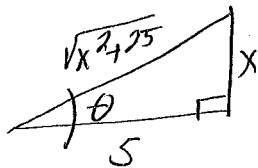


(#6) $\int \frac{x^2}{\sqrt{x^2+25}} dx$

Trig sub:



choose:

$$\tan(\theta) = \frac{x}{5}$$

$$\Rightarrow 5 \tan(\theta) = x$$

$$\Rightarrow 5 \sec^2(\theta) d\theta = dx$$

choose:

$$\sec(\theta) = \frac{\sqrt{x^2+25}}{5}$$

$$\Rightarrow 5 \sec(\theta) = \sqrt{x^2+25}$$

so we have:

$$\int \frac{x^2}{\sqrt{x^2+25}} dx = \int \frac{(5 \tan(\theta))^2 \cdot 5 \sec(\theta)}{5 \sec(\theta)} d\theta$$

$$= \int 25 \tan^2(\theta) \cdot \sec(\theta) d\theta$$

$$= 25 \int [\sec^2(\theta) - 1] \cdot \sec(\theta) d\theta$$

$$= 25 \int [\sec^3(\theta) - \sec(\theta)] d\theta$$

$$= 25 \left[\left[\frac{1}{2} (\sec(\theta) \tan(\theta) + \ln |\sec(\theta) + \tan(\theta)|) \right] - \left[\ln |\sec(\theta) + \tan(\theta)| \right] \right] + C$$

$$= 25 \left[\left[\frac{1}{2} (\sec(\theta) \tan(\theta) + \ln |\sec(\theta) + \tan(\theta)|) \right] - \left[\ln |\sec(\theta) + \tan(\theta)| \right] \right] + C$$

#8

$$\int \frac{x^2 - x + 2}{(x+1)(x-1)^2} dx =$$

partial frac

$$\frac{(x+1)(x-1)^2}{1} \left[\frac{x^2 - x + 2}{(x+1)(x-1)^2} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \right]$$

$$\Rightarrow x^2 - x + 2 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$$\text{Let } x = 1$$

$$\Rightarrow (1)^2 - (1) + 2 = 0 + 0 + C((1)+1)$$

$$\Rightarrow$$

$$2 = C(2)$$

$$\Rightarrow$$

$$C = 1$$

this was my error

$$\text{Let } x = -1$$

$$\Rightarrow (-1)^2 - (-1) + 2 = A((-1)-1)^2 + 0 + 0$$

$$4 = A(4)$$

$$\Rightarrow$$

$$A = 1$$

$$\text{Let } x = 0$$

$$\Rightarrow (0)^2 - 0 + 2 = (1)(0-1)^2 + B(0+1)(0-1) + (1)(0+1)$$

$$\Rightarrow 2 = 1 + B(-1) + 1$$

$$\Rightarrow 0 = -B$$

$$\Rightarrow B = 0$$

so we have:

$$\int \left[\frac{1}{x+1} + \frac{1}{(x-1)^2} \right] dx$$

so this gives:

$$\boxed{\ln|x+1| - \frac{1}{(x-1)} + C}$$

now.

$$\int \frac{1}{(x-1)^2} dx = \int (x-1)^{-2} dx$$

$$\text{Let } u = x-1$$

$$\frac{du}{dx} = 1$$

$$\int u^{-2} du = \frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{(x-1)} + C$$

#9. $\int \frac{7x^2+12}{x^3+4x} dx = \int \frac{7x^2+12}{x(x^2+4)} dx$
par frac

$$\frac{x(x^2+4)}{1} \left[\frac{7x^2+12}{x(x^2+4)} = \frac{A}{x} + \frac{(Bx+C)}{(x^2+4)} \right]$$

$$\Rightarrow 7x^2+12 = A(x^2+4) + (Bx+C)(x)$$

$$= Ax^2 + 4A + Bx^2 + Cx$$

$$= (A+B)x^2 + Cx + 4A$$

EQUATE COEFFICIENTS:

$$A+B = 7 \Rightarrow (3)+B=7 \Rightarrow B=4$$

$$C=0 \Rightarrow C=0$$

$$4A = 12 \Rightarrow A=3$$

my error was here

so we have:

$$\int \left[\frac{3}{x} + \frac{4x}{x^2+4} \right] dx$$

$$= 3\ln|x| + 2\ln|x^2+4| + C$$

Now:

$$\int \frac{4x}{x^2+4} dx$$

$$\text{Let } u = x^2+4$$

$$\Rightarrow du = 2x dx$$

$$\Rightarrow 2du = 4x dx$$

$$= 2 \int \frac{1}{u} du = 2\ln|u| + C$$

$$= 2\ln|x^2+4| + C$$