

Lesson 17: Exponential Functions

An exponential function is a function where a _____ is raised to a _____.

$$f(x) = b^x \text{ where } b > 0 \text{ and } b \neq 1$$

Examples of Exponential Functions

Non-examples of Exponential Functions

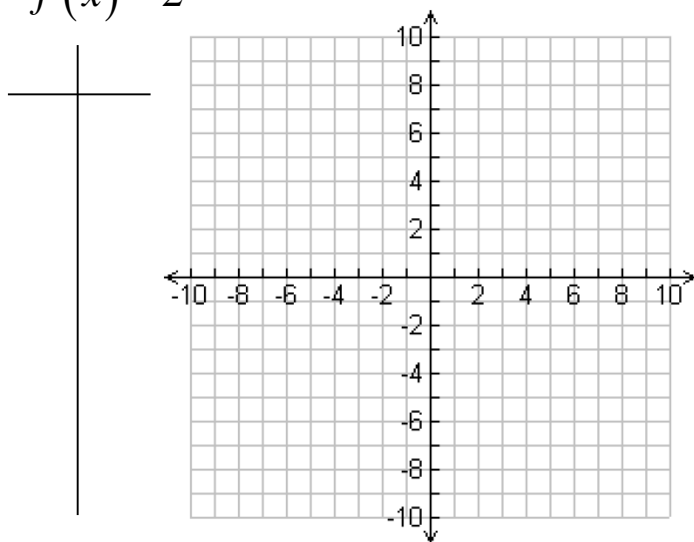
--	--

Remember from Intermediate Algebra

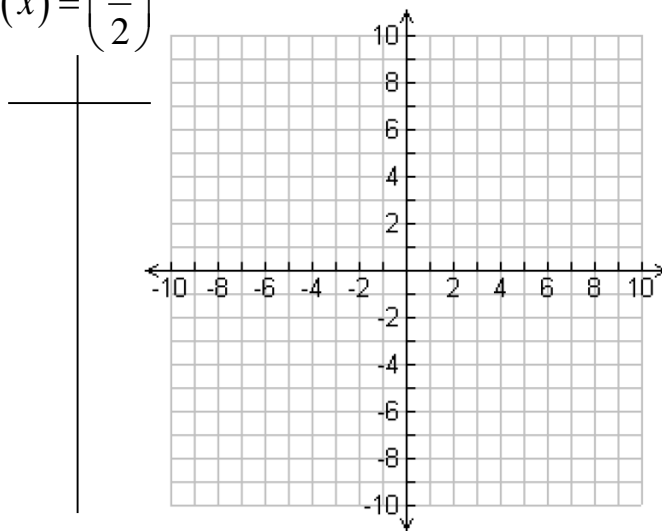
$$x^0 = \quad x^1 = \quad x^{-1} = \quad \left(\frac{a}{b}\right)^{-1} =$$

Graph the following Exponential Functions:

$$f(x) = 2^x$$



$$f(x) = \left(\frac{1}{2}\right)^x$$



Domain: _____

Range: _____

X-int: _____

Y-Int: _____

Asymptote: _____

Domain: _____

Range: _____

X-int: _____

Y-Int: _____

Asymptote: _____

Transformations

Parent Function: $f(x) = b^x$

All exponential functions (in “basic” form) have 3 point on their graphs:

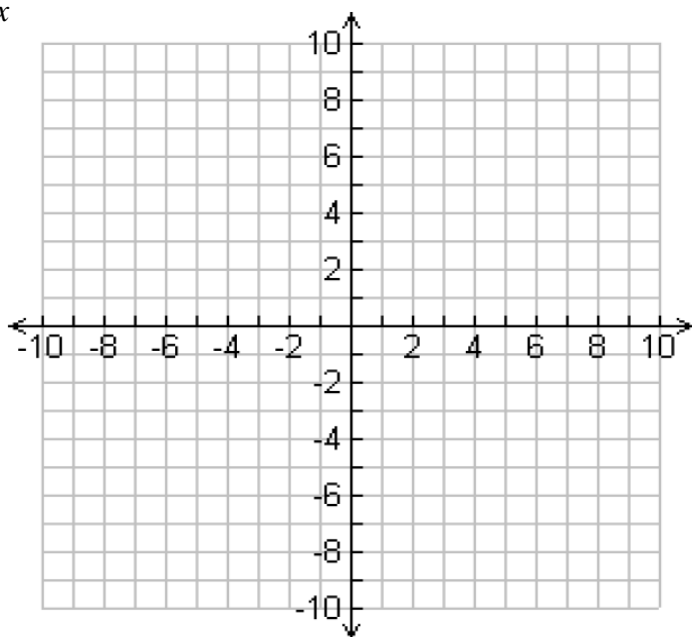
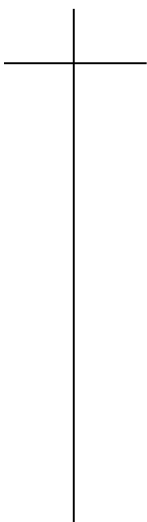
Vertical Shift	$f(x) = b^x + c$	
	$f(x) = b^x - c$	
Horizontal Shift	$f(x) = b^{(x+c)}$	
	$f(x) = b^{(x-c)}$	
Reflections	$f(x) = -b^x$	
	$f(x) = b^{-x}$	
Vertical Stretch and Compress	$f(x) = c \cdot b^x$ Note that $c \cdot b^x \neq (cb)^x$	$c > 1$
		$0 < c < 1$

If you have more than one transformation, use the _____ or _____ to determine the order of the transformations.

Transformations Examples:

Sketch

$$f(x) = 3^x$$



Then, sketch the following

$$f_2(x) = 3^{x-5}$$

$$f_3(x) = 3^x - 4$$

$$f_4(x) = -3^x$$

$$f_5(x) = 2 \cdot 3^x$$

Natural Number e:

Of all possible choices of bases, the most preferred or most natural base is the number e . The number e has important significance in science and mathematics. It is often called Euler's number named after Leonhard Euler.

$$e = 2.7182818284590452353602874713527...$$

The number e is defined by:

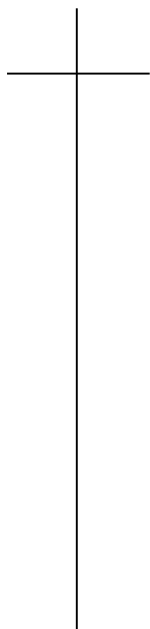
If n is a positive integer, then $\left(1 + \frac{1}{n}\right)^n \rightarrow e$ as $n \rightarrow \infty$

n	$(1 + 1/n)^n$
1	2.00000
2	2.25000
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
10,000	2.71815
100,000	2.71827

NOTE: The number e is a _____, not a _____.

Sketch

$$f(x) = e^x$$



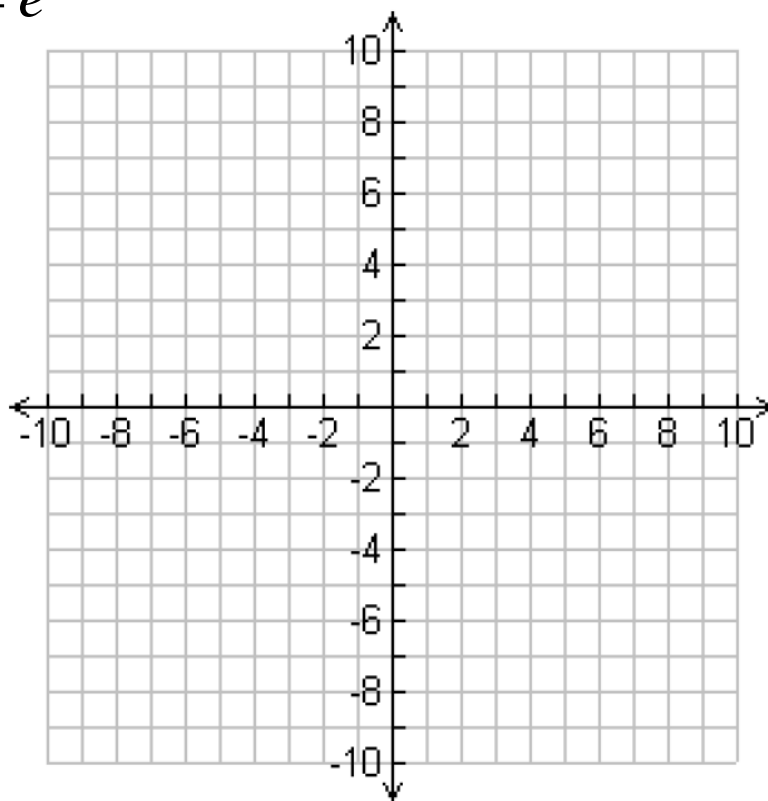
Then, sketch the following

$$f_2(x) = e^{x-4}$$

$$f_2(x) = e^x + 3$$

$$f_2(x) = e^{-x}$$

$$f_2(x) = 3e^x$$



Lesson 18: Logarithmic Functions

Exponential Function: $f(x) = 2^x$

Find the Inverse:

We cannot find the _____ of the exponential equation because we have not yet learned to _____ for the _____ in the exponent. We need a new _____ for the _____ of exponential.

Just like _____ is the opposite of addition,

And _____ is the opposite of multiplication,

And _____ is the opposite of square,

The exponential function has an opposite (or _____), called _____.

Logarithmic Function:

_____ if and only if _____.

The base stays the same for both but we _____ the x and the y (because they are _____.)

Note: Some bases are used frequently, and have simplified notation.

Common Log with Base 10: $\log_{10} x =$

Natural Log with Base e: $\log_e x =$

Logarithm Form	Exponential Form
$y = \log_b x$	
$y = \log x$	
$y = \ln x$	

Change from Logarithmic Form to Exponential Form

$2 = \log_5 x$	$3 = \log_4 64$	$\ln 7 = y$
----------------	-----------------	-------------

Change from Exponential Form to Logarithmic Form

$12^2 = x$	$b^3 = 8$	$e^y = 9$
------------	-----------	-----------

Evaluate Logs WITHOUT Calculator

1. Set log = _____.
2. Switch log equation to _____.
3. _____ for x.
4. Remove _____ from your answer.

$\log_2 16$	$\log_3 9$	$\log_{25} 5$
$\log_7 7$	$\log_7 7^2$	$\log_5 1$

Properties of Logs:

• $\log_a a = \underline{\hspace{2cm}}$	• $\ln e = \underline{\hspace{2cm}}$
• $\log_a 1 = \underline{\hspace{2cm}}$	• $\ln 1 = \underline{\hspace{2cm}}$
• $\log_a a^r = \underline{\hspace{2cm}}$	• $\ln e^r = \underline{\hspace{2cm}}$
• $a^{\log_a r} = \underline{\hspace{2cm}}$	• $e^{\ln r} = \underline{\hspace{2cm}}$

Evaluate:

$\log_{15} 15 =$	$\log_9 1 =$	$\log_7 7^8 =$	$3^{\log_3 17} =$
------------------	--------------	----------------	-------------------

Graphing Logarithmic Functions

Let's try to answer the first question we posed in this section again using _____.

Find the inverse of $f(x) = 2^x$ algebraically.

so, $f(x) = 2^x$ and _____ are inverses.

More generally, $f(x) = b^x$ and _____ are inverses.

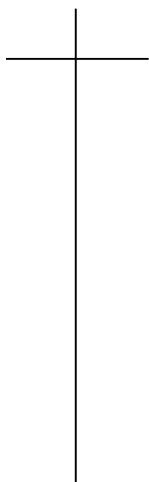
$f(x) = 3^x$ and _____ are inverses.

$g(x) = e^x$ and _____ are inverses.

$h(x) = \log_6 x$ and _____ are inverses.

$p(x) = \log x$ and _____ are inverses.

Sketch $f(x) = 2^x$ and $f(x) = \log_2 x$



$$f(x) = 2^x$$

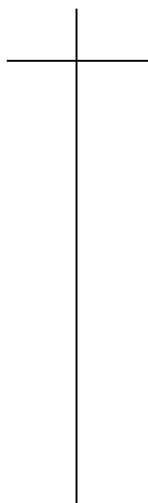
Domain: _____

Range: _____

X-int: _____

Y-Int: _____

Asymptote: _____



$$f(x) = \log_2 x$$

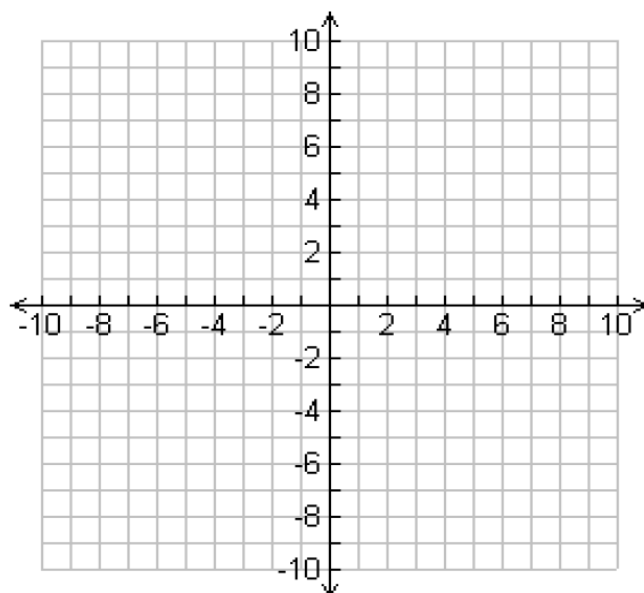
Domain: _____

Range: _____

X-int: _____

Y-Int: _____

Asymptote: _____



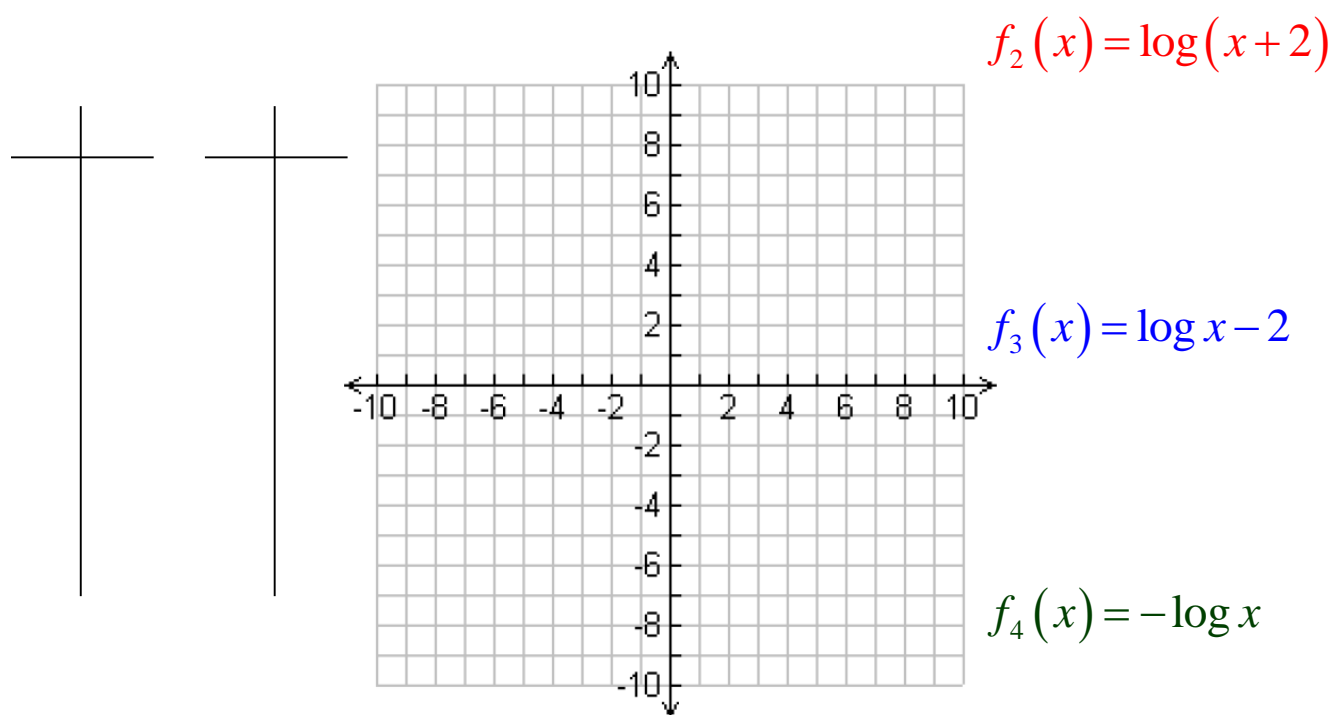
Transformations

Parent Function: $f(x) = \log_b x$

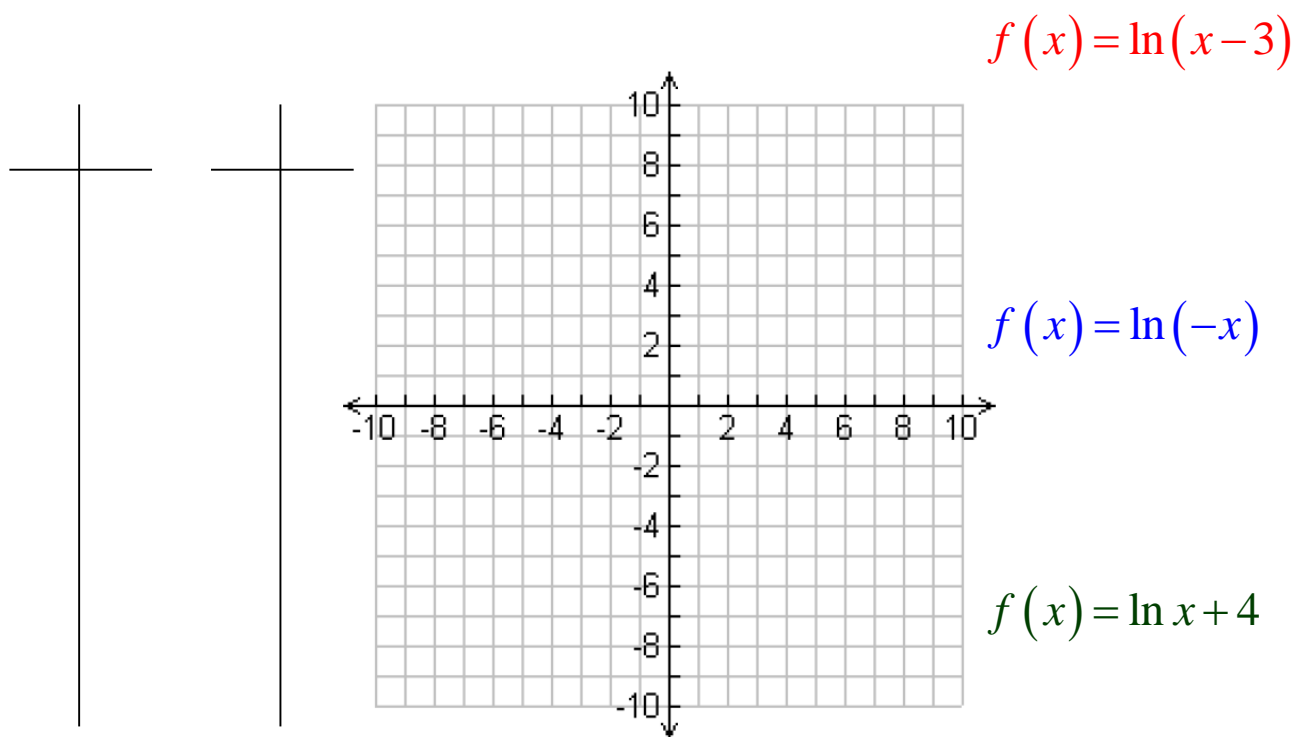
Vertical Shift	$f(x) = \log_b x + c$	
	$f(x) = \log_b x - c$	
Horizontal Shift	$f(x) = \log_b (x + c)$	
	$f(x) = \log_b (x - c)$	
Reflections	$f(x) = -\log_b x$	
	$f(x) = \log_b (-x)$	
Vertical Stretch and Compress	$f(x) = c \log_b x$	$c > 1$
		$0 < c < 1$

Examples:

Sketch: $f(x) = \log x$



Sketch: $f(x) = \ln x$



Lesson 19: Properties of Logarithms

In this section, we will be studying the _____ or _____ of logarithms.

The first 3 properties will be used to _____ long expressions with logs, called _____, as well as _____ short expressions with logs, called _____.

The last property will help us calculate _____ of logs using the _____.

Log Rules

Product Rule: $\log_b mn =$

Expand:

$\log_4(7x)$	$\log_8(xyz)$
--------------	---------------

Condense:

$\log_4 2 + \log_4 5$	$\log 25 + \log x$
-----------------------	--------------------

Quotient Rule: $\log_b \frac{m}{n} =$

Expand:

$\log_7\left(\frac{19}{x}\right)$	$\ln\left(\frac{e^3}{7}\right)$
-----------------------------------	---------------------------------

Condense:

$\log(4x-3) - \log x$	$\log_4 16 - \log_4 2$
-----------------------	------------------------

Power Rule: $\log_b m^p =$

Expand:

$\log_5 x^4$	$\log(4x)^5$	$\ln \sqrt{x}$
--------------	--------------	----------------

Condense:

$2\log x$	$4\log_5 2$
$5\log_b x - 2\log_b 6 - \frac{1}{2}\log_b y$	$3\ln(x+7) - \ln x$

On graphing and scientific calculators, there are logarithm buttons for the _____ or _____ and _____ or _____ .

Notice that there are no logarithm buttons for any bases other than ____ or _____. If you need to estimate a logarithm in decimal form with a different base, then you need to use the _____ formula.

Change of Base Formula: $\log_b m =$

Approximate:

$\log_5 140 =$	$\log_4 15 =$	$\log_2 100 =$

In summary, the 4 properties of logarithms that we learned this lesson are:

• Product Rule:	$\log_b m + \log_b n =$ _____
• Quotient Rule:	$\log_b m - \log_b n =$ _____
• Power Rule :	$p \cdot \log_b m =$ _____
• Change of Base Formula:	$\log_b m =$ _____

We will use these properties in the next section to _____ logarithmic equations.

Lesson 20: Exponential and Logarithmic Equations

There are 2 types of _____ equations and 2 types of _____ equations we will be solving in this section.

Exponential Equations:

	Property	Examples	Steps
Type 1	Bases can be expressed as the _____		<ol style="list-style-type: none"> 1. Change both expressions to the _____. 2. Set the exponents _____ to each other and _____.
Type 2	Bases can _____ be expressed as the _____		<ol style="list-style-type: none"> 1. Isolate the _____. 2. Take the _____ of both sides of equation. This will cause the variable in the exponent to move to the coefficient of expression (_____ rule). 3. _____ for x.

Type 1: Exponential Equations with Common Bases

Change of Base Solving Method:

If $b^m = b^n$, then

$4^x = 16$	$2^{3x-8} = 16$	$27^{x+3} = 9^{x-1}$
$9^x = \frac{1}{3}$	$9^x = \frac{1}{\sqrt{3}}$	

Type 2: Exponential Equations with Uncommon Bases

$4^x = 15$	$9^{-x} = 2$	$40e^{3x} - 3 = 237$
------------	--------------	----------------------

Logarithmic Equations:

If there are more than one log in the equation, then the bases must be _____.

	Property	Examples	Steps
Type 1	Equation can be expressed as the _____ or _____		<ol style="list-style-type: none"> 1. _____ to a single logarithm using the _____ of logarithms. 2. Rewrite _____ equation as a _____ equation. 3. Solve and _____ all solutions do Not make the arguments _____.
Type 2	Equation can be expressed as the _____ or _____		<ol style="list-style-type: none"> 1. _____ to a single logarithm using the _____ of logarithms on each side of equation. 2. Set _____ equal to each other. 3. Solve and _____ all solutions do Not make the arguments _____.

Type 1: Solving Log Equations (Log = #)

$\log(x-3)=1$	$\log_4(x+3)=2$	$3\ln(2x)=12$
$\log_2 x + \log_2(x-7)=3$	$\log_4(x-3)=2+\log_4(x+3)$	

Type 2: Solving Log Equations (Log = Log)

If $\log_b m = \log_b n$, then $m = n$.

$\log_2 5 + \log_2(x-3) = \log_2(x+5)$	$\ln(x-1) + \ln(x) = \ln(2x)$
$\log(x+1) = \log(x+9) - \log x$	

Lesson 21: Matrices

Definitions

Matrix: A rectangular array of numbers arranged in _____ and _____ placed inside brackets

Matrices: _____ of Matrix

Elements: The _____ inside the brackets of a matrix

Examples:

Today we will be using matrices to solve Linear System of Equations. In the past we have used Graphing, Substitution and Elimination to solve Linear Equations (usually in 2 variables). In this lesson we will be solving linear equations in 3 variables with Matrices.

2-Variable Linear system Example	3-Variable system Example

Augmented Matrix: A Matrix with a _____ separating the columns into 2 groups. We usually use an augmented matrix to rewrite a system of Linear Equations.

Rewrite the following system of equations in an augmented matrix

System of Equation	Augmented Matrix
$3x + y - z = 9$ $-x - y + 3z = 3$ $x + 2y - z = 0$	
$x - 3y + 2z = -12$ $y + 4z = -3$ $z = 2$	
$3x - 5z = -12$ $4y - z = 5$ $2x - 3y = -4$	

Echelon Form: a Matrix is said to be in Row Echelon form if there are _____ down the main diagonal (upper-left to lower-right) and _____ below the ones

$$\left[\begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \end{array} \right]$$

example: $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right]$

So, if the matrix is in row echelon form, then use _____ to solve the system.

Our goal: To Solve linear system of equations by Matrices

System of equations	$\begin{aligned} 3x + y - z &= 9 \\ -x - y + 3z &= 3 \\ x + 2y - z &= 0 \end{aligned}$
Rewrite system of equations as a augmented matrix	$\left[\begin{array}{ccc c} 3 & 1 & -1 & 9 \\ -1 & -1 & 3 & 3 \\ 1 & 2 & -1 & 0 \end{array} \right]$
Use row operations to change matrix to a row equivalent matrix in row echelon form. (We will learn how to use row operations next.)	$\dots \left[\begin{array}{ccc c} 1 & 2 & -1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right]$
Rewrite matrix from row echelon form to system of equations.	$\begin{aligned} x + 2y - z &= 0 \\ y + 2z &= 3 \\ z &= 2 \end{aligned}$
Solve the system using back substitution	$(4, -1, 2)$

Row Operations

There are 3 row operations that produce matrices that represent systems with the same solution set.

Row Operation	Notation	Example
Interchange 2 Rows	$R_1 \leftrightarrow R_3$	$\left[\begin{array}{ccc c} 3 & 1 & -1 & 9 \\ -1 & -1 & 3 & 3 \\ 1 & 2 & -1 & 0 \end{array} \right]$
Multiply a Row by a non-zero number	$2R_3 \rightarrow R_3$	$\left[\begin{array}{ccc c} 1 & 2 & -1 & 0 \\ -1 & -1 & 3 & 3 \\ 3 & 1 & -1 & 9 \end{array} \right]$
Multiply a Row by a non-zero number and then add the product to any other row	$R_1 + R_2 \rightarrow R_2$ $-6R_1 + R_3 \rightarrow R_3$	$\left[\begin{array}{ccc c} 1 & 2 & -1 & 0 \\ -1 & -1 & 3 & 3 \\ 6 & 2 & -2 & 18 \end{array} \right]$

Gauss/Jordan Elimination:

Use row Operations to Simplify the Matrix into a Row Equivalent Matrix is Row Echelon Form

$\left[\begin{array}{cccc c} 1 & \# & \# & \# & \# \\ \# & \# & \# & \# & \# \\ \# & \# & \# & \# & \# \end{array} \right]$	$\left[\begin{array}{cccc c} 1 & \# & \# & \# & \# \\ 0 & \# & \# & \# & \# \\ 0 & \# & \# & \# & \# \end{array} \right]$	$\left[\begin{array}{cccc c} 1 & \# & \# & \# & \# \\ 0 & 1 & \# & \# & \# \\ 0 & \# & \# & \# & \# \end{array} \right]$	$\left[\begin{array}{cccc c} 1 & \# & \# & \# & \# \\ 0 & 1 & \# & \# & \# \\ 0 & 0 & \# & \# & \# \end{array} \right]$	$\left[\begin{array}{cccc c} 1 & \# & \# & \# & \# \\ 0 & 1 & \# & \# & \# \\ 0 & 0 & 1 & \# & \# \end{array} \right]$
Get a 1 in the upper right hand corner	Get 0's below the 1 in the first column	Get a 2 in the second row, second column	Get a 0 below the 1 in the second column	Get a 1 in the third row, third column

Steps to solve Systems of Equations using Matrices

1. Write system as an augmented matrix
2. Use row operations to get row equivalent matrix in Row Echelon Form
3. Use Substitution to solve for the variables. Answer solution in an ordered triple.

Solve the following System of Equations

$$3x + y - z = 9$$

$$-x - y + 3z = 3$$

$$x + 2y - z = 0$$

$$2x + y - z = 10$$

$$-x + 2y + 2z = 6$$

$$x - 3y + 2z = -15$$

Special Cases

Just as in system of equations in 2 variables, you may encounter systems with no solution or infinitely many solutions.

Matrix is Row Echelon Form	System of Equation	How many solutions	Solution
$\left[\begin{array}{ccc c} 1 & 2 & -1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right]$			
$\left[\begin{array}{ccc c} 1 & 3 & -4 & 8 \\ 0 & 1 & -11 & 5 \\ 0 & 0 & 0 & 4 \end{array} \right]$			
$\left[\begin{array}{ccc c} 1 & -2 & 1 & 4 \\ 0 & 1 & -3 & 7 \\ 0 & 0 & 0 & 0 \end{array} \right]$			

$$2x + 4y - 6z = 10$$

$$x - y + z = 4$$

$$x + 2y - 3z = 7$$

$$\begin{aligned}-3x + y + z &= 1 \\ 4x - 3y + 2z &= 2 \\ 5x - 3y + z &= 1\end{aligned}$$