10.4: The Comparison Tests

In the several comparison tests in this section, the main idea is to compare a given series with another series that is known to be convergent or divergent.

The (Direct) Comparison Test

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.

- If $\sum b_n$ is convergent and $a_n \le b_n$ for all *n*, then $\sum a_n$ is also convergent.
- If $\sum b_n$ is divergent and $a_n \ge b_n$ for all *n*, then $\sum a_n$ is also divergent.

Example 1: Test the series $\sum_{n=1}^{\infty} \frac{1}{n^2+2n+5}$ for convergence or divergence.

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Example 2: Test the series $\sum_{n=1}^{\infty} \frac{7}{5n-3}$ for convergence or divergence.

Example 3: Test the series $\sum_{n=1}^{\infty} \frac{2}{3^n + 4}$ for convergence or divergence.

NOTE: The terms of the series being tested must be ***smaller*** than the terms of a convergent series, or ***larger*** than the terms of a divergent series. Otherwise, the Direct Comparison Test is useless.

Example 4: Test the series $\sum_{n=1}^{\infty} \frac{3}{4n^3 - 3n}$ for convergence or divergence.

The Limit Comparison Test

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n \to \infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.

<u>Example 5:</u> Test the series $\sum_{n=1}^{\infty} \frac{3}{4n^3 - 3n}$ for convergence or divergence.

<u>Example 6:</u> Test the series $\sum_{n=1}^{\infty} \frac{2n^2 + n + 3}{n^4 + n^3 - n^2}$ for convergence or divergence.

<u>Example 7:</u> Test the series $\sum_{n=1}^{\infty} \frac{2}{3 + \sqrt{n}}$ for convergence or divergence.