Lesson 1: Quadratic Equations

Quadratic Equation:

The quadratic equation in _________________________ form is _______________________________.

In this section, we will review 4 methods of ____________________ quadratic equations, and when it is most

_________________________ to use each method.

1. ________________________________
2. ________________________________
3. ________________________________
4. ________________________________

Method 1: Factoring

When to Use Factoring: ____

Steps: 1. Put the quadratic equation in ____________________________________.

2. ______________________________ the expression.

3. Use ________________________________Property to Solve.

Zero Product Property:

If a·b=0, then either _______________ or __________________.

$2x^2-5x-12=0$	$5x^2-20=0$	$4x^2-9x+8=3x-1$

Method 2: Square Root Method or Extraction of Roots Method

When to Use Square Root Method: In order to use square root method, the equation must be in the format:

____________________ = ___________.

Notice that if there is no ___________________ term in the standard form of the quadratic equation or if b = ______, then it is _________________ to put in this form.

Steps: 1. Put the quadratic equation in the form _____________________________________.

2. Take the _______________________________ of both sides of equation and ________________.

3. When you take _______________________ of both sides, you MUST take the _______________ parts.

$5x^2 = 9$	$(3x+1)^2 = -9$	$2(2x+1)^2 + 3 = 11$

Method 3: Completing the Square

Investigation of Perfect Squares

$(x+1)^2 =$

$(x+2)^2 =$

$(x+3)^2 =$

$(x+4)^2 =$

$\vdots$

$(x+7)^2 =$

$\vdots$

$(x-8)^2 =$

$(x\ \_\_\_\_)^2 = x^2 + 12x + \_\_\_\_\_$

$(x\ \_\_\_\_)^2 = x^2 - 18x + \_\_\_\_\_$

Method 3: Completing the Square (continued)

When to use Completing the Square Method: This method will ________________ work, but I would only use this method if I was unable to ____________________ or use ____________________________.

The most __________________________ equations to use Completing the Square Method, have a = ______ and b is ______________.

Steps: 1. From ____________________ form, make a = ______ by dividing each term by _______,

2. Move the _______________ term to the right side of equation and add ___________ to each side.

3. Complete the Square by _________________the linear term by ______ and ____________________. Put this value in the blanks. The right side will now factor into a _________________________.

4. Finish Solving by using _______________________________ method.

$x^2 + 10x + 21 = 0$	$2x^2 + 14x + 4 = 0$	$3x^2 - 18x + 21 = 0$

Method 4: Quadratic Formula

When to Use Quadratic Formula: ____

Steps: 1. Put the quadratic equation in ______________________________________.

2. Find the values of ________, _________, and _________.

3. ____________________values in quadratic formula, which is:

4. Reduce.

$x^2+9x+11=3x-2$	$9x^2-18x+7=0$

Choosing the Best Method

In summary, when choosing a method to solve a quadratic equation, follow this order.

1. ____________________ try to ________________ first.

2. If b = _____ or if ___________________________, then use ______________________________.

3. If a = _____ and b is _____________, then it is convenient to use _________________________________.

4. As a _______________ resort, use ____, which will solve

________ quadratic equations.

Lesson 2: Miscellaneous Equations

1. Higher Order Equations - Factoring

Steps: 1. Get equations in general form, or set ___________________________________

2. Factor out ____, if possible.

3. Factor the remaining expression depending on the number of terms left

   a. 2 Terms:____

   b. 3 Terms:____

   c. 4 Terms: ____

4. Make sure all factors are _______________________. If they are not _________________, then repeat step 3.

5. Set each factor ______________________ and ________________ for the variable.

Solve.

$6x^3 + 22x^2 - 8x = 0$	$5x^4 - 20x^2 = 0$	$x^4 + 4x^3 - 8x = 32$

2. Rational Exponents

Review from Intermediate Algebra: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Examples:

$27^{\frac{2}{3}} =$

$16^{\frac{3}{4}} =$

Solve:

$x^2 = 9$	$x^3 = 8$
$x^2 = -9$	$x^3 = -8$

Solving Rational Exponent Equations $\quad x^{\frac{m}{n}} = k \qquad (\qquad\qquad)^{\frac{m}{n}} = k$

Steps: 1. Isolate the _________________________ with the rational exponent.

2. Raise both sides to the ________________________of the exponent, or ________.

a. If ___________________ of exponent or _____is ________, then put ______ sign on value.

b. If ___________________ of exponent or _____is ________, then DO NOT put ____ sign on value.

3. You MUST __________________ your solution(s) and eliminate ___________________ solutions.

$x^{\frac{m}{n}} = k$

If m is even $\qquad$ if m is odd

$x = \pm k^{\frac{n}{m}} \qquad x = k^{\frac{n}{m}}$

$x^{\frac{3}{2}} = 27$	$x^{\frac{2}{3}} = 4$

More Examples: Solve.

$\left(x^2-3x+3\right)^{\frac{3}{2}}-1=0$	$\left(x+5\right)^{\frac{2}{3}}=4$

3. Solving Equations of the Quadratic Form (using Substitution)

The following are examples of the quadratic form. What makes these seemingly different equations similar?

$$x^4-8x^2-9=0$$

1.

$$5x^{\frac{2}{3}}+11x^{\frac{1}{3}}+2=0$$

2.

$$\left(x+3\right)^2+7\left(x+3\right)-18=0$$

3.

Each of these 5 similarities to the right

4.

__________________ the equations to be

in the _________________ form.

5.

3. Solving Equations of the Quadratic Form (using Substitution) continued

Steps: 1. Identify the equation as a ___________________________ and set equation equal to _____.

2. Let some variable, _____ be equal to the original equation's ___________ term variable part. This equation is important to write down, because we will use it in step 5.

3. Find the _______________of the new variable ____, which will always be the first term's variable part.

4. ____________________ the new variable____ into the equation to get a quadratic equation and ___________ for the new variable, ____.

5. To solve for original variable, ________________ solution(s) into equation from step 2.

6. ____________ your solution(s). This is mandatory, if your equation includes ________________ exponents.

Solve:

$x^4 - 8x^2 - 9 = 0$	$5x^{\frac{2}{3}} + 11x^{\frac{1}{3}} + 2 = 0$	$(x+3)^2 + 7(x+3) - 18 = 0$

Lesson 3: Absolute Value Equations and Inequalities

Remember, the _________________ means distance from _____. Often we say, the absolute value makes the value inside the bars ________________. Examples: $|15| =$ $|-15| =$

1. Solving Absolute Equations

Solve in your HEAD

$\|x\| = 3$	$\|x\| = -3$

Notice that the __________ on the value determines if the equation has __________________.

Steps: 1. Isolate the ________________________ in the ________________________.

$$|\qquad\qquad\qquad| = c$$

2. If c is ___________________, which means _____ or ______then split the absolute value equation into two ____________without _____________; or and then ________________.	3. If c is ___________________, then the answer is ________________, because the absolute value of ______ expression can ___________ be equal to a negative value.

Solve:

$\|2x-3\| = 11$	$4\|x-2\|+7=3$

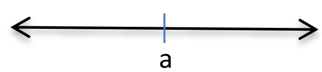
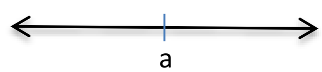
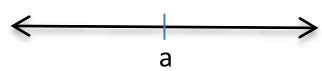
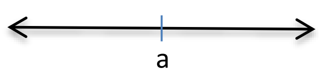
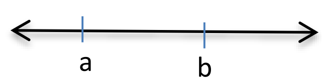
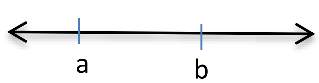
Solve:

$7\|3x\|+2=16$	$2\|3x-1\|+7=7$

2. Interval Notation

Interval notation represents the set of _______ numbers between two ____________________. If you would like to ________________ the endpoint, then use a _____ or _____ symbol. If you would NOT like to include the endpoint, then use a _____ or _____ symbol. We always write interval notation as follows:

________________, _________________

Inequality Notation	Graph	Interval Notation
$x<a$		
$x>a$		
$x\leq a$		
$x\geq a$		
$a<x<b$		
$a\leq x\leq b$		
$a<x\leq b$		
$a\leq x<b$		
All Real Numbers		
No Solution		

3. Solving Linear Inequalities

Remember: ____________________ the inequality sign when you multiply or divide by a ______________________ or if you __________________the equation.

$-4x+3 \le 27$	$-2 < x$

4. Solving Inequalities with Absolute Value

When solving inequalities with ___________________________, first _____________________ the absolute value expression. $|\qquad\qquad\qquad| < c$ or $|\qquad\qquad\qquad| > c$

Next, identify if the ___________ on the opposite side of the absolute value is _________________, or __________________.

A. Let's explore when c is ____________________________.

Inequality w/ Absolute Value	Graph of Solutions	Solution in Inequality form	Solution in Interval Notation
$\lvert x \rvert < 2$			
$\lvert x \rvert > 2$			

Let x be algebraic expression and **<u>let c be a positive number</u>**.

If $|x| < c$, then.

If $|x| > c$, then OR

And Examples:

Or Examples:

$\lvert x \rvert \le 3$	$\lvert x \rvert > 4$

$\|3x-2\|<8$	$\|3x-2\|\geq 8$
$\left\|\frac{x-2}{3}\right\|<4$	$\|1-2x\|-4\geq -1$

B. Let's explore when c is ____________________.

Inequality w/ Absolute Value	Graph of Solutions	Solution in Interval Notation
$\|x\|<-2$		
$\|x\|>-2$		

Let x be algebraic expression and **<u>let c be a negative number</u>**.

If $|x|<c$, then the solution is ______________________.

If $|x|>c$, then the solution is ______________________.

Solve

$\lvert 3x-2\rvert \le -8$	$\lvert 3x-2\rvert \ge -8$	$4\lvert 7-x\rvert + 9 < 5$

Lesson 4: Basics of Functions and Their Graphs

A relation is a ____.

The set of all ____elements in a relation is called the ____________________ and

the set of all ____ elements a relations is called the __________________. The following is a relation.

Student	Color of their Shirt
April	
Bob	
Carlos	
Dion	
Eva	

Let us define the following sets as:

$A = \{$ $\}$

$B = \{$ $\}$

The domain of A is : A_D

The range of A is: A_R

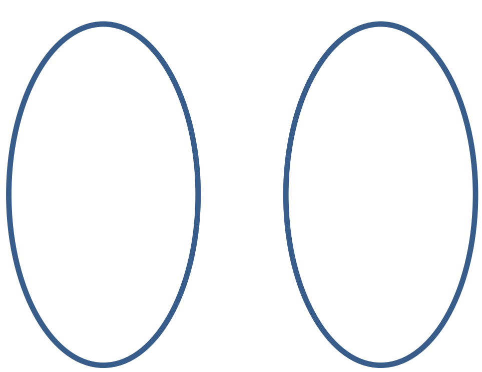
The domain of B is : B_D

The range of B is: B_R

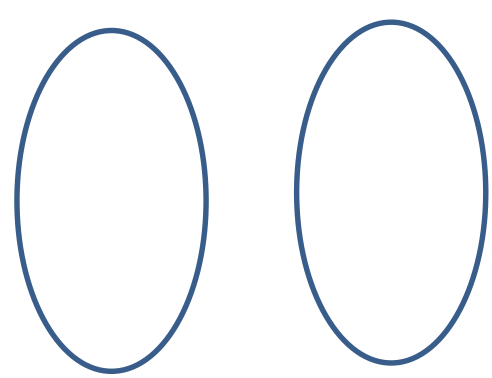
A __________________ is a __________________where each element in the ________________ corresponds

to __________________ element in the ________________.

A



B



Functions can be expressed several ways.

Functions as _____________	Functions as _____________	Functions as _____________

Functions as Sets

Determine if the following relations are function? Find domain and Range

$I=\{(10,8),(6,4),(2,0),(-2,-4)\}$	$K=\{(3,4),(3,5),(8,9),(1,0)\}$	$J=\{(4,5),(6,8),(8,8),(6,8)\}$
Is the relation a Function? _______	Is the relation a Function? _______	Is the relation a Function? _______
Domain:______________	Domain:______________	Domain:______________
Range:_______________	Range:_______________	Range:_______________

Functions as Equations

____________________________ is another way of writing an ___________________.

Function notation defines the __________________, or ___________ of the function by using any value of the ___________, (x). If an equation is a function, then we ____________________ y with ________ pronounced ___________________.

y = 3x + 1 ↔ ___________________

To find the value of a function at a given _________________________________, we _______________ the ______________________________ into the equation and _________________ or _______________.

Let $f(x)=2x-3$

Find $f(5)=$	Find $f(-x)=$	Find $f(x+1)=$

Let $g(x) = x^2 + 3x + 5$

Find $g(-2) =$	Find $g(-x) =$	Find $g(x+3) =$

Let $h(x) = \dfrac{x^2}{x-1}$

Find $h(-1) =$	FInd $h(4) =$	Find $h(-x) =$

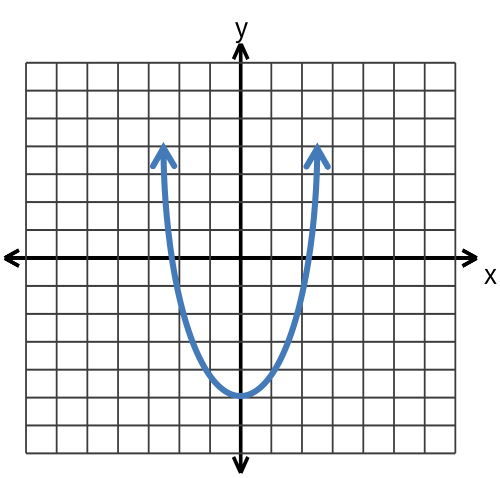
Functions as Graphs

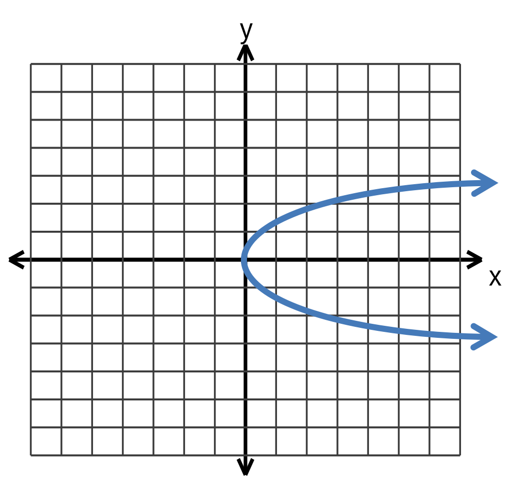
The ___________ of a ____________________ is the picture that represents all the ______________ or ___________________for the equation/function.

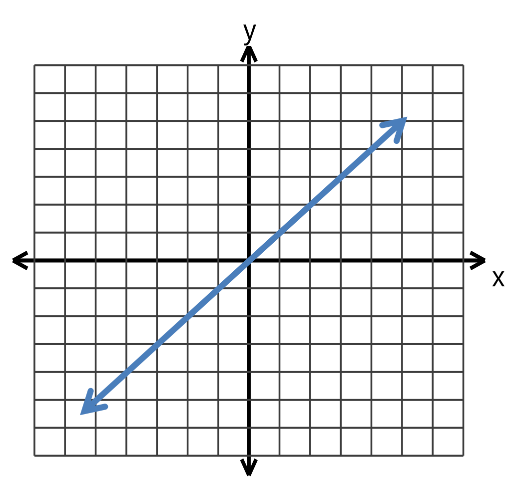
Remember: If every value in the ______________ corresponds to only ______ value in the ______________, then the graph is a __________________. If ______ value in the domain corresponds to more than ______ value in the ____________, then the graph is not a function.

To determine if a graph is a function, we will use the ________________________, which states that if a ________________ intersects the graph at ________ than one point, then the graph is ____ a function.

Are these relations also functions?

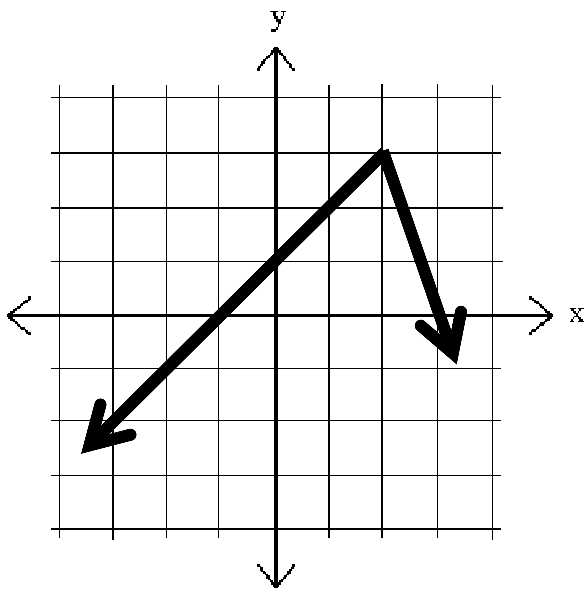


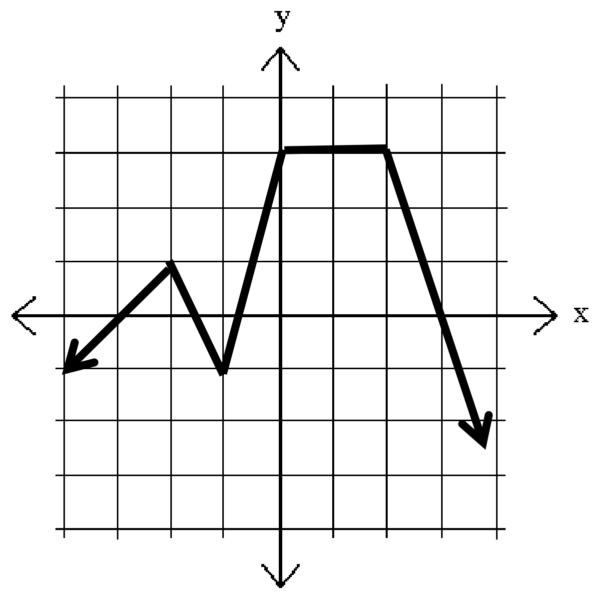




Functions as Graphs: Finding Values

We can also find __________of a function by looking at the ____________.
To find a function value, go to the given _______________________________ on the ___ axis.
Your ________________ is the ____coordinate at that input.





Find f(-2) = ______
Find f(-1) = ______
Find f(0) = _______
Find f(1) = _______
Find f(2) = _______

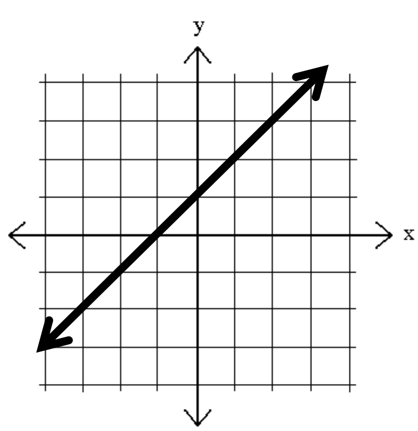
Find f(-3) = _______
Find f(-2) = _______
Find f(0) = ________
Find f(2) = ________
Find f(3) = ________

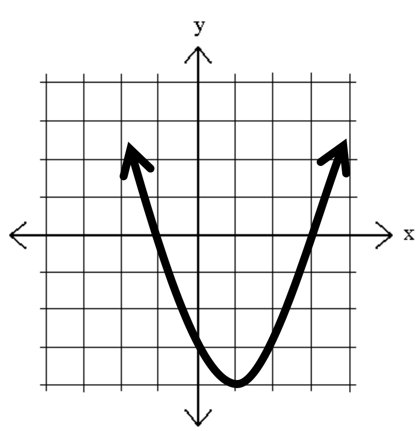
Functions as Graphs: X and Y Intercepts

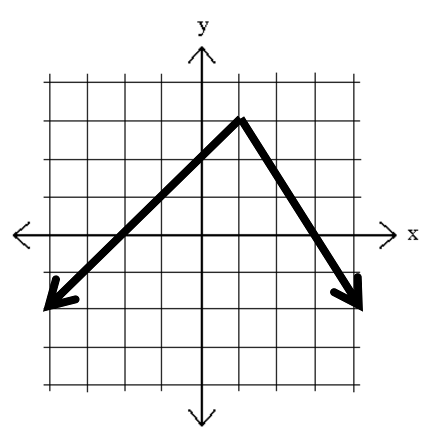
____________________________ are where the graph _____________ the ________________.
Algebraically, you find it by setting _______________ and solving for ____.

The ____________________________ is where the graph _____________ the ________________.
Algebraically, you find it by setting _______________ and solving for ____.

Find the x and y intercepts of the following graphs:







X Intercept(s):__________

Y Intercept: ____________

X Intercept(s):__________

Y Intercept: ____________

X Intercept(s):_________

Y Intercept: ___________

Functions as Graph: Domain and Range

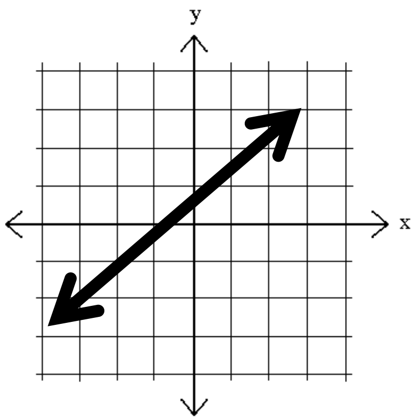
We will write the domain and range using ____.

Remember: The ____________of a relation is all the ________ or ___ values that relation includes.

In order to find the _______________of the graph, look at the end points of the relation graphed from___________ to ___________.

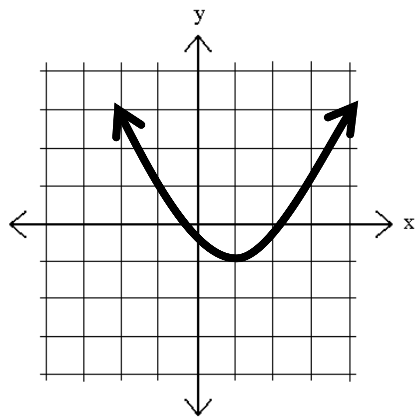
The ____________of a relation is all the ________ or ___ values that relation includes.

In order to find the _______________of the graph, look at the end points of the relation graphed from___________________ to ___________________.



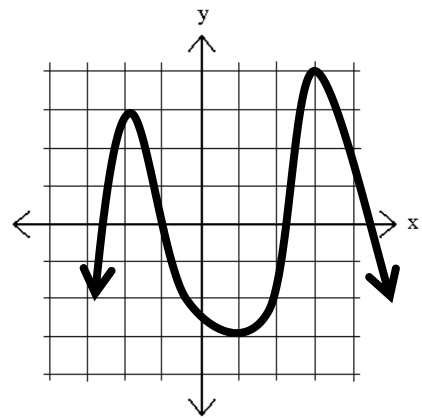
Domain:___________

Range:____________



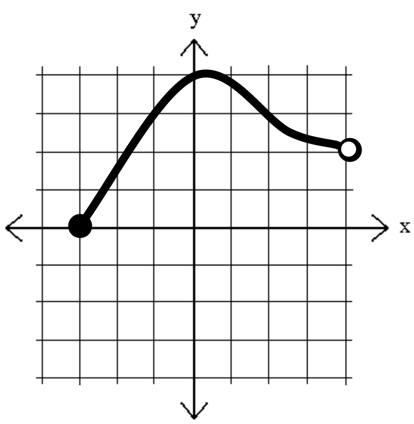
Domain:___________

Range:____________



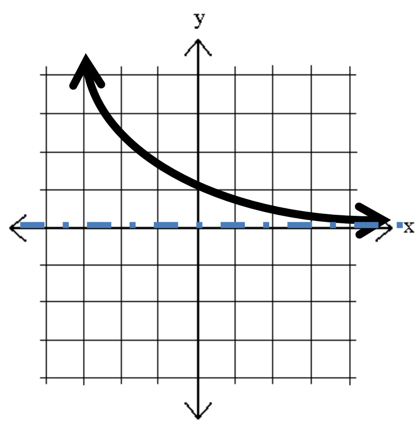
Domain:___________

Range:____________



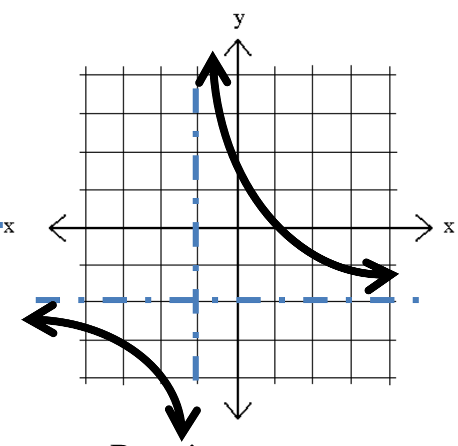
Domain:___________

Range:____________



Domain:___________

Range:____________



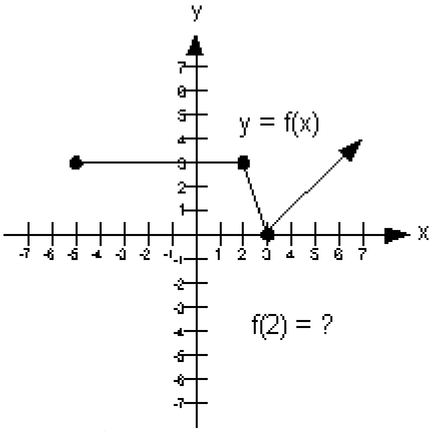
Domain:___________

Range:____________

Functions as Graphs Summary

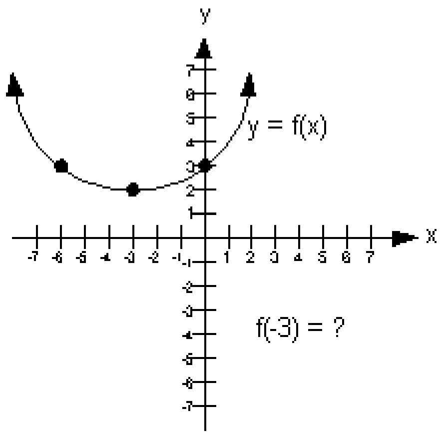
Zeros: The value of _____ when f(x) = _______, or the _____ coordinate of the ____ intercept.

1.



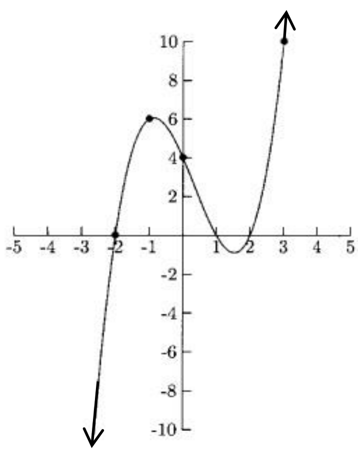
Domain ________________________
Range __________________________
X Intercept(s) ___________________
Y intercept ____________________
Zeros __________________________
Find f(2) = ____________________
Find f(3) = _____________________

2.



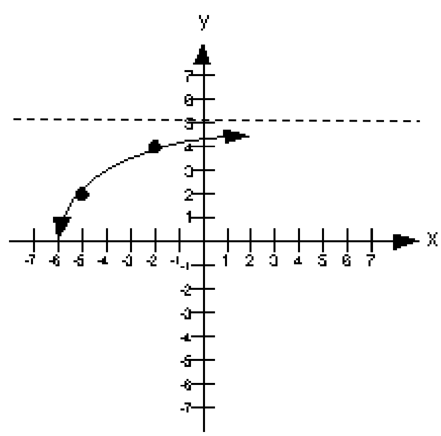
Domain ________________________
Range __________________________
X Intercept(s) __________________
Y intercept ____________________
Zeros __________________________
Find f(-3) = ___________________
Find f(-6) = ____________________

3



Domain ________________________
Range __________________________
X Intercept(s) ___________________
Y intercept ____________________
Zeros __________________________
Find f(-1) = ____________________
Find f(0) = _______________________

4.



Domain ________________________
Range __________________________
X Intercept(s) ___________________
Y intercept ____________________
Zeros __________________________
Find f(-2) = ____________________
Find f(-5) = ______________________

Lesson 5: More on Basics of Functions and their Graphs

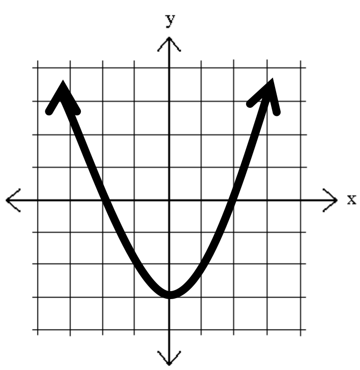
Even and Odd functions and their Symmetry

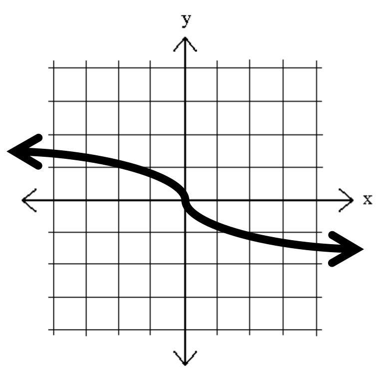
TYPE	GRAPH	SYMMETRY	ALGEBRAIC DETERMINATION
			If f(-x) = ________ for all x in the domain, then the functions is _______.
			If f(-x) = ________ for all x in the domain, then the functions is _______.

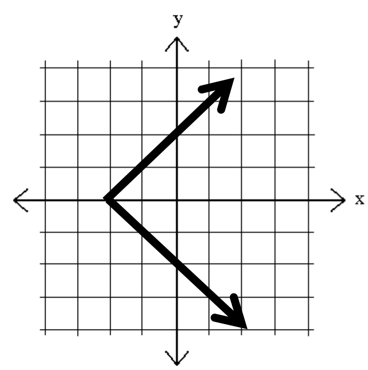
Algebraically determine if the following functions are even, odd or neither.

$f(x) = 3x^2 + 1$	$f(x) = 4x^3 - x$	$f(x) = x^5 + 1$

Use possible symmetry to determine whether the following graphs are even, odd or neither.







Piecewise Defined Functions:

A ______________________________ function is a ________________ that is defined by more than __________equation over a specified ____________________.

Example: For a Cell Phone Plan, you will pay $20 for the first 60 minutes, and then $0.40 per additional minute.

$$C(t) = \begin{cases} \\ \\ \end{cases}$$

Find C(30)=

Find C(120)=

To find a function value with a piecewise defined function you must look for the ________________the function value belongs in. Then you substitute that value in to the corresponding ________________________.

Example:

Let $f(x) = \begin{cases} 6x-1 & if \;\; x<0 \\ 7x+3 & if \;\; x \geq 0 \end{cases}$

Find f(-3)=	Find f(0)=	Find f(4)=

Let $g(x) = \begin{cases} x^2-5 & if \;\; x \neq 0 \\ 4 & if \;\; x = 0 \end{cases}$

Find g(-3)=	Find g(0)=	Find g(3)=

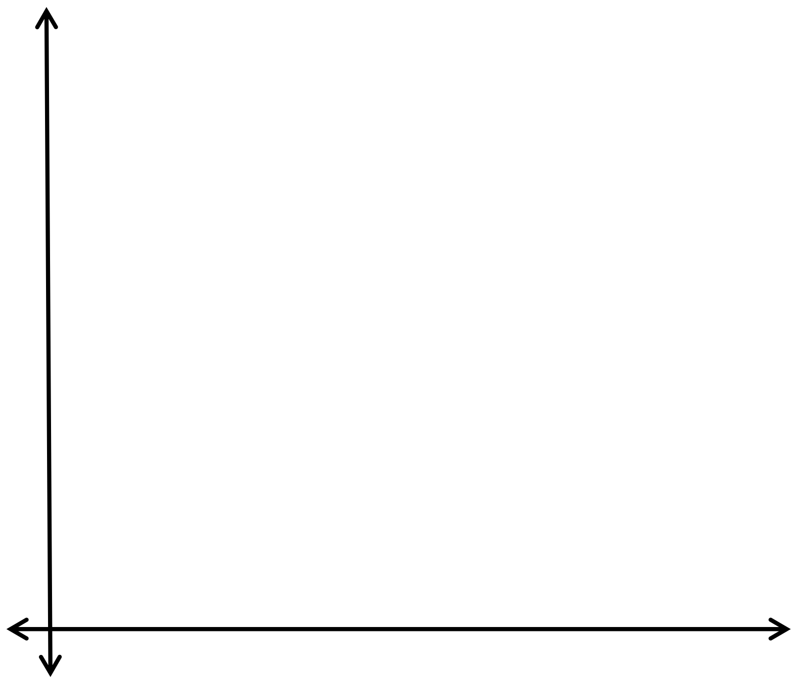
Graphing Piecewise Functions

To __________________ a piecewise defined function, choose _____________values for _______________, including the _________________________ of each domain, whether or not that the endpoint is _____________________ in the domain. Label each endpoint as _______________ or not. Sketch the ________________ of the function.

Remember our Cell Phone Plan Function:

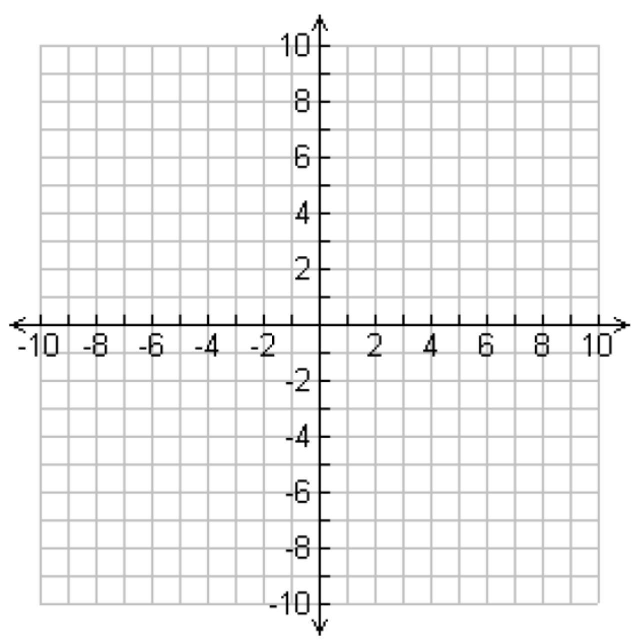
$$C(t)=\begin{cases}20 & if\ 0\le t\le 60\\ 20+.40(t-60) & if\ \ t>60\end{cases}$$

t	C(t)

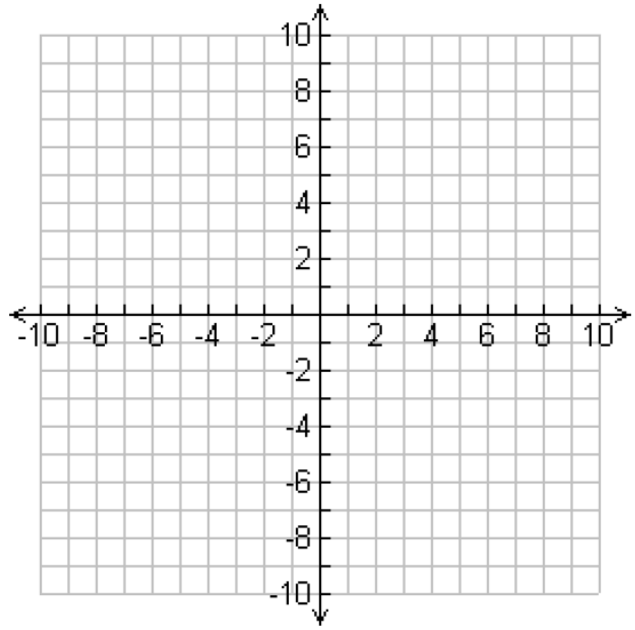


Graph the following piecewise defined functions.

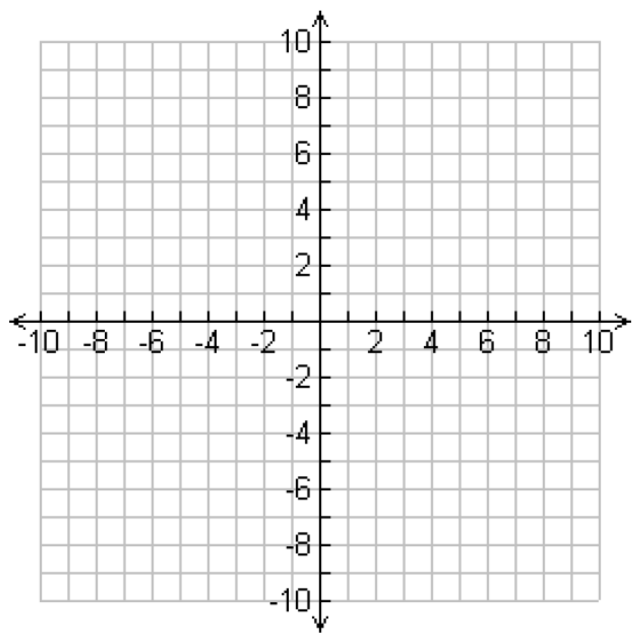
$$f(x)=\begin{cases}1+x & if\ x<0\\ x^2 & if\ x\ge 0\end{cases}$$



$$f(x) = \begin{cases} x+3 & if\ -3 \le x < 0 \\ 2 & if\ x = 0 \\ \sqrt{x} & if\ x > 0 \end{cases}$$



$$f(x) = \begin{cases} |x| & if \quad x \ne 0 \\ 1 & if \quad x = 0 \end{cases}$$



Difference Quotient

The difference quotient is used to understand the rate at which functions change, which is covered heavily in future courses. For this College Algebra course, we will need to understand how to evaluate this ratio.

Definition: $\dfrac{f(x+h)-f(x)}{h} \qquad where\ h \neq 0$

Examples: Find the difference quotient for the following functions

$f(x)=6x+1$
$f(x)=-2x^2+x+5$
$f(x)=\dfrac{1}{2x}$