8.1: Integration by Parts

The Product Rule of differentiation states that if f and g are differentiable functions, then

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + f'(x)g(x)$$

Integrating this equation gives:

$$\int\left[f(x)g'(x) + g(x)f'(x)\right]dx = f(x)g(x)$$

$$\int\left[f(x)g'(x)\right]dx + \int\left[g(x)f'(x)\right]dx = f(x)g(x)$$

After rearranging terms:

$$\int\left[f(x)g'(x)\right]dx = f(x)g(x) - \int\left[g(x)f'(x)\right]dx$$

This last equation is the formula for integration by parts. Convenient substitutions can make the equation simpler. Let $u = f(x)$ and $v = g(x)$. Then, $du = f'(x)dx$ and $dv = g'(x)dx$. So, by substitution, we obtain the following formula.

Integration by Parts Formula:

$$\int u\,dv = uv - \int v\,du$$

Example 1: Find $\int x\cos x\,dx$.

Example 2: Find $\int \ln x \, dx$.

Example 3: Find $\int x^2 e^{3x} dx$.

Example 4: $\int e^x \cos x \, dx$.

Example 5: $\int_1^4 x^3 \ln x \, dx$

Example 6: $\int \tan^{-1} x \, dx$.