

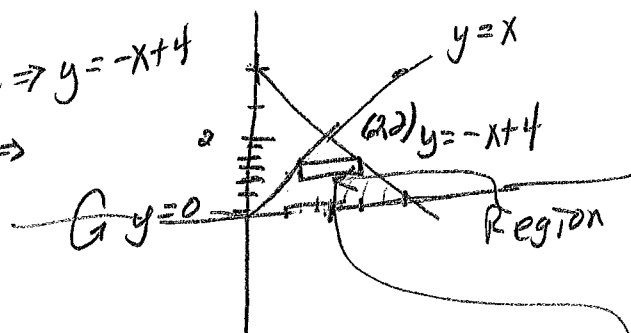
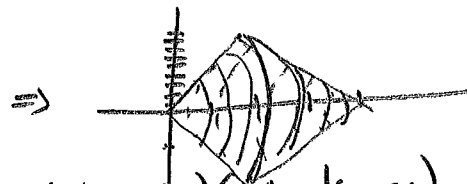
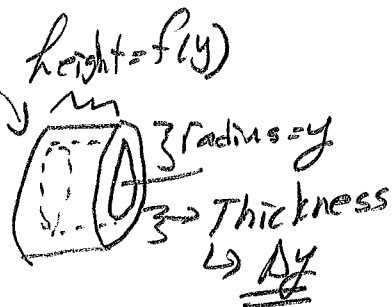
pg 470 (cont)

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#19. $x+y=4 \Rightarrow y=-x+4$

$y=x \Rightarrow$

Revolve about
x-axis
shell method.



$$V = 2\pi \int_c^d (\text{Radius})(\text{Height})(\text{Thickness})$$

$$= 2\pi \int_0^2 (y) \left(\begin{array}{l} \text{need height} \\ \text{as function of } y \end{array} \right)$$

$$\begin{aligned} \hookrightarrow y = -x + 4 &\Rightarrow y - 4 = -x \\ &\Rightarrow -y + 4 = x \\ &\Rightarrow f(y) = -y + 4 \end{aligned}$$

so we have:

$$V = 2\pi \int_0^2 (y)(-y+4) dy$$

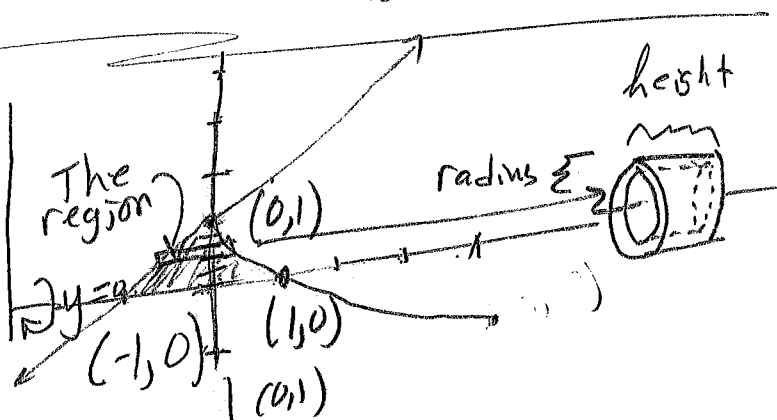
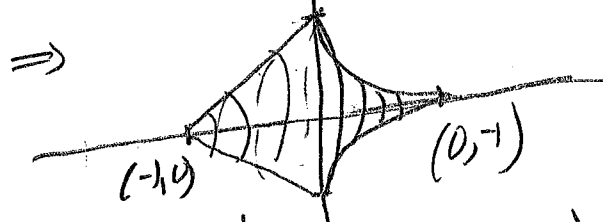
#21. SHELL METHOD.
Given: $y = 1 - \sqrt{x}$
 $y = x + 1 \Rightarrow$

Rotate about the x-axis:

we need the equations as
functions of y .

$$\begin{aligned} y &= 1 - \sqrt{x} & y &= x + 1 \\ \Rightarrow y - 1 &= -\sqrt{x} & \Rightarrow y - 1 &= x \\ \Rightarrow -y + 1 &= \sqrt{x} & & \\ \Rightarrow (-y + 1)^2 &= x & \text{"Left"} & \end{aligned}$$

"Right"



$$V = 2\pi \int_c^d (\text{Radius})(\text{Height})(\text{Thickness})$$

(Right - Left)

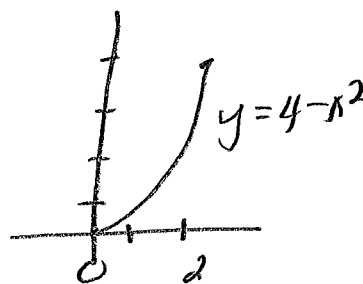
so we have

$$V = 2\pi \int_0^1 (y)((-y+1)^2 - (y-1)) dy$$

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#21 Find arc length of
 $y = 4 - x^2$ on $[0, 2]$

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Arc length =

$$s = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

we have

$$f(x) = 4 - x^2$$

$$\Rightarrow f'(x) = -2x$$

$$\Rightarrow [f'(x)]^2 = (-2x)^2$$

$$\stackrel{\text{so}}{=} s = \int_0^2 \sqrt{1 + (-2x)^2} dx$$

#23 $y = \frac{1}{x}$ on $[1, 3]$ $\stackrel{\text{so}}{=}$

$$f(x) = \frac{1}{x} = x^{-1}$$

$$\Rightarrow f'(x) = -x^{-2}$$

$$s = \int_1^3 \sqrt{1 + (-x^{-2})^2} dx$$

#24 $y = \frac{1}{x+1}$ on $[0, 1]$
 $= (x+1)^{-1}$ $\stackrel{\text{so}}{=}$

$$\Rightarrow f(x) = (x+1)^{-1}$$

$$\Rightarrow f'(x) = -(x+1)^{-2}$$

$$s = \int_0^1 \sqrt{1 + (-(x+1)^{-2})^2} dx$$

#25 $y = \sin(x)$ on $[0, \pi]$ $\stackrel{\text{so}}{=}$

$$f(x) = \sin(x)$$

$$\Rightarrow f'(x) = \cos(x)$$

$$s = \int_0^{\pi} \sqrt{1 + (\cos(x))^2} dx$$

#26 $y = \cos(x)$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$ $\stackrel{\text{so}}{=}$

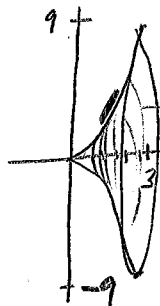
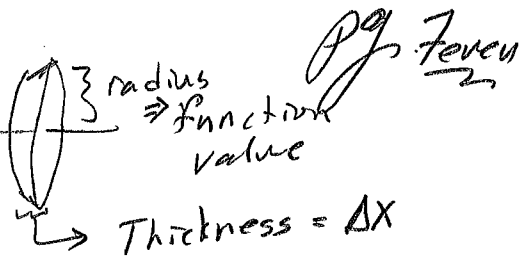
$$\Rightarrow f(x) = \cos(x)$$

$$\Rightarrow f'(x) = -\sin(x)$$

$$s = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 + (-\sin(x))^2} dx$$

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#39. $y = \frac{1}{3}x^3$ on $[0, 3]$



$$S = 2\pi \int_a^b (\text{Radius}) \sqrt{1 + [f'(x)]^2} dx$$

so $S = 2\pi \int_0^3 (\frac{1}{3}x^3) \sqrt{1 + (x^2)^2} dx$

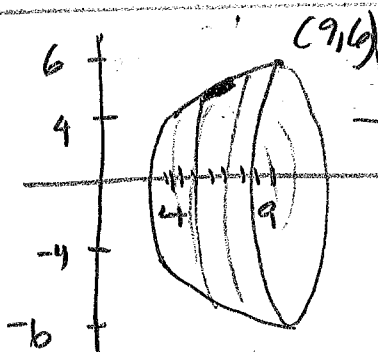
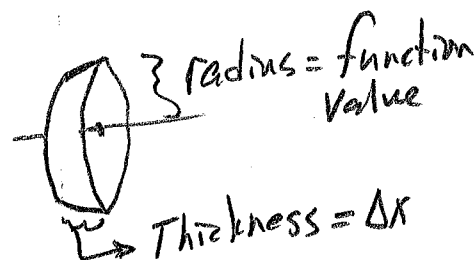
$$f(x) = \frac{1}{3}x^3$$

$$\Rightarrow f'(x) = 3(\frac{1}{3})x^2 = x^2$$

#40 $y = 2\sqrt{x}$ on $[4, 9]$

$$f(x) = 2x^{1/2}$$

$$\Rightarrow f'(x) = 2(\frac{1}{2})x^{-1/2} = x^{-1/2}$$



$$S = 2\pi \int_a^b (\text{radius}) \sqrt{1 + (f'(x))^2} dx$$

so $S = 2\pi \int_4^9 (2x^{1/2}) \sqrt{1 + (x^{-1/2})^2} dx$

#43 $y = \sqrt{4 - x^2}$ on $[-1, 1]$

$$f(x) = (4 - x^2)^{1/2}$$

$$\Rightarrow f'(x) = \frac{1}{2}(4 - x^2)^{-1/2}$$

so we have

$$S = 2\pi \int_a^b (\text{radius}) (\sqrt{1 + (f'(x))^2}) dx$$

so $S = 2\pi \int_{-1}^1 (4 - x^2)^{1/2} \sqrt{1 + [\frac{1}{2}(4 - x^2)^{-1/2}]^2} dx$

#44 $y = \sqrt{9 - x^2}$ on $[-2, 2]$

$$f(x) = (9 - x^2)^{1/2}$$

$$f'(x) = \frac{1}{2}(9 - x^2)^{-1/2}$$

$$S = 2\pi \int_a^b (\text{radius}) \sqrt{1 + (f'(x))^2} dx$$

so $S = 2\pi \int_{-2}^2 (9 - x^2)^{1/2} \sqrt{1 + (\frac{1}{2}(9 - x^2)^{-1/2})^2} dx$

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#9 $F=5$ when $x=3$
 $\Rightarrow F(3)=5$

$$\Rightarrow F(x) = kx$$

$$\Rightarrow F(3) = k(3)$$

$$\Rightarrow 5 = k(3)$$

$$\Rightarrow k = \frac{5}{3}$$

How much work is done
 from 0 to 7 inches?
 $\rightarrow [0, 7]$

$$W = \int_a^b F(x) dx$$

$$= \int_0^7 \frac{5}{3} x dx$$

$$= \left[\frac{5}{(2)3} x^2 \right]_0^7$$

$$= \left[\frac{5}{6} x^2 \right]_0^7$$

$$= \left[\left(\frac{5}{6} (7)^2 \right) - \left(\frac{5}{6} (0)^2 \right) \right]$$

$$= \boxed{\frac{245}{6} \text{ units}}$$

#11. $F(x) = kx$

$$\Rightarrow 20 = k(9)$$

$$\Rightarrow k = \frac{20}{9}$$

How much work is done
 stretching the spring
 1 ft = 12 inches?

$$\Rightarrow W = \int_a^b F(x) dx$$

$$= \int_0^{12} \frac{20}{9} x dx$$

$$= \left[\frac{20}{9(2)} x^2 \right]_0^{12}$$

$$= \left[\left(\frac{10}{9} (12)^2 \right) - (0) \right]$$

$$= \boxed{\frac{1440}{9} \text{ units}}$$