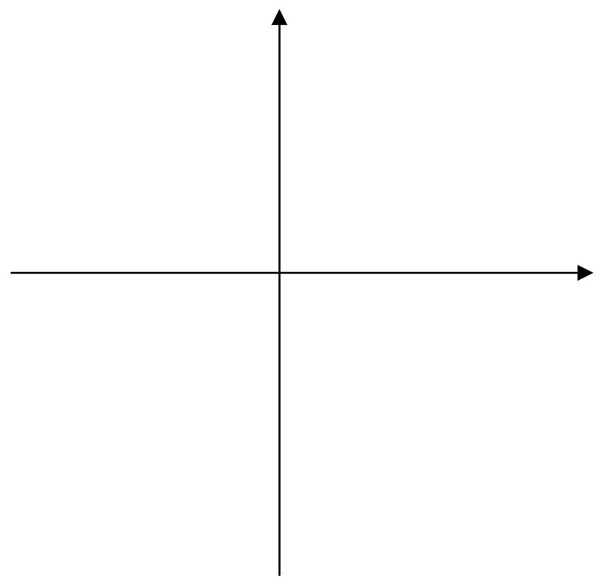
11.3: Polar Coordinates

The polar coordinate system, introduced by Isaac Newton, is often more convenient in some applications than the more traditional Cartesian, or rectangular, coordinate system. A point in the plane is chosen to be the pole, usually denoted as *O* and is equivalent to the origin in rectangular coordinates. Beginning at the pole, the polar axis is a ray than usually extends to right from the pole. (Often, the polar axis can be thought of as the "non-negative" part of the *x*-axis.) For any point *P* in the plane, let *r* be the distance between the pole *O* and the point *P*. Let θ be the measure (in radians) of the angle formed by the polar axis to the line *OP*. The point P can then be represented in polar coordinates as (r,θ). In this coordinate system, both *r* and θ can be any real number. In particular, this means both *r* and θ can be negative. The points (r,θ) and $(-r,\theta)$ lie on the same line, but on opposite sides of the pole *O*.

Example 1: Plot each of the following points in the polar coordinate system.

(a) $(2,\ \pi/4)$ (b) $(3,\ -\pi/3)$ (c) $(-2,\ \pi/4)$ (d) $(7, 3\pi)$



In the rectangular coordinate system, every point in the plane is represented by a ***unique*** ordered pair. However, in polar coordinates, this is not the case. In fact, every point can be represented by an infinite number of ordered pairs.

Example 2: All of the following ordered pairs represent the same point.

(a) $(7,\ \pi/6)$ (b) $(7, 13\pi/6)$ (c) $(-7,\ 7\pi/6)$ (d) $(7,\ -11\pi/6)$

Conversions:

<u>Equations relating Polar and Rectangular (Cartesian) Coordinates:</u>

$x = r\cos\theta$ $\quad y = r\sin\theta$ $\quad r^2 = x^2 + y^2$ $\quad \tan\theta = \dfrac{y}{x}$

<u>Example 3:</u> Convert the polar ordered pair $(8,\ 3\pi/4)$ to rectangular coordinates.

<u>Example 4:</u> Convert the polar ordered pair $(-3, 2)$ to rectangular coordinates.

<u>Example 5:</u> Convert the Cartesian ordered pair $(-6, 6)$ to polar coordinates.

<u>Example 6:</u> Convert the Cartesian ordered pair $(-3, -7)$ to polar coordinates.

Example 7: Convert the polar equation to an equivalent Cartesian equation. Sketch the graph.

$r\sin\theta = 3$

Example 8: Convert the polar equation to an equivalent Cartesian equation. Sketch the graph.

$r = 6\sin\theta$

Example 9: Convert the Cartesian equation to an equivalent polar equation.

$x^2 + y^2 = 5$

Example 10: Convert the Cartesian equation to an equivalent polar equation.

$x^2 = 5y$