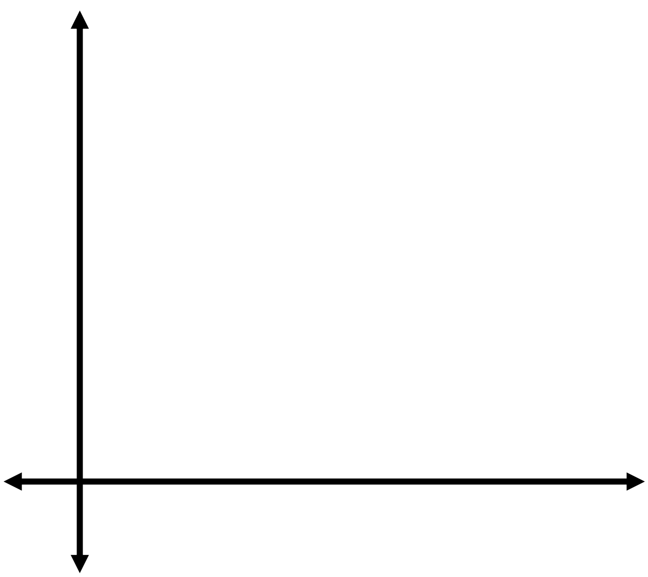
Lesson 6: Linear Functions and their Slope

A linear function is represented by a _____________ line when graph, and represented in an ________________ where the variables have no whole number exponent higher than _____.

Forms of a Linear Equation

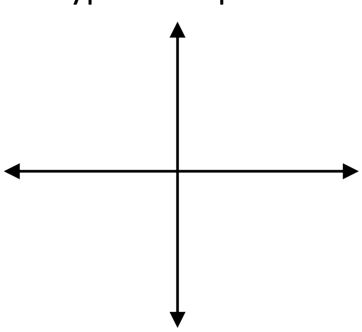
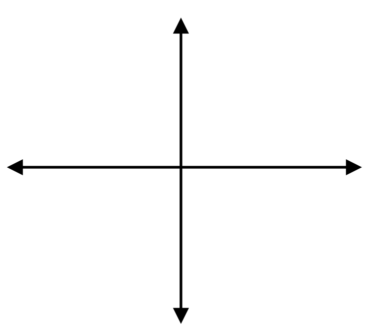
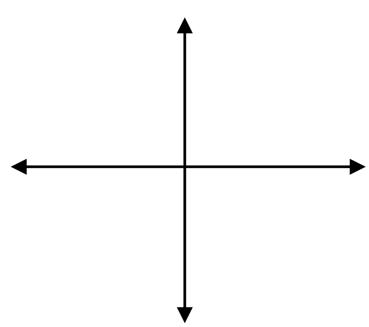
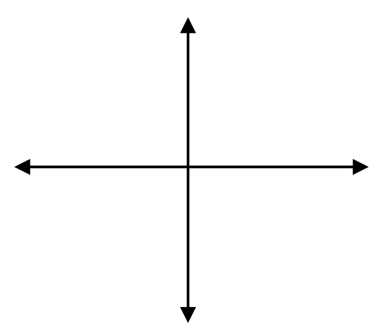
Name	Equation	When is it useful to use this form?
Standard Form		
Slope-Intercept Form		
Point Slope Form		

<u>Slope</u> – Steepness of a Line



Formula:

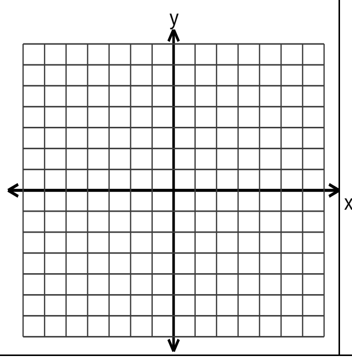
Type of Slope

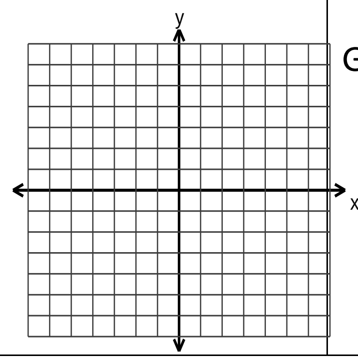
Examples:

Find the slope for the following pairs of points. Then plot the points and draw the linear function containing the two points

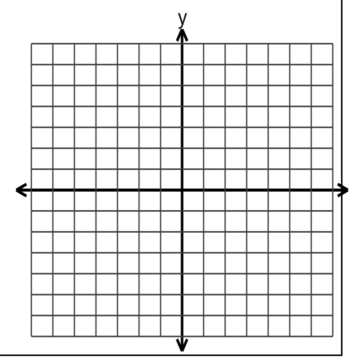
(6, -4) and (-4, 2)
Slope:

Graph: 

(-3, 4) and (2, 4)
Slope:

Graph: 

(1, 5) and (1, 2)
Slope:

Graph: 

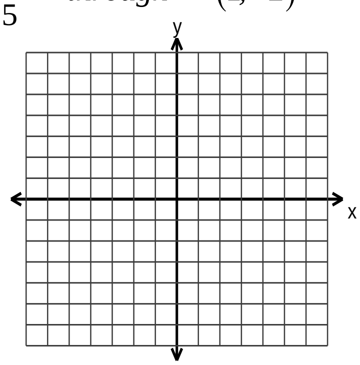
Special Cases: Use HOY VUX

H	V
O	U
Y	X

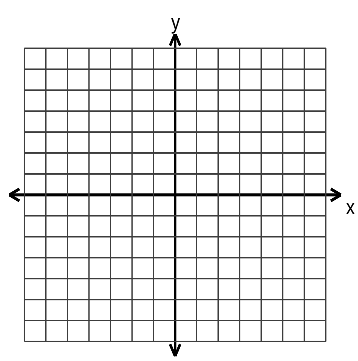
Graph using a Point and a Slope

1. Plot the ___________.
2. Use ______________ to find another point. Think numerator = _____________ and denominator = __________.
3. Connect points with a _________________ line using __________________ at the ends.

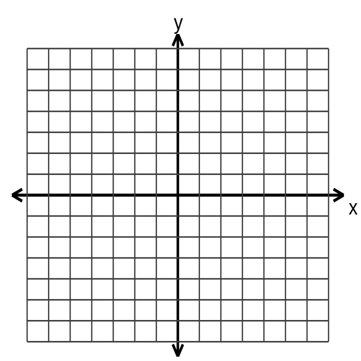
$m=\frac{3}{5}$ *through* $(1,-2)$



$m=0$ *through* $(1,-2)$



m *undefined* *through* $(1,-2)$

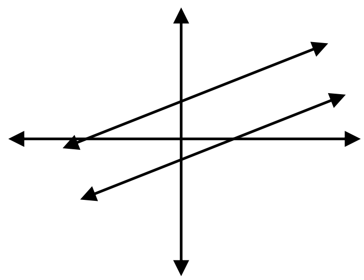
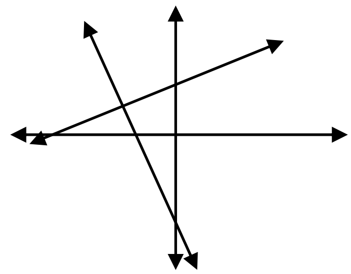
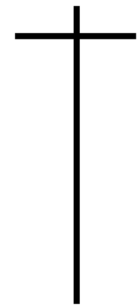


Writing Equations of Linear Functions:

Given Information	Steps:
A Point and a Slope	1. Plug information into ______________________ form, when possible. 2. Solve for ____ to answer in __________________ form.
Special Cases where slope is undefined or 0 Use HOY VUX	1. If slope is _____, then the line is _________________ and the equation is ___________. 2. If slope is _____, then the line is _________________ and the equation is ___________.
2 Points	1. Use 2 points to solve for ________________. 2. Plug ___________ and one of the ____________ into point-slope form. 3. Solve for ____ to answer in __________________ form.

Write the following equations:

Slope of 6, and passes through (-2, 5)	Slope of $-\frac{3}{5}$, and passes through (10, -4)
Slope of 0, and passing through (3,1)	Undefined Slope, and passing through (3,1)
Line passing through the points (-3, 1) and (2, 4)	Line passing through the points (3, 5) and (8, 15)

Parallel Slopes	Perpendicular Slopes
	 
Parallel slopes are ____________________.	Perpendicular slopes are _____________________.

Writing Equations using Parallel and Perpendicular Slopes

1. Put the given equation into _______________________ form to find ______________.
2. If the lines are parallel, then slope of the new line is the ____________. If the lines are perpendicular, then slope the new line is ___________________.
3. Plug this new ______________ and the given point into ________________________ form.
4. Solve for ______ to get answer in _______________________ form.

Find the equation of a line passing through (-2, -7) and *parallel* to the line $10x+2y=7$.
Find the equation of a line passing through (-4, 2) and *perpendicular* to the line $y=\frac{1}{3}x+7$.

Special Cases: Use HOY VUX

Find equation passing through (-2, 6) perpendicular to x = -4.

Find equation passing through (4, 8) perpendicular to y = 9.

Find equation passing through (-1, 5) parallel to x = 3.

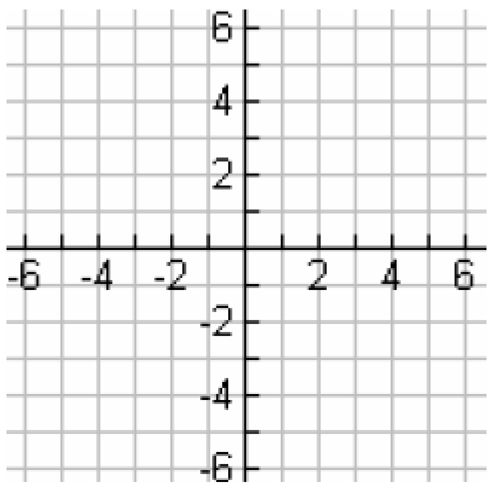
Find equation passing through (11, -3) parallel to y = -7.

Graphing Linear Functions

Form	Steps:
Slope Intercept Form	1. Identify the _________ and _______. 2. Plot the _______________ point. 3. Find the next point by using __________ or ______ over ______. 4. Connect points using ______________.
Standard Form	Translate the equation into _______________ form and repeat above steps OR 1. Find and plot the ______________ by setting y = _____. (,) 2. Find and plot the ______________ by setting x = _____. (,) 3. Connect points using ___________.

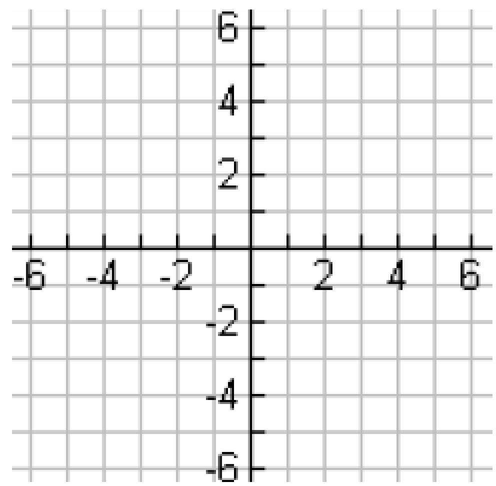
Graph $y = -3x + 2$

m=____ b=____ or ________



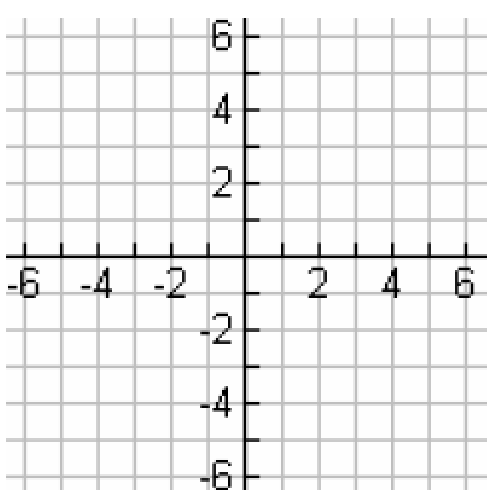
Graph $y = \frac{3}{4}x - 3$

m=____ b=____ or ________



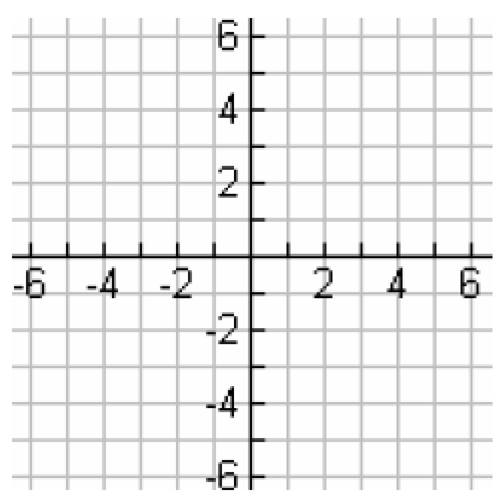
$$6x - 9y = 18$$

x-int: (,0)   y-int: (0,   )

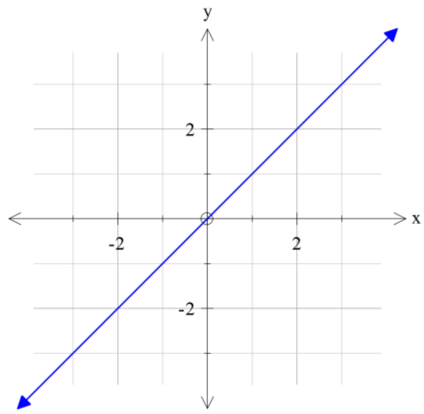
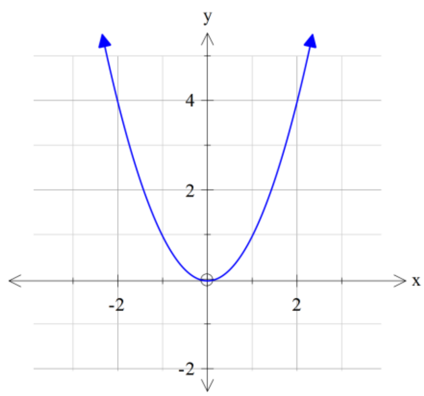
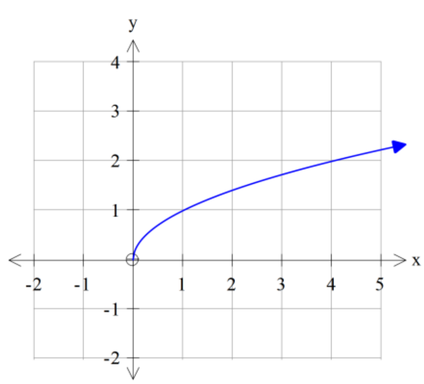


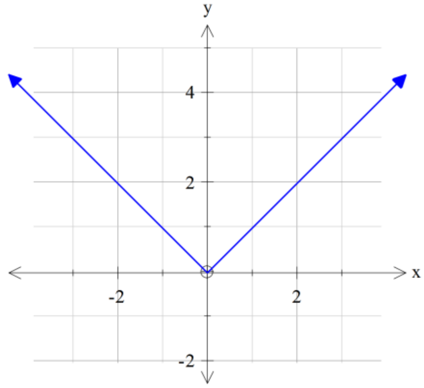
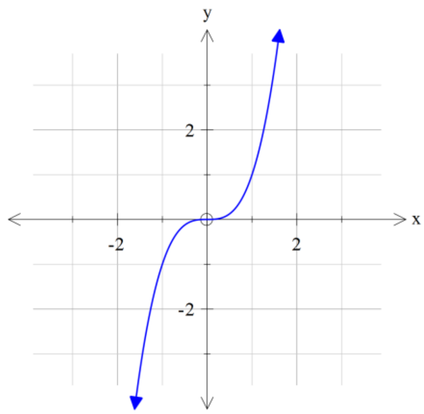
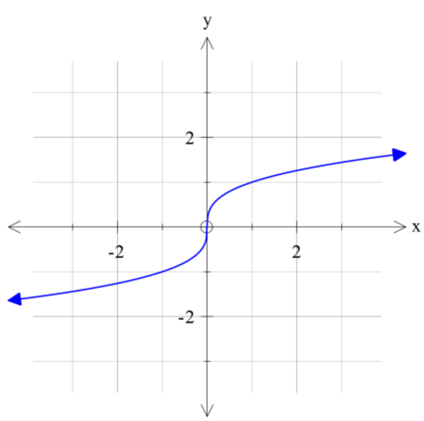
$$4x + y = 6$$

x-int: (,0)   y-int: (0,   )



Lesson 7: Transformation of Graphs

Function	Graph	Table/Key Points
Basic Linear Function: $f(x) = x$		Vertex: (0,0) Key Points: (-1,-1) and (1,1)
Basic Quadratic function: $f(x) = x^2$		Vertex: (0,0) Key Points: (-1,1) and (1,1)
Basic Root Function: $f(x) = \sqrt{x} = x^{\frac{1}{2}}$		Vertex: (0,0) Key Points: (1,1)

Function	Graph	Table/Key Points
Basic Absolute Value function: $f(x) = \lvert x \rvert$		Vertex: (0,0) Key Points: (-1,1) and (1,1)
Basic Cubic Function: $f(x) = x^3$		Vertex: (0,0) Key Points: (-1,1) and (1,1)
Basic Cube Root Function: $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$		Vertex: (0,0) Key Points: (-1,1) and (1,1)

Vertical Shifting

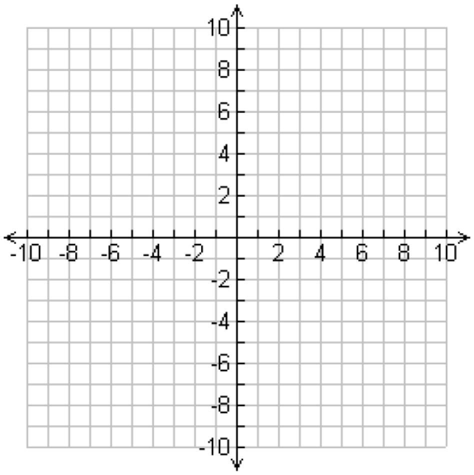
Let $f(x)$ be a function, and k be a number.

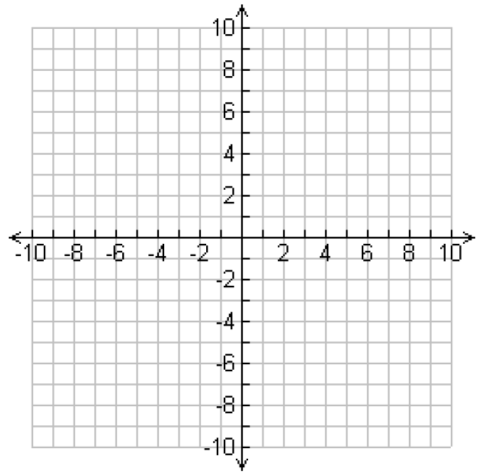
$y = f(x) + k$	When you _____ k to the ________, the graph shifts __________________.
$y = f(x) - k$	When you _____ k to the ________, the graph shifts __________________.

Examples:

Graph $f(x) = \lvert x \rvert$, $g(x) = \lvert x \rvert + 2$, $h(x) = \lvert x \rvert - 3$

Graph $f(x) = x^2$, $g(x) = x^2 + 3$, $h(x) = x^2 - 5$





Horizontal Shifting

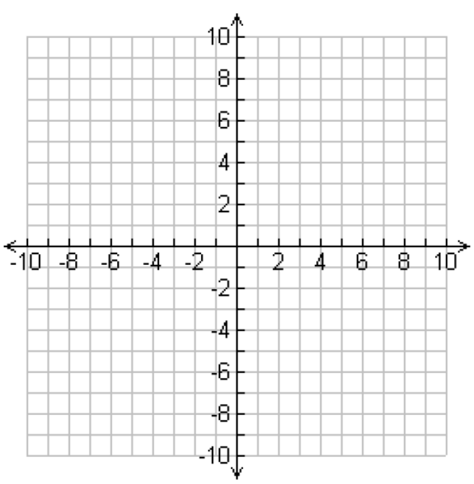
Let $f(x)$ be a function, and h be a number.

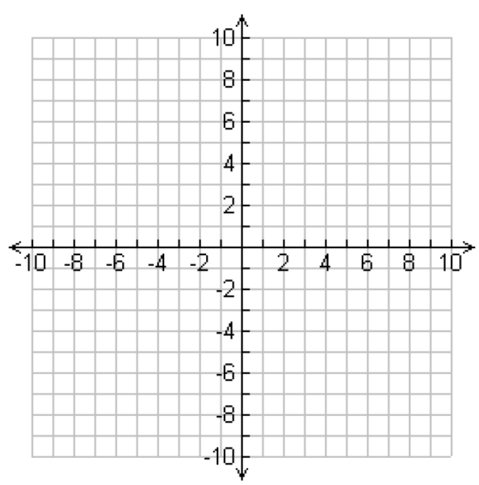
$y = f(x+h)$	When you _____ h to the ________, the graph shifts __________________.
$y = f(x-h)$	When you _____ h to the ________, the graph shifts __________________.

Examples

Graph $f(x) = \sqrt{x}$, $g(x) = \sqrt{x+1}$, $h(x) = \sqrt{x-3}$

Graph $f(x) = x^3$, $g(x) = (x+3)^3$, $h(x) = (x-2)^3 + 4$





Vertical Stretch and Compress $y = a \cdot f(x)$

When multiplying the function by a ___________value, the graph gets ___________________ vertically. To graph, we will ___________ the y value of the key points by the value ____.

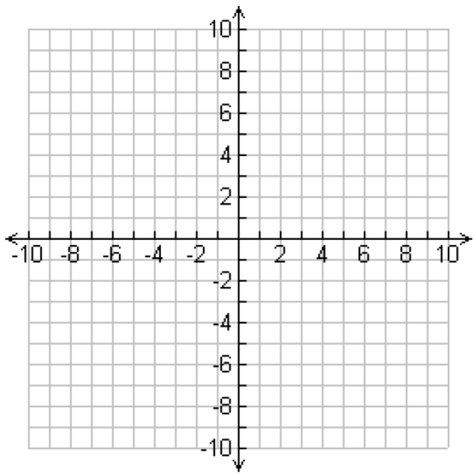
When multiplying the function by a ___________value, the graph gets _______________.

Examples:

$f(x) = \sqrt[3]{x}$

Graph $g(x) = 2\sqrt[3]{x}$

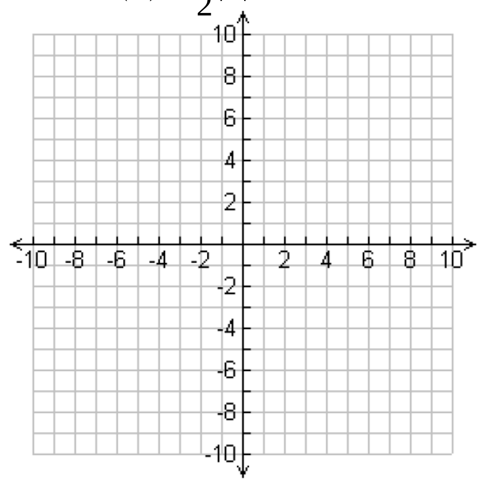
$h(x) = -\sqrt[3]{x}$



$f(x) = |x|$

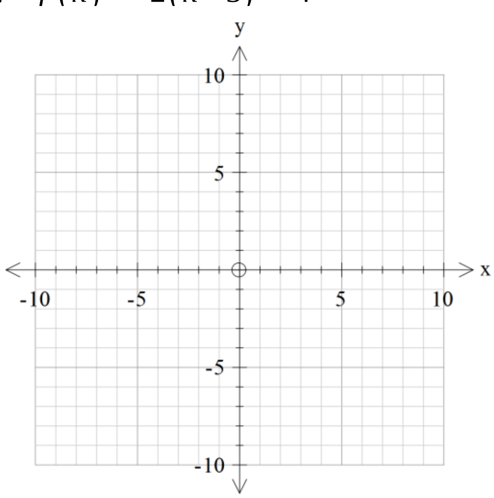
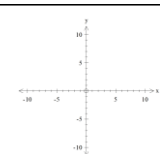
Graph $g(x) = -|x|$

$h(x) = \frac{1}{2}|x|$

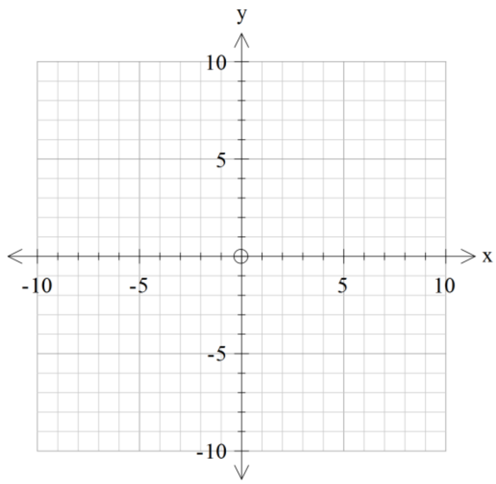


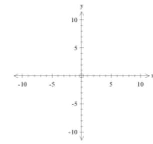
Steps to Graph with more than one transformation: $y = a(x-h)^2 + k$

1. Determine the ____________________ of the graph.
2. Find the horizontal shift (____), and vertical shift (___), to find the translated ____________ (___,___).
3. Use the stretch and compress (____) to find new key points in relation to your new vertex.
4. Connect all points. Make sure that your graph looks like the ____________________.

1. $f(x) = -2(x-3)^2 - 4$ 	Basic Shape:  Horizontal Shift:___________ Vertical Shift: ___________ Vertex: _____________ Compress/Stretch: ___________

2. $f(x)=\frac{1}{2}\sqrt{x+5}+2$



Basic Shape: 

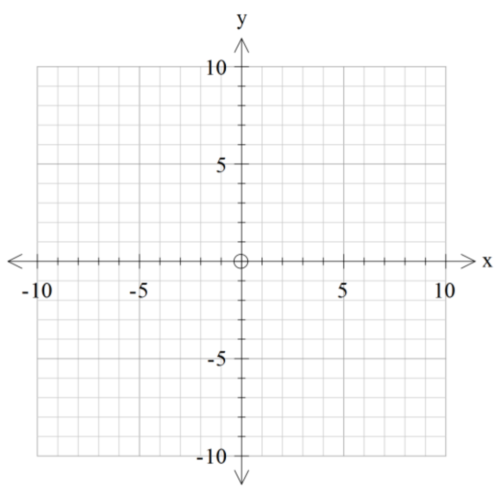
Horizontal Shift:____________

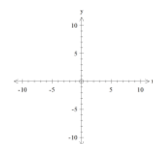
Vertical Shift: _____________

Vertex: ______________

Compress/Stretch: ____________

3. $f(x)=-(x-1)^3+5$



Basic Shape: 

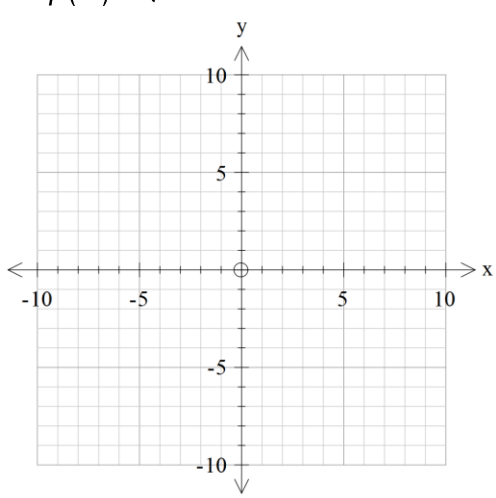
Horizontal Shift:____________

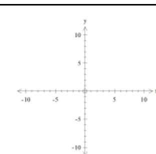
Vertical Shift: _____________

Vertex: ______________

Compress/Stretch: ____________

4. $f(x)=\sqrt[3]{x+3}-1$



Basic Shape: 

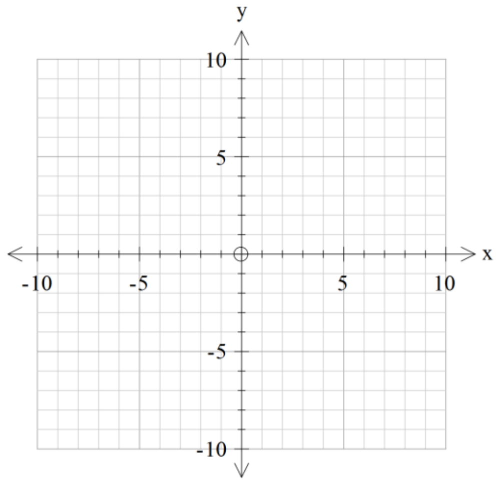
Horizontal Shift:____________

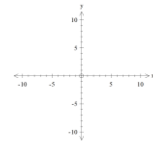
Vertical Shift: _____________

Vertex: ______________

Compress/Stretch: ____________

5. $f(x) = -|x-4| + 2$



Basic Shape: 

Horizontal Shift:____________

Vertical Shift: _____________

Vertex: ______________

Compress/Stretch: ____________

Lesson 8: Operations of Functions

Finding Domain from an Equation

<u>Domain:</u> Remember that the domain of a function is the set of all _____________ .

When finding the domain of a function, begin by assuming that the domain is ____________________. Then determine if there is any place where the function is not _______________, and _______________ those values from the domain.

The 2 types of functions who have excluded values are:

1. Rational Functions because ____. To identify these "bad" values that DO NOT belong to the domain, set the _____________________ ≠ 0 and solve. Write your domain in _________________ notation without these excluded values.
2. EVEN root function because ____. To identify the qualifying values that DO belong to the domain, set the expression _____________ the radical (called the radicand) ≥ 0. Write your domain in _________________ notation.

Find Domain:

$f(x)=x^2-7x$	$g(x)=\dfrac{3x+2}{x^2-2x-3}$	$h(x)=\dfrac{x-4}{x^3+9x^2+8x}$
$h(x)=\sqrt{3x+2}$	$p(x)=\sqrt{x+3}+\sqrt{x-5}$	$q(x)=\dfrac{\sqrt{x+8}}{x-3}$

Combination of Functions

Given 2 functions $f(x)$ and $g(x)$ we can determine:

1. Sum: $(f+g)(x)$ =______________________. This operation makes a _______ function by ___________ the two functions.

2. Difference: $(f-g)(x)$ =_________________. This operation makes a _______ function by ___________ the two functions.

3. Product: $(f \cdot g)(x)$=______________________. This operation makes a _______ function by ___________ the two functions.

4. Quotient: $\left(\frac{f}{g}\right)(x)$ =______________________. This operation makes a _______ function by ___________ the two functions.

The domain of the new "combined" functions will be the intersection, or all values in __________, of the domains of $f(x)$ and $g(x)$, excluding any new "bad" values created by combining the functions. This usually occurs with the _______________ of two functions.

Given: $f(x)=x^2+2$ and $g(x)=x-7$. Find the new combined functions and their new domain.

$(f+g)(x)=$	$(f-g)(x)=$
$(f \cdot g)(x)=$	$\left(\dfrac{f}{g}\right)(x)=$

Given $f(x)=\sqrt{x+2}$ and $g(x)=x-4$. Find the new combined functions and their new domain.

$(f+g)(x)=$	$(f-g)(x)=$
$(f \cdot g)(x)=$	$\left(\dfrac{f}{g}\right)(x)=$

Composition of functions

Function composition is applying one function to the result of another. For example:

g	f

Given 2 functions $f(x)$ and $g(x)$ we can determine:

$(f \circ g)(x) = f(g(x))$ which means to ____

$(g \circ f)(x) = g(f(x))$ which means to ____

Given $f(x) = x - 7$ and $g(x) = x^2 + 2$. Find the following

$(f \circ g)(x) =$	$(g \circ f)(x) =$
$(f \circ g)(3) =$	$(g \circ f)(3) =$

Given $f(x) = x^2 + 2$ and $g(x) = \sqrt{x-4}$. Find the following

$(f \circ g)(x) =$	$(g \circ f)(x) =$
$(f \circ g)(5) =$	$(g \circ f)(5) =$

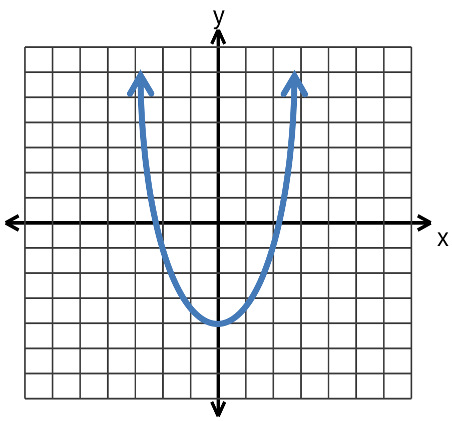
Lesson 9: Inverses Functions

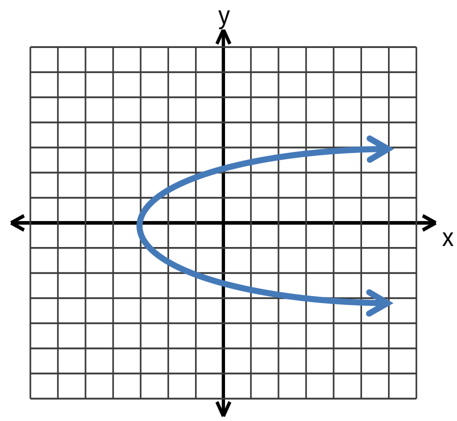
Definition of a Function

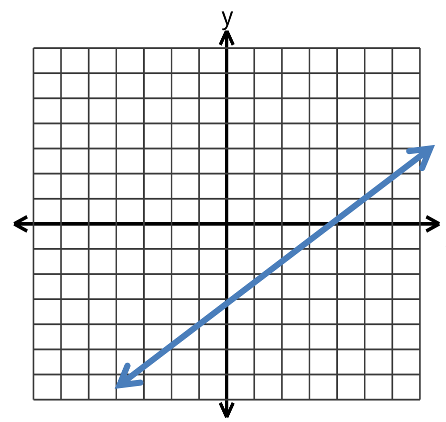
A relation is said to be a Function if and only if each ____value corresponds to only one ____value.

Graphically: Use the ______________________________ to determine if the graph is a function.

Determine if the following graphs are functions?





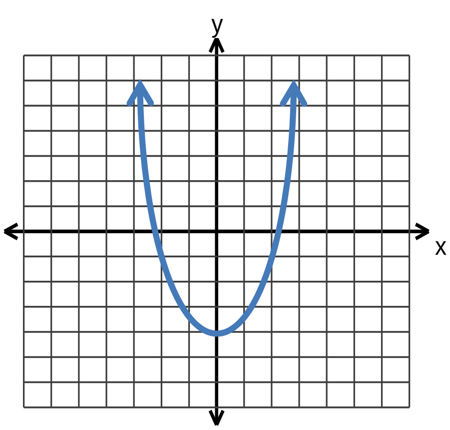


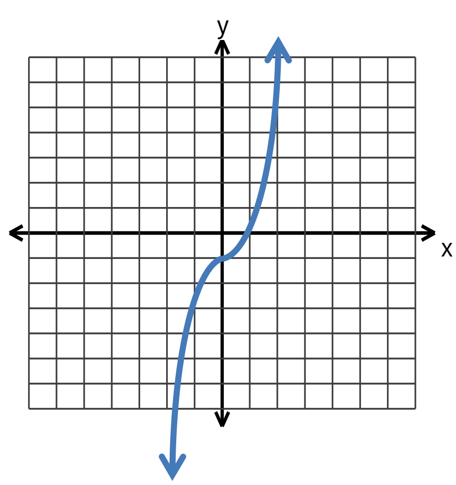
One to One Functions (1 – 1 functions)

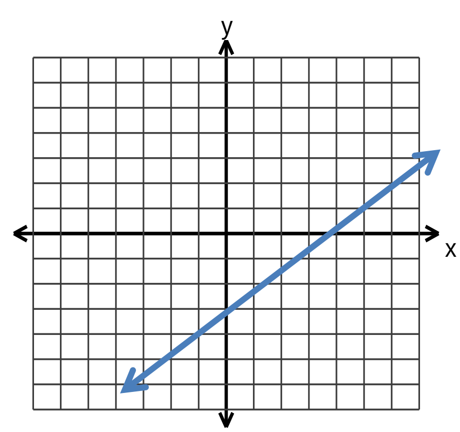
A Function is said to be one to one if each ______ value corresponds to only one value for ____.

Graphically: Use the _______________________to determine if the following functions are One to One Functions.

Determine if the following funcitons are One to One Functions



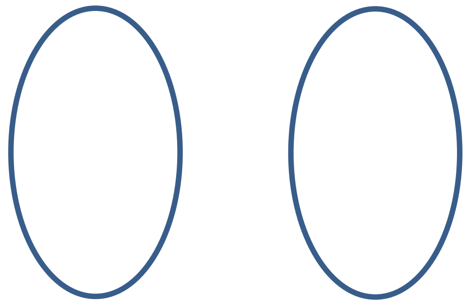




What is an Inverse?

Only One to One Functions have Inverse Functions.

Domain of f Range of f

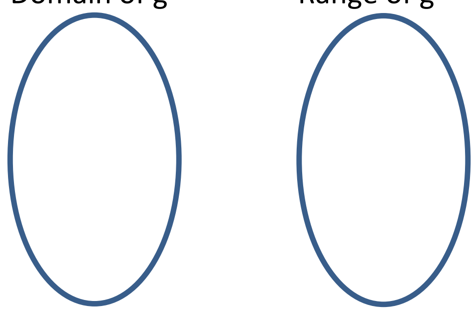


Is f a function? ________

Is f one to one? ________

Let us suppose that there exists a function g with the following properties:

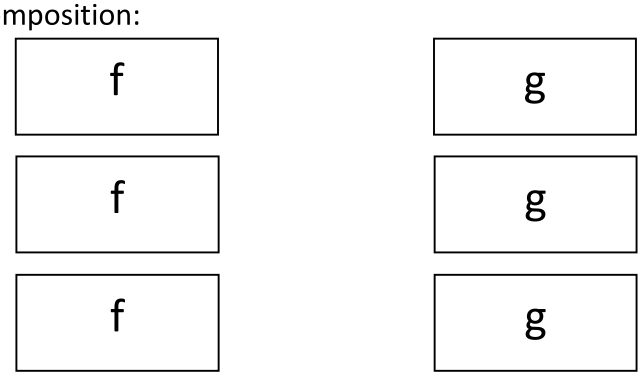
Domain of g Range of g



Is g a function? ________

Is g one to one? ________

Function composition:



Similarly,

Definition of Inverse Functions:

If f is a one to one function, then g is the <u>inverse function</u> of f if:

____ for every x in the domain

and

____ for every x in the domain

To verify if 2 given functions are inverses

f and g are inverses if: $f\left(g\left(x\right)\right)=x$ AND $g\left(f\left(x\right)\right)=x$

Determine if the following 2 functions inverses of each other

$f\left(x\right)=3x+2$ and $g\left(x\right)=\frac{x-2}{3}$	$f\left(x\right)=\frac{1}{4}x-5$ and $g\left(x\right)=4x-20$	$f\left(x\right)=\frac{2}{x-5}$ and $g\left(x\right)=\frac{2}{x}+5$

To Find an Inverse from an Equation (given the function is 1 – 1)

Steps:

1. Change $f\left(x\right)$ $to\ y$
2. Switch all x's and y's
3. Solve for y
4. Change y to $f^{-1}\left(x\right)$

Note: $f^{-1}\left(x\right)$ is notation for the inverse of f(x).

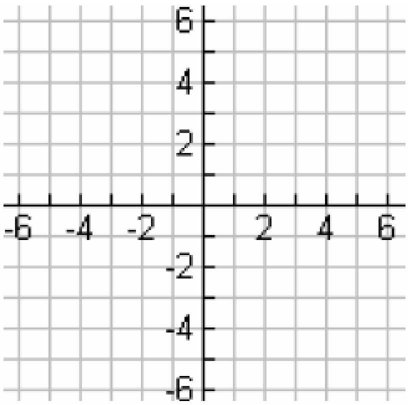
Pronounced: "f inverse of x".

$f^{-1}\left(x\right)$ is NOT "f to the negative 1 exponent"

Find $f^{-1}\left(x\right)$ for $f\left(x\right)=2x+7$	Find $f^{-1}\left(x\right)$ for $f\left(x\right)=4x^3-1$	Find $f^{-1}\left(x\right)$ for $f\left(x\right)=\frac{2}{x}$

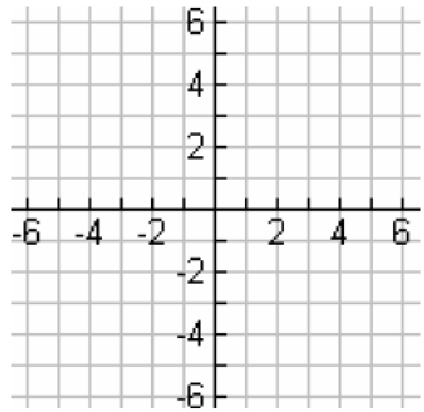
To Sketch an Inverse from a Graph

$y = x^2$



$y = x^2$ INVERSE

INVERSE

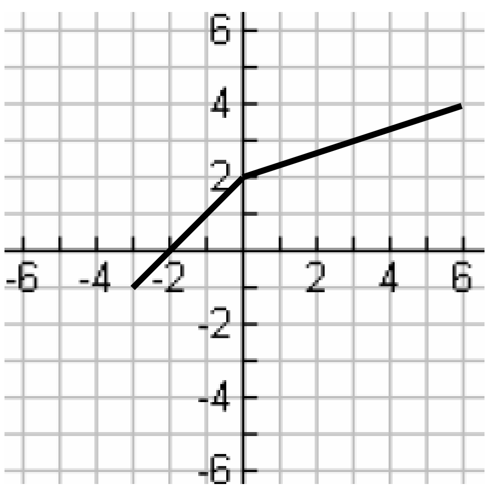


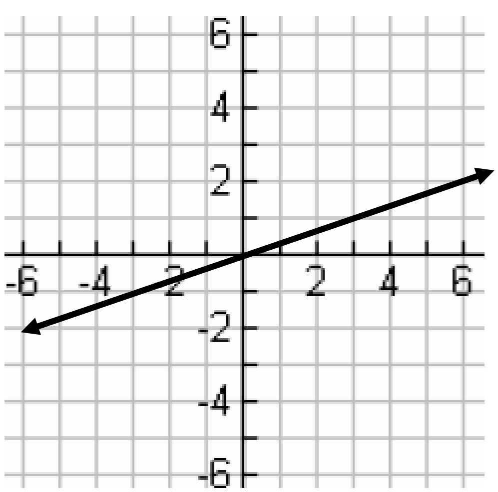
Is the original graph a function?

Is the inverse graph a function? Why?

Is the original graph 1 - 1?

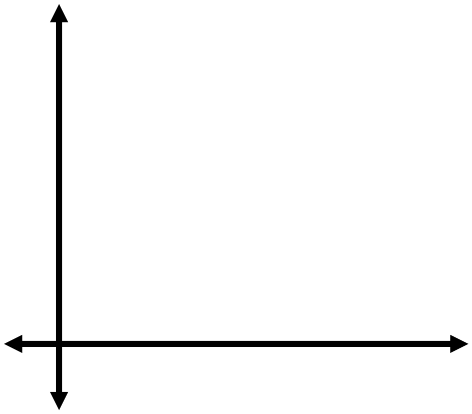
Sketch an Inverse for the following graphs





Lesson 10: Distance, Midpoint and Circles

Distance Formula (Derivation)



Distance Formula

The ____________________, d, between two point $A(x_1, y_1)$ to $B(x_2, y_2)$ can be found by:

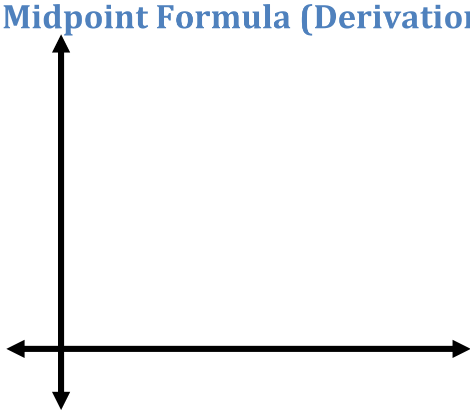
$$d = \sqrt{(\qquad\qquad)^2 + (\qquad\qquad)^2}$$

Note that the distance is a ___________.

Examples: Find the <u>distance</u> between the following pairs of points

$(5,1)$, $(8,5)$	$(-4,1)$, $(2,-3)$

Midpoint Formula (Derivation)



Midpoint Formula

The ______________________ of a line segment, from $A(x_1, y_1)$ to $B(x_2, y_2)$ can be found by the formula:

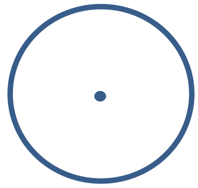
$$\left(\underline{\qquad} \;,\; \underline{\qquad} \right)$$

Note that the midpoint is a ___________, so the answer should be written as an __________________.

Examples: Find the <u>midpoint</u> between the following pairs of points

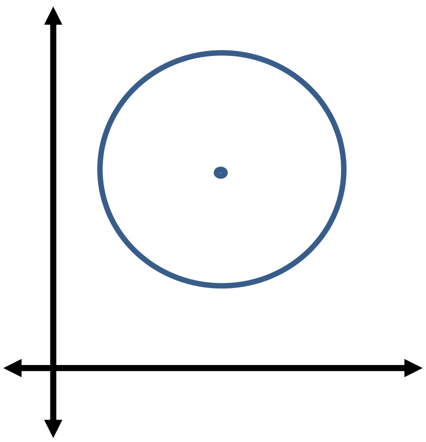
$(-4,-7)$, $(-1,-3)$	$(7\sqrt{3},-6)$, $(3\sqrt{3},-2)$

Circles



A ____________________ is a set of points in a plane that are located a fixed distance, called the __________, from a given point, called the _____________.

Equation of a Circle (derivation)



The equation of a circle is standard form is:_________________________________

Where (h,k) is the _________ of the circle and r is the __________ of the circle.

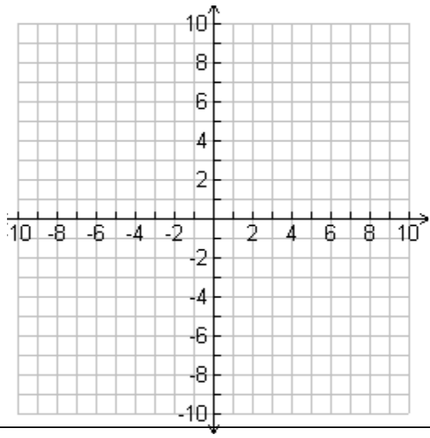
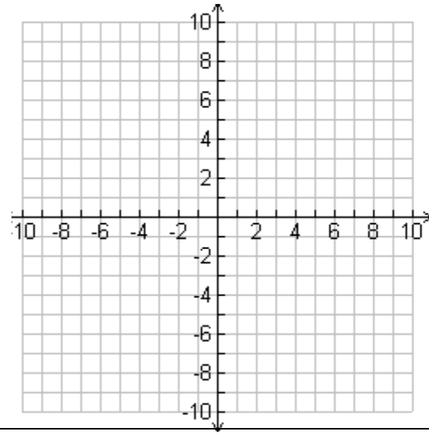
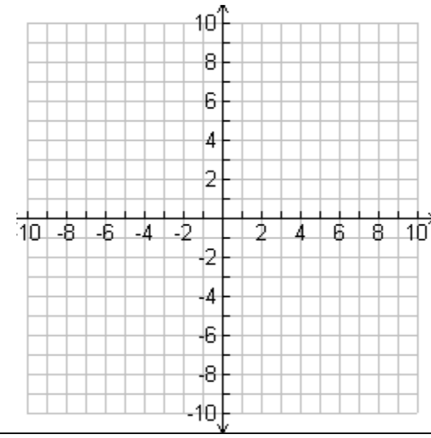
1. Given a center and a radius, write the equation

$C:(2,-1)$ $r=4$	$C:(-3,5)$ $r=3$	$C:(-4,2)$ $r=\sqrt{5}$

Steps to graph a circle:

1. Determine the _______________________ of the circle and draw the point on your graph
2. Determine the ____________________of the circle and move up, down, left and right __________units from the center
3. Connect the points giving it a ______________ shape

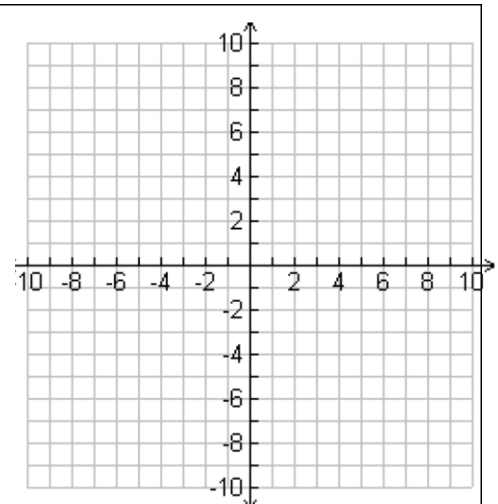
2. Graph given an equation in standard form

$(x-2)^2+(y+3)^2=16$	$x^2+y^2=49$	$(x+3)^2+(y-5)^2=20$
Center: __________	Center: __________	Center: __________
Radius = _________	Radius = _________	Radius = _________
		

3. Complete the square and graph

When the equation of a circle is not in ____________________ form, we must ______________________________ to find the radius, center and graph.

$x^2+y^2+8x+4y+16=0$

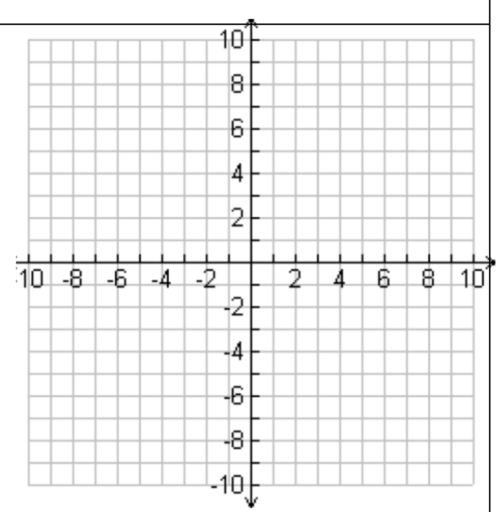


Standard Form: ___________________________________

Center: _____________

Radius =_____________

$x^2+y^2+12x-6y+29=0$

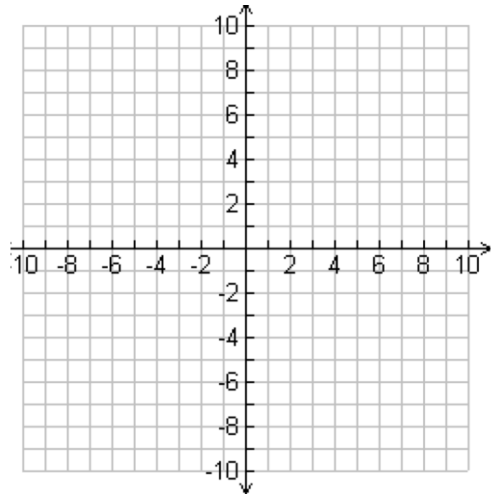


Standard Form: ___________________________________

Center: _____________

Radius =_____________

$x^2 + y^2 - 6y - 7 = 0$



Standard Form: ______________________________

Center: ___________

Radius =___________