

7.8: Relative Rates of Growth

Rates of growth as $x \rightarrow \infty$:

(These apply when $f(x)$ and $g(x)$ are positive, at least for large values of x .)

1. We say that “ f grows faster than g ”

$$\text{if } \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \infty, \text{ or, equivalently,}$$

$$\text{if } \lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = 0.$$

2. We say that “ f and g grow at the same rate ”

$$\text{if } \lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = L,$$

provided that L is finite and positive.

Example 1: Describe the “relative growth rates” of x^2 and e^x .

Example 2: Describe the “relative growth rates” of x^2 and x^3 .

Example 3: Describe the “relative growth rates” of x and $\ln x$.

Example 4: Describe the “relative growth rates” of e^x and e^{3x} .

Example 5: Describe the “relative growth rates” of $5x^3 - 2x$ and $7x^3 + \sin x$.

Example 6: Describe the “relative growth rates” of $\sqrt{x^4 - 23}$ and $2x - 5x^2$.