - \frac{40}{3}\sqrt{2}\right]} = \boxed{\frac{22}{3} - \frac{10}{3}\sqrt{2}}$

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69. $2\pi\int_0^1 (x+1)\sqrt{1-x}\,dx$

Double Linear

$= 2\pi\int_0^1 (x+1)(1-x)^{1/2}\,dx$

Let $u = 1-x \Rightarrow u-1 = -x$

$\Rightarrow du = -dx \Rightarrow 1-u = x$

$\Rightarrow -du = dx$ ✓ $\Rightarrow 2-u = x+1$ ✓

Limits

upper $u = 1-1 = 0$

lower $u = 1-0 = 1$

$= -2\pi\int_1^0 (2-u)u^{1/2}\,du$

$= 2\pi\int_0^1 (2u^{1/2} - u^{3/2})\,du$

$= 2\pi\left[\frac{2u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2}\right]_0^1$

$= 2\pi\left[\left(\frac{4}{3}(1)^{3/2} - \frac{2}{5}(1)^{5/2}\right) - [0-0]\right]$

$= 2\pi\left[\frac{4}{3} - \frac{2}{5}\right] = 2\pi\left[\frac{20-6}{15}\right] = \boxed{\frac{28\pi}{15}}$

70. $2\pi\int_{-1}^0 x^2\sqrt{x+1}\,dx = 2\pi\int_{-1}^0 x^2(x+1)^{1/2}\,dx$

Let $u = x+1 \Rightarrow u-1 = x$

$du = dx$ ✓ $\Rightarrow (u^2-2u+1) = x^2$ ✓

Limits

upper: $u = 0+1 = 1$

lower $u = -1+1 = 0$

$= 2\pi\int_0^1 (u^2-2u+1)u^{1/2}\,dx$

$= 2\pi\int_0^1 \left[u^{5/2} - 2u^{3/2} + u^{1/2}\right]dx$

$= 2\pi\left[\frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_0^1$

$= 2\pi\left[\left(\frac{2}{7}(1)^{7/2} - \frac{4}{5}(1)^{5/2} + \frac{2}{3}(1)^{3/2}\right) - (0-0+0)\right]$

$= 2\pi\left[\frac{2}{7} - \frac{4}{5} + \frac{2}{3}\right]$

$= 2\pi\left[\frac{30-84+70}{105}\right]$

$= 2\pi\left[\frac{16}{105}\right] = \boxed{\frac{32\pi}{105}}$