

①. ①. $\int \cos^2(x) dx = \frac{1}{2} \int [1 + \cos(2x)] dx$
 $= \boxed{\frac{1}{2} \left[x + \frac{1}{2} \sin(2x) \right] + C}$

②. $\int 8x^2 \sin(2x) dx =$
parts: $u = 8x^2$ $v = -\frac{1}{2} \cos(2x)$
 $du = 16x dx$ $dv = \sin(2x) dx$
 $= (8x^2) \left(-\frac{1}{2} \cos(2x)\right) - \int (16x) \left(-\frac{1}{2} \cos(2x)\right) dx$
 $= -4x^2 \cos(2x) + \int (8x) (\cos(2x)) dx$
parts: $u = 8x$ $v = \frac{1}{2} \sin(2x)$
 $du = 8 dx$ $dv = \cos(2x) dx$
 $= -4x^2 \cos(2x) + (8x) \left(\frac{1}{2} \sin(2x)\right) - \int (8) \left(\frac{1}{2} \sin(2x)\right) dx$
 $= -4x^2 \cos(2x) + 4x \sin(2x) - \int 4 \sin(2x) dx$
 $= -4x^2 \cos(2x) + 4x \sin(2x) - 4 \left(-\frac{1}{2} \cos(2x)\right) + C$
 $= \boxed{-4x^2 \cos(2x) + 4x \sin(2x) + 2 \cos(2x) + C}$

③. $\int \sin^3(x) \cos^2(x) dx = \int \sin^2(x) \cos^2(x) \sin(x) dx$
Trig reduction $= \int [1 - \cos^2(x)] \cos^2(x) \sin(x) dx$
u-sub: Let $u = \cos(x)$
 $\Rightarrow du = -\sin(x) dx$
 $\Rightarrow -du = \sin(x) dx$
 $= - \int [1 - u^2] u^2 du$
 $= - \int [u^2 - u^4] du$
 $= - \left[\frac{u^3}{3} - \frac{u^5}{5} \right] + C$
 $= \boxed{- \left[\frac{\cos^3(x)}{3} - \frac{\cos^5(x)}{5} \right] + C} \quad \text{OR} \quad \boxed{\frac{\cos^5(x)}{5} - \frac{\cos^3(x)}{3} + C}$

④. $\int \tan^3(x) \cdot \sec^4(x) dx$
Trig Reduction Note there are 2 options

OPTION I

$$\begin{aligned} \int \tan^3(x) \cdot \sec^4(x) dx &= \\ &= \int \tan^3(x) \cdot \sec^2(x) \cdot \sec^2(x) dx \\ &= \int \tan^3(x) [\tan^2(x) + 1] \sec^2(x) dx \\ &= \int [\tan^5(x) + \tan^3(x)] \sec^2(x) dx \\ &\quad \underline{u\text{-sub: } \text{Let } u = \tan(x)} \\ &\quad \Rightarrow du = \sec^2(x) dx \\ &= \int [u^5 + u^3] du \\ &= \frac{u^6}{6} + \frac{u^4}{4} + C \\ &= \boxed{\frac{\tan^6(x)}{6} + \frac{\tan^4(x)}{4} + C} \end{aligned}$$

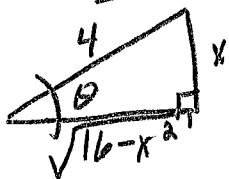
OPTION II

$$\begin{aligned} \int \tan^3(x) \sec^4(x) dx &= \\ &= \int \tan^2(x) \cdot \sec^3(x) \cdot \sec(x) \tan(x) dx \\ &= \int [\sec^2(x) - 1] \sec^3(x) \cdot \sec(x) \tan(x) dx \\ &= \int [\sec^5(x) - \sec^3(x)] \sec(x) \tan(x) dx \\ &\quad \underline{u\text{-sub: } \text{Let } u = \sec(x)} \\ &\quad \Rightarrow du = \sec(x) \tan(x) dx \\ &= \int [u^5 - u^3] du \\ &= \frac{u^6}{6} - \frac{u^4}{4} + C \\ &= \boxed{\frac{\sec^6(x)}{6} - \frac{\sec^4(x)}{4} + C} \end{aligned}$$

⑤.

$$\int \frac{x^3}{16\sqrt{16-x^2}} dx =$$

Trig. sub:



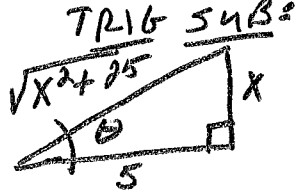
choose $\sin(\theta) = \frac{x}{4}$
 $\Rightarrow 4 \sin(\theta) = x$
 $\Rightarrow 4 \cos(\theta) d\theta = dx$

choose: $\cos(\theta) = \frac{\sqrt{16-x^2}}{4}$
 $\Rightarrow 4 \cos(\theta) = \sqrt{16-x^2}$

$$\begin{aligned} &= \int \frac{(4 \sin(\theta))^3 \cdot (4 \cos(\theta))}{16 (4 \cos(\theta))} d\theta \\ &= \int \frac{64 \sin^3(\theta)}{16} d\theta \end{aligned}$$

$$\begin{aligned} &= 4 \int \sin^3(\theta) d\theta \\ &= 4 \int \sin^2(\theta) \cdot \sin(\theta) d\theta \\ &= 4 \int [1 - \cos^2(\theta)] \cdot \sin(\theta) d\theta \\ &\quad \underline{u\text{-sub: } \text{Let } u = \cos(\theta)} \\ &\quad \Rightarrow du = -\sin(\theta) d\theta \\ &\quad \Rightarrow du = \sin(\theta) d\theta \\ &= (-4) \int [1 - u^2] du \\ &= -4 \left[u - \frac{u^3}{3} \right] + C \\ &= -4 \left[\cos(\theta) - \frac{\cos^3(\theta)}{3} \right] + C \\ &= \boxed{-4 \left[\frac{\sqrt{16-x^2}}{4} - \frac{1}{3} \frac{(\sqrt{16-x^2})^3}{(4)^3} \right] + C} \\ &\quad \underline{\text{or}} \\ &= \boxed{\frac{(\sqrt{16-x^2})^3}{48} - \sqrt{16-x^2} + C} \end{aligned}$$

⑥ $\int \frac{x^2}{\sqrt{x^2+25}} dx =$

TRIG SUB: 

choose $\tan(\theta) = \frac{x}{5}$
 $\Rightarrow 5 \tan(\theta) = x$
 $\Rightarrow 5 \sec^2(\theta) d\theta = dx$
 pg 3 here choose $\sec(\theta) = \frac{\sqrt{x^2+25}}{5}$

$$= \int \frac{(5 \tan(\theta))^2 \cdot 5 \sec^2(\theta)}{5 \sec(\theta)} d\theta$$

$$= \int 25 \tan^2(\theta) \cdot \sec(\theta) d\theta$$

$$= 25 \int [\sec^2(\theta) - 1] \cdot \sec(\theta) d\theta$$

$$= 25 \int [\sec^3(\theta) - \sec(\theta)] d\theta$$

$$= 25 \left[\frac{1}{2} (\ln |\sec(\theta) + \tan(\theta)| + \sec(\theta) \tan(\theta)) - \ln |\sec(\theta) + \tan(\theta)| \right] + C$$

$$= 25 \left[\frac{1}{2} \left(\ln \left| \frac{\sqrt{x^2+25}}{5} + \frac{x}{5} \right| + \left(\frac{\sqrt{x^2+25}}{5} \right) \left(\frac{x}{5} \right) \right) - \ln \left| \frac{\sqrt{x^2+25}}{5} + \frac{x}{5} \right| \right] + C$$

⑦ $\int \frac{x^2 - x + 2}{(x+1)(x-1)^2} dx$

Par. Frac:

$$\frac{(x+1)(x-1)^2}{1} \left[\frac{x^2 - x + 2}{(x+1)(x-1)^2} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \right]$$

$$\Rightarrow x^2 - x + 2 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

Let $x = 1$

$$\Rightarrow (1)^2 - (1) + 2 = 0 + 0 + C(1+1)$$

$$\Rightarrow 2 = C(2)$$

$$\Rightarrow C = 1$$

Let $x = -1$

$$\Rightarrow (-1)^2 - (-1) + 2 = A(0) + 0 + 0$$

$$\Rightarrow 4 = A(4)$$

$$\Rightarrow A = 1$$

Let $x = 0$

$$\Rightarrow 0 - 0 + 2 = (1)(0-1)^2 + B(1)(-1) + (1)(0+1)$$

$$\Rightarrow 2 = 1 - B + 1$$

$$\Rightarrow B = 0$$

so we have:

$$\int \frac{x^2 - x + 2}{(x+1)(x-1)^2} dx =$$

$$= \int \left[\frac{1}{(x+1)} + \frac{1}{(x-1)^2} \right] dx$$

$$= \int \frac{1}{(x+1)} dx + \int (x-1)^{-2} dx$$

$$= \ln |x+1| + \frac{(x-1)^{-1}}{-1} + C$$

or

$$= \ln |x+1| - \frac{1}{(x-1)} + C$$

so we
have:

8. $\int \frac{7x^2+12}{x(x^2+4)} dx$
PAR. FRAC.

we have:

$$\frac{7x^2+12}{x(x^2+4)} = \frac{A}{x} + \frac{(Bx+C)}{(x^2+4)}$$

$$\begin{aligned} \Rightarrow 7x^2+12 &= A(x^2+4) + (Bx+C)(x) \\ &= Ax^2+4A+Bx^2+Cx \\ &= (A+B)x^2+Cx+4A \end{aligned}$$

EQUATE
COEFFICIENTS

$$A+B = 7 \Rightarrow 4+B=7 \Rightarrow B=3$$

$$C = 0 \Rightarrow C=0$$

$$4A = 12 \Rightarrow A=3$$

$$\begin{aligned} \int \frac{7x^2+12}{x(x^2+4)} dx &= \int \left[\frac{3}{x} + \frac{4x}{x^2+4} \right] dx \\ &= \int \frac{3}{x} dx + \int \frac{4x}{x^2+4} dx \end{aligned}$$

u-sub

$$\text{Let } u = x^2+4$$

$$\Rightarrow du = 2x dx$$

$$\Rightarrow 2du = 4x dx$$

$$\begin{aligned} &= \int \frac{3}{x} dx + 2 \int \frac{1}{u} du \\ &= 3 \ln|x| + 2 \ln|u| + C \\ &= 3 \ln|x| + 2 \ln|x^2+4| + C \\ &\text{OR} \\ &= 3 \ln|x| + 2 \ln(x^2+4) + C \end{aligned}$$

9. $\lim_{x \rightarrow 0} \frac{\cos(x)-1}{e^x-1} \xrightarrow{\text{D.S.}} \frac{\cos(0)-1}{e^0-1} = \frac{0}{0} \rightarrow \text{indeterminate form}$
 $\rightarrow \text{apply L'Hopital's Rule.}$

so we have:

$$\lim_{x \rightarrow 0} \frac{\cos(x)-1}{e^x-1} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0} \frac{\sin(x)}{e^x} \xrightarrow{\text{D.S.}} \frac{\sin(0)}{e^0} = \frac{0}{1} = \boxed{0}$$

10. $\lim_{x \rightarrow (-\infty)} \frac{x^2}{e^{-x}} \xrightarrow{\text{D.S.}} \frac{(-\infty)^2}{e^{-(-\infty)}} = \frac{\infty}{\infty} \Rightarrow \text{in determinate form.}$
 $\rightarrow \text{Apply L'Hopital's Rule.}$

$$\lim_{x \rightarrow (-\infty)} \frac{x^2}{e^{-x}} \xrightarrow{\text{L'H}} \lim_{x \rightarrow (-\infty)} \frac{2x}{-e^{-x}} \xrightarrow{\text{D.S.}} \frac{2(-\infty)}{-e^{-(-\infty)}} = \frac{-\infty}{-\infty} \Rightarrow \text{again indeterminate}$$

 $\rightarrow \text{apply L'Hopital again.}$

$$\lim_{x \rightarrow (-\infty)} \frac{2x}{-e^{-x}} \xrightarrow{\text{L'H}} \lim_{x \rightarrow (-\infty)} \frac{2}{e^{-x}} \xrightarrow{\text{D.S.}} \frac{2}{e^{-(-\infty)}} = \frac{2}{\infty} = \boxed{0}$$



$$\begin{aligned}
 \textcircled{11}. \int_0^{\infty} \frac{6}{(x-1)^4} dx &= \lim_{b \rightarrow \infty} \left[\int_2^b \frac{6}{(x-1)^4} dx \right] \\
 &= \lim_{b \rightarrow \infty} \left[\int_2^b 6(x-1)^{-4} dx \right] \\
 &= \lim_{b \rightarrow \infty} \left[\frac{6(x-1)^{-3}}{-3} \right]_2^b \\
 &= \lim_{b \rightarrow \infty} \left[\left(-\frac{6}{3}(b-1)^{-3} \right) - \left(-\frac{6}{3}(2-1)^{-3} \right) \right] \\
 &= \lim_{b \rightarrow \infty} \left[\frac{-2}{(b-1)^3} + \frac{2}{(1)} \right] \\
 &= \underline{\underline{\text{D.S}}} \left[\frac{-2}{\infty} + 2 \right] \\
 &= 0 + 2 \\
 &= \boxed{2}
 \end{aligned}$$

#11-21 $\Rightarrow$ see basic forms sheet.

Bonus: (22) $\int \frac{(2x-m)}{(x)(x-m)} dx$

we have par frac

$$\frac{(x)(x-m)}{1} \left[\frac{(2x-m)}{(x)(x-m)} = \frac{A}{x} + \frac{B}{(x-m)} \right]$$

$$\Rightarrow (2x-m) = A(x-m) + B(x)$$

Let $x=0$

$$\Rightarrow -m = A(0-m) + 0$$

$$\Rightarrow -m = A(-m)$$

$$\Rightarrow A = 1 \checkmark$$

Let $x=m$

$$\Rightarrow 2m-m = 0 + B(m)$$

$$\Rightarrow m = B(m)$$

$$\Rightarrow B = 1 \checkmark$$

so we have:

$$\int \frac{(2x-m)}{(x)(x-m)} dx =$$

$$= \int \left[\frac{1}{x} + \frac{1}{(x-m)} \right] dx$$

$$= \boxed{\ln|x| + \ln|x-m| + C}$$