

- I
- ① E
 - ② B
 - ③ D
 - ④ B
 - ⑤ A

- ⑥ C
- ⑦ C
- ⑧ D
- ⑨ C
- ⑩ A

- ⑪ E
- ⑫ E
- ⑬ E
- ⑭ A
- ⑮ C

20. $f(x) = \begin{cases} \frac{-8}{x^2-4} & \text{if } x \leq 0 \\ 3-x^2 & \text{if } x > 0 \end{cases}$

* Not continuous if $x^2-4=0 \Rightarrow x=\pm 2 \Rightarrow$ disc. at $x=-2$ ✓
not at $x=2$

* Must check at $x=0$

① $f(0) = \frac{-8}{(0)^2-4} = \frac{-8}{-4} = 2$ ✓

② $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{-8}{x^2-4} = \frac{-8}{(0)^2-4} = 2$ ✓

$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (3-x^2) = (3-0) = 3$ ✓

$\therefore \lim_{x \rightarrow 0} f(x)$ D.N.E.

$\therefore$ not continuous at $x=0$

so Discontinuities at

$$\begin{aligned} x &= -2 \\ x &= 0 \end{aligned}$$

* Note: since $x=2$ is not in the interval $x \leq 0$, there is no discontinuity there.

21. $f(x) = \begin{cases} 3x-7 & \text{if } x \leq 4 \\ x^2-kx+1 & \text{if } x > 4 \end{cases}$

we require

$\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x)$

$\Rightarrow \lim_{x \rightarrow 4^-} (3x-7) = \lim_{x \rightarrow 4^+} (x^2-kx+1)$

$\Rightarrow 3(4)-7 = (4)^2 - k(4) + 1$

$\Rightarrow 12-7 = 16-4k+1$

$\Rightarrow 5 = 17-4k$

$\Rightarrow -12 = -4k$

$\Rightarrow k = 3$

$$(22) \lim_{x \rightarrow 5} \frac{(\sqrt{x-1} - 2)}{(x-5)} \stackrel{\text{D.S.}}{=} \frac{\sqrt{(5-1)} - 2}{(5-5)} = \frac{0}{0}$$

$$\stackrel{\text{D.S.}}{=} \lim_{x \rightarrow 5} \frac{(\sqrt{x-1} - 2)}{(x-5)} \cdot \frac{(\sqrt{x-1} + 2)}{(\sqrt{x-1} + 2)} = \lim_{x \rightarrow 5} \frac{(x-1-4)}{(x-5)(\sqrt{x-1} + 2)}$$

$$= \lim_{x \rightarrow 5} \frac{\cancel{(x-5)}}{(\cancel{x-5})(\sqrt{x-1} + 2)}$$

$$= \lim_{x \rightarrow 5} \frac{1}{\sqrt{x-1} + 2}$$

$$\stackrel{\text{D.S.}}{=} \frac{1}{\sqrt{5-1} + 2}$$

$$= \frac{1}{2+2} = \frac{1}{4}$$

$$(23) f(x) = \begin{cases} -\sin(x) & \text{if } x \leq 0 \\ \tan(x) & \text{if } x > 0 \end{cases} \text{ on } (-2\pi, 2\pi)$$

① $-\sin(x)$ is continuous everywhere.

② $\tan(x)$ discontinuities at $x = \pi/2 \neq x = 3\pi/2$

③ Must check at $x=0$

$$\text{① } f(0) \stackrel{\text{top}}{=} -\sin(0) = 0 \checkmark$$

$$\text{(ii)} \lim_{x \rightarrow 0} f(x) \stackrel{\text{top}}{=} \lim_{x \rightarrow 0^-} (-\sin(x)) \stackrel{\text{D.S.}}{=} -\sin(0) = 0 \checkmark$$

$$\lim_{x \rightarrow 0^-} f(x) \stackrel{\text{D.S.}}{=} \lim_{x \rightarrow 0^-} (-\sin(x)) = -\sin(0) = 0 \checkmark$$

$$\lim_{x \rightarrow 0^+} f(x) \stackrel{\text{BOT}}{=} \lim_{x \rightarrow 0^+} (\tan(x)) \stackrel{\text{D.S.}}{=} \tan(0) = 0 \checkmark$$

$$\therefore \lim_{x \rightarrow 0} f(x) = 0 \checkmark$$

$$\text{(iii)} \text{ These are equal: } \lim_{x \rightarrow 0} f(x) = f(0) \checkmark$$

$\therefore$ continuous at $x=0$

$\therefore$ The only discontinuities are at

$$x = \pi/2 \quad \& \quad x = 3\pi/2$$

(24) $\lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x^2 - 2x - 15} \xrightarrow{D.S.} \frac{(5)^2 - 7(5) + 10}{(5)^2 - 2(5) - 15} = \frac{25 - 35 + 10}{25 - 10 - 15} = \frac{0}{0}$ pg 3hree

$\stackrel{SO}{=} \lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x^2 - 2x - 15} = \lim_{x \rightarrow 5} \frac{(x-5)(x-2)}{(x-5)(x+3)}$
 $= \lim_{x \rightarrow 5} \frac{(x-2)}{(x+3)}$
 $= \frac{5-2}{5+3} = \boxed{\frac{3}{8}}$

(25) $\lim_{x \rightarrow 0} \frac{5 \tan(3x)}{4x} = \frac{5}{4} \lim_{x \rightarrow 0} \frac{\tan(3x)}{x}$
 $= \frac{5 \cdot 3}{4} \lim_{x \rightarrow 0} \frac{\tan(3x)}{3x}$
 $= \frac{15}{4} \cdot 1$
 $= \boxed{\frac{15}{4}}$

(26) $f(x) = \begin{cases} 4x - x^2 & x \leq -1 \\ \frac{15}{x+4} & x > -1 \end{cases}$

(i) $f(-1) \stackrel{TOP}{=} 4(-1) - (-1)^2 = -4 - 1 = -5$

(ii) $\lim_{x \rightarrow (-1)^-} f(x) \stackrel{TOP}{=} \lim_{x \rightarrow (-1)^-} (4x - x^2) \stackrel{D.S.}{=} 4(-1) - (-1)^2 = -4 - 1 = -5$

$\lim_{x \rightarrow (-1)^+} f(x) \stackrel{BOT}{=} \lim_{x \rightarrow (-1)^+} \left(\frac{15}{x+4} \right) \stackrel{D.S.}{=} \frac{15}{(-1)+4} = \frac{15}{3} = 5$

$\therefore \lim_{x \rightarrow (-1)} f(x) \text{ D.N.E.}$

$\therefore f(x) \text{ is NOT CONTINUOUS AT } x = -1$

27) Prove $\lim_{x \rightarrow -4} (-3x - 6) = 6$

I Let $\epsilon > 0$

II Search for δ

we want

$$|f(x) - L| < \epsilon \Rightarrow |(-3x - 6) - 6| < \epsilon$$

$$\Rightarrow |-3x - 12| < \epsilon$$

$$\Rightarrow |-3(x + 4)| < \epsilon$$

$$= |-3| |x - (-4)| < \epsilon$$

$$\Rightarrow |x - (-4)| < \frac{\epsilon}{|-3|}$$

Choose $\delta = \frac{\epsilon}{|-3|}$

III Verify δ :

$$0 < |x - a| < \delta \Rightarrow |x - (-4)| < \frac{\epsilon}{|-3|}$$

$$\Rightarrow |-3| |x + 4| < \epsilon$$

$$\Rightarrow |-3(x + 4)| < \epsilon$$

$$\Rightarrow |-3x - 12| < \epsilon$$

$$\Rightarrow |-3x - 6 + 6 - 12| < \epsilon$$

$$\Rightarrow |(-3x - 6) - 6| < \epsilon$$

$$\Rightarrow |f(x) - L| < \epsilon$$